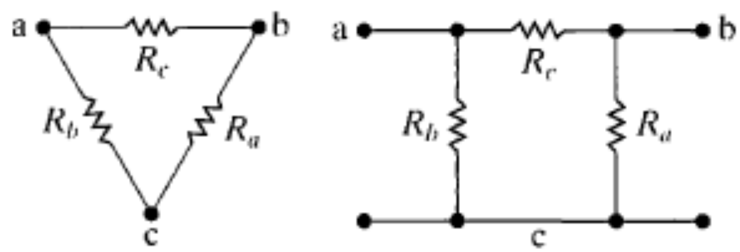
Delta-Wye Transformations

As we have seen so far, it is often convenient to simplify circuits to an equivalent resistance using the series and parallel combining rules. Unfortunately, in some cases the resistors are not in the right pattern for combining.

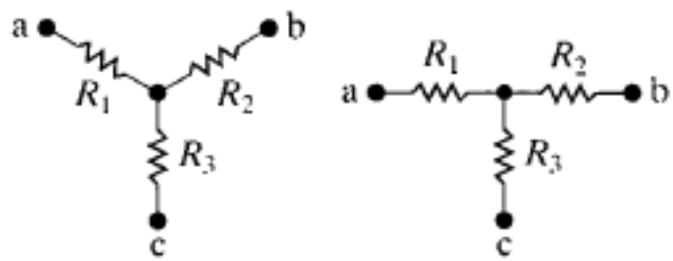
What can we do in such a case? One useful result, called the Delta-Wye (or Pi-Tee) transformation allows us to replace three resistors that form a triangular configuration (the Delta or Pi) by three other resistors in a Y-shaped (the Wye or Tee) configuration. The reverse rule, the Wye-Delta (or Tee-Pi) transformation takes three resistors in a Y configuration and replaces them by three in a triangular configuration. In both cases this replacement often allows us to then continue the simplification of the resistive circuit using series/parallel rules.

This handout starts with the circuit notation and states the transformation rules, then works several examples, and ends with a few problems for you to try (with answers given).

<u>Notation</u>: The Delta/Pi and Wye/Tee configurations are shown below. In both cases the three nodes in the circuit are labeled a, b, and c (in the Delta/Pi configuration the labels match the resistance *opposite* the node)



The Delta/Pi configurations:



The Wye/Tee configurations:

<u>Transformation</u>: The equations relating the values are

$$R_1 = \frac{R_\mathrm{b} R_\mathrm{c}}{R_\mathrm{a} + R_\mathrm{b} + R_\mathrm{c}} \qquad R_\mathrm{a} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

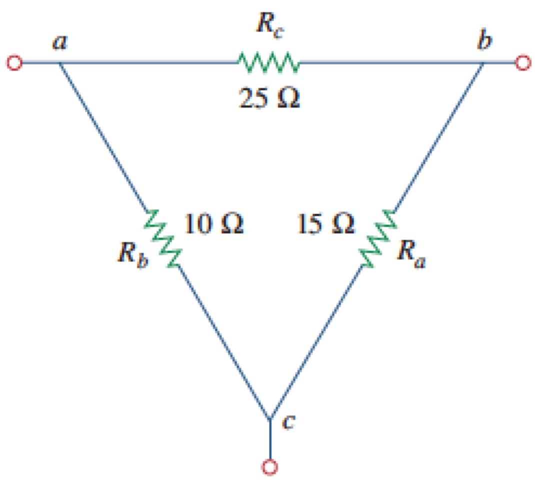
$$R_2 = \frac{R_\mathrm{a} R_\mathrm{c}}{R_\mathrm{a} + R_\mathrm{b} + R_\mathrm{c}} \qquad R_\mathrm{b} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

