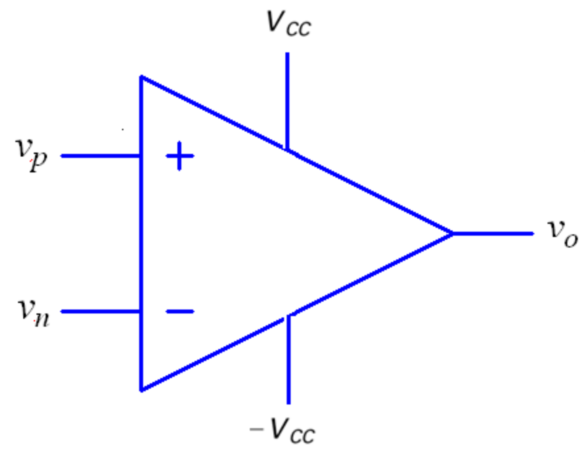
Op Amps 1

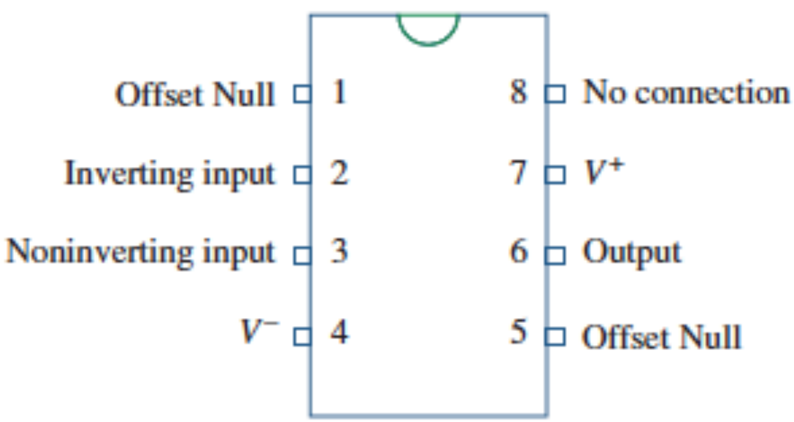
concept, full analysis, simple model

Op Amps

- A 3-terminal device
  - Inverting input v_n
  - Non-inverting input v_p
  - Output v_o

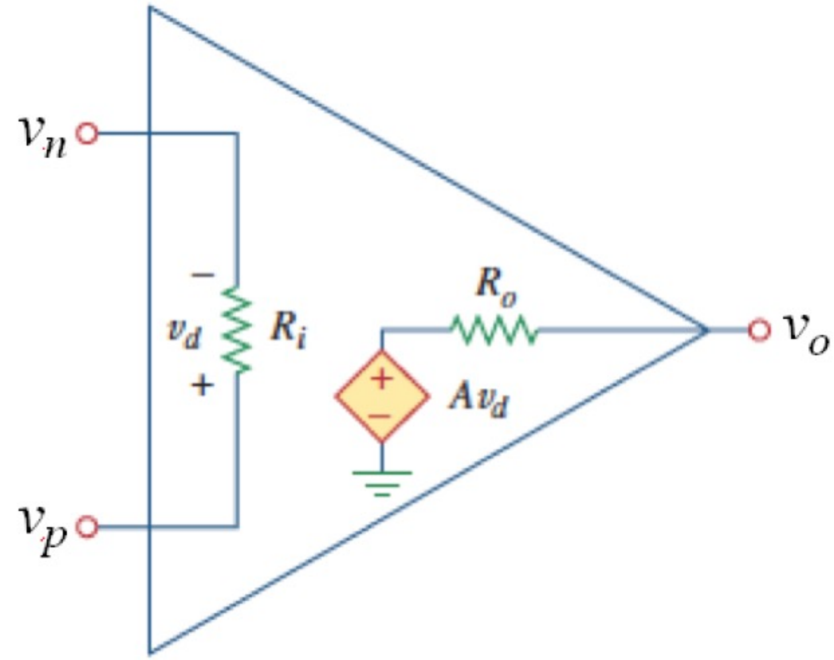


- Is a powered device, not unlike gates in ELE 201/202
  - Introduced in the 1960's
  - Lots of applications
- Common example is the 741
  - Cost is < $1



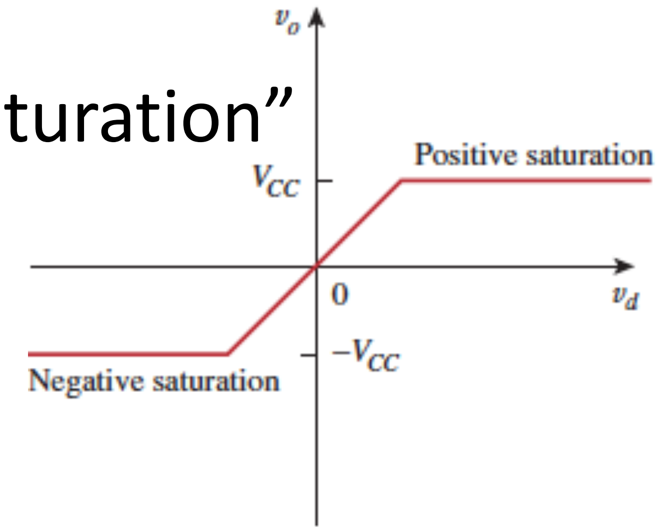
pin configuration

- Nominally behaves like a voltage dependent voltage source

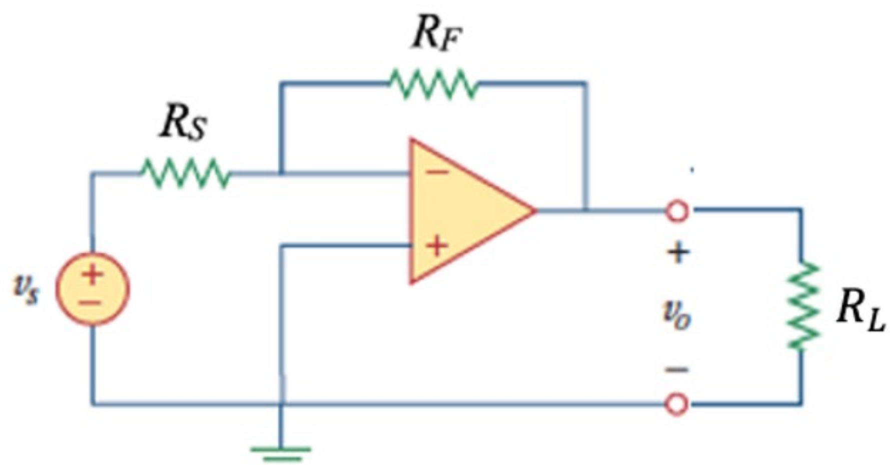


- Simplest circuit model:
  - Dependent voltage source
    - Control voltage is $v_d = v_p - v_n$
    - "Open loop" gain: A (is large, 10^5 or more)
  - Two resistors
    - "Input resistance" R_i : large (10^5 Ω or more)
    - "Output resistance" R_o : small (100 Ω or less)

- Experiences open loop "saturation"



  – Usually configured with "negative feedback"



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