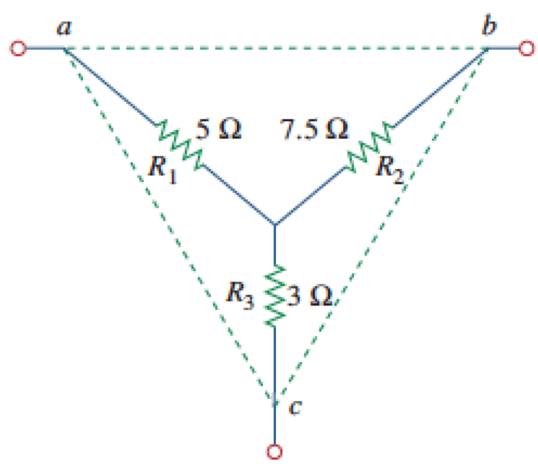
$$R_3 = \frac{R_\mathrm{a} R_\mathrm{b}}{R_\mathrm{a} + R_\mathrm{b} + R_\mathrm{c}} \qquad R_\mathrm{c} = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

The pattern is as follows:

- Delta to Wye: the Wye resistor connected to a particular node (say node a, resistor R_1) is equal to the product of the Delta resistors connected to the same node ($R_b R_c$) divided by the sum of the Delta resistors.
- Wye to Delta: the Delta resistor between two nodes (say nodes a and b, resistor R_c) is equal to the sum of products of pairs of resistors from the Wye divided by the Wye resistor connected to the remaining node (R_3).

<u>Example 1</u>: Convert the Delta on the left to the Wye on the right:

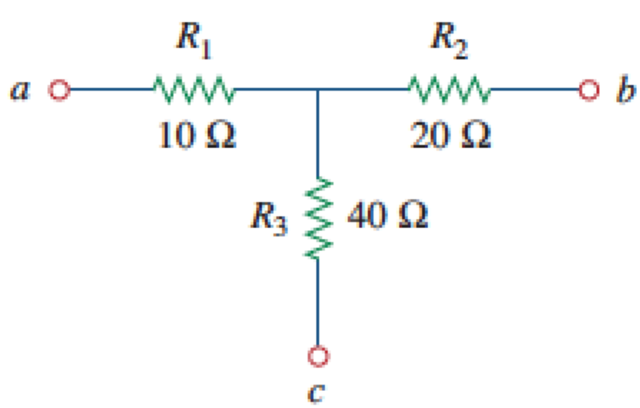




Using the Delta-Wye equations with $R_a = 15\ \Omega$, $R_b = 10\ \Omega$, and $R_c = 25\ \Omega$ yields

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 * 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$
$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{15 * 25}{50} = \frac{375}{50} = 7.5\ \Omega$$
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 * 10}{50} = \frac{150}{50} = 3\ \Omega$$

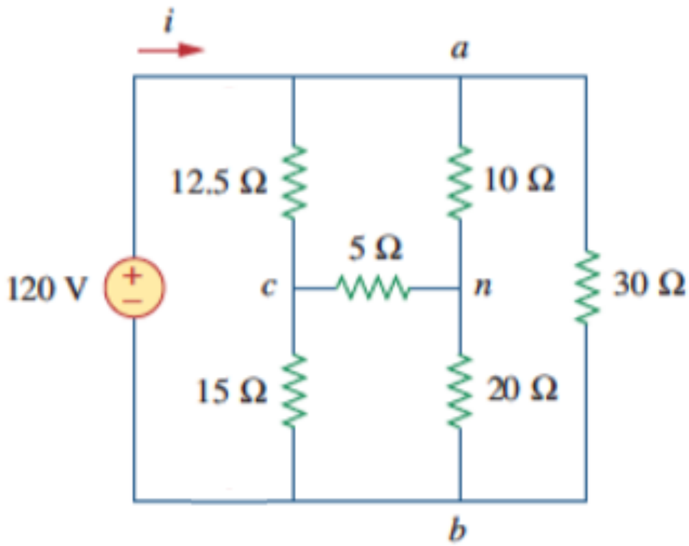
<u>Example 2</u>: Convert the Wye shown to a Delta:



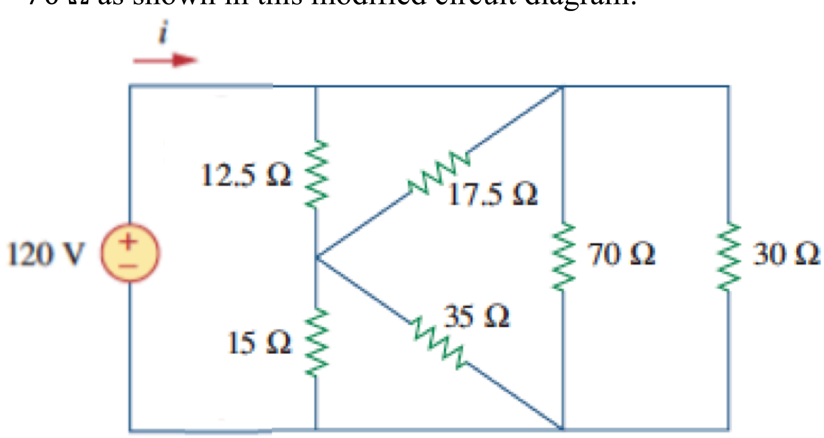
Using the Wye-Delta equations with $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $R_3 = 40\ \Omega$ yields

$$R_a = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1} = \frac{10 * 20 + 10 * 40 + 20 * 40}{10} = \frac{1400}{10} = 140\ \Omega$$
$$R_b = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2} = \frac{1400}{20} = 70\ \Omega$$
$$R_c = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3} = \frac{1400}{40} = 35\ \Omega$$

Example 3: Using Delta-Wye and/or Wye-Delta transformations, fine the current *i* in the circuit shown:



Clearly there are multiple Wyes and Deltas in this diagram; hindsight on this problem suggests that converting the Wye composed of the 10, 20, and 5 Ohm resistors (with center marked *n*) is the most efficient method. Using the node labelling of *a*, *b*, and *c* as shown we have $R_1 = 10\ \Omega$, $R_2 = 20\ \Omega$, and $R_3 = 5\ \Omega$, so solving the equations the Delta replacement has $R_a = 35\ \Omega$, $R_b = 17.5\ \Omega$, and $R_c = 70\ \Omega$ as shown in this modified circuit diagram:

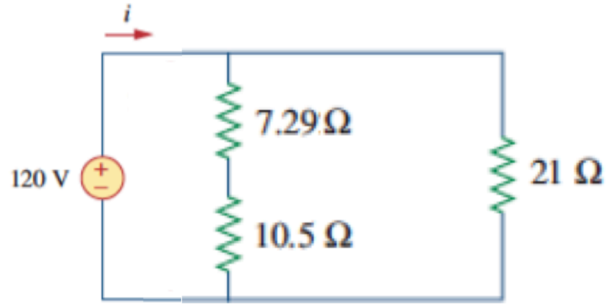


From here we can use series and parallel combining:

- On the right, 30 Ω in parallel with Ω is 21 Ω

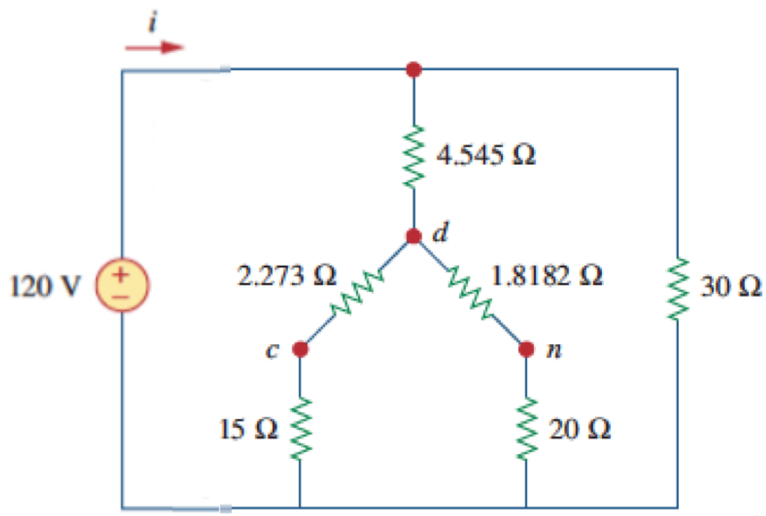
- On the top, 12.5 Ω in parallel with 17.5 Ω is 7.29 Ω
- On the bottom, 15 Ω in parallel with 35 Ω is 10.5 Ω

to yield:



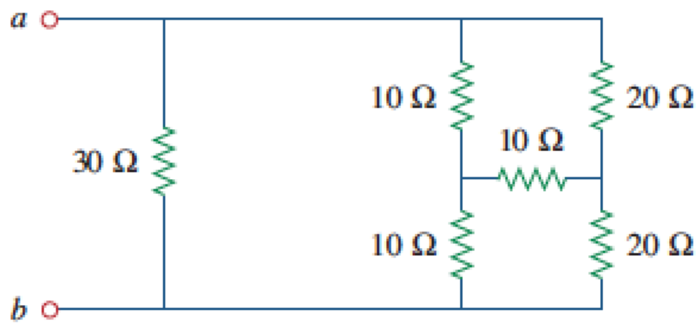
Now combining the 7.29 Ω and 10.5 Ω in series and then with the 21 Ω in parallel yields an equivalent resistance of 9.63 Ω, so $i = \frac{120\text{ V}}{9.63\ \Omega} = 12.5\text{ A}$.

An alternative approach might convert the Delta with nodes *c*, *a*, and *n* to a Wye:

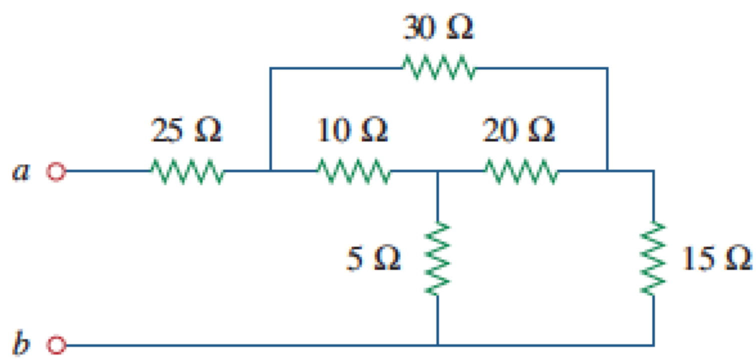


At this point series and parallel combining yields the same result as above. In a similar way one could start by converting the Delta formed by nodes *c*, *n*, and *b* to a Wye and then use series and parallel combining.

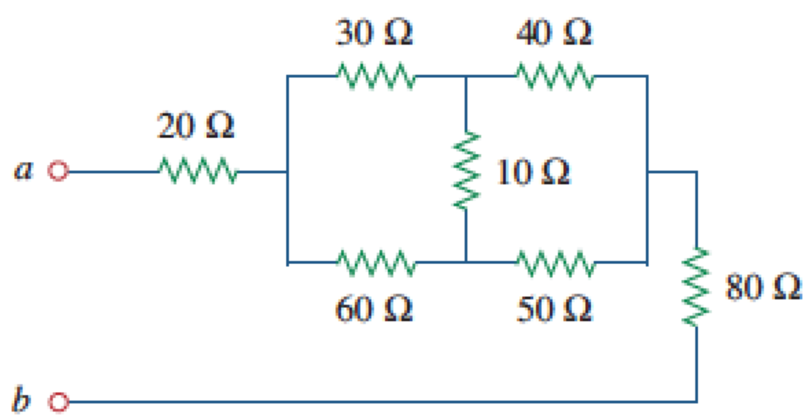
Practice problem 1: Find the equivalent resistance at terminals *a* and *b*. Hint – transforming one of the deltas on the right should allow for the use of series parallel combining. (answer 9.23 Ω):



Practice problem 2: Find the equivalent resistance at terminals *a* and *b*. (answer 36.25 Ω):



Practice problem 3: Find the equivalent resistance at terminals *a* and *b*. Hint – transform one of the wyes in the middle (either 10, 30, 40 or 10, 50, 60) and follow through with series/parallel combining. Note also that this circuit layout matches the on-line homework. (answer 142 Ω):



Practice problem 4: Assuming that all of the resistors are equal to 30 Ω, find the equivalent resistance at terminals *a* and *b*. (answer 33.3 Ω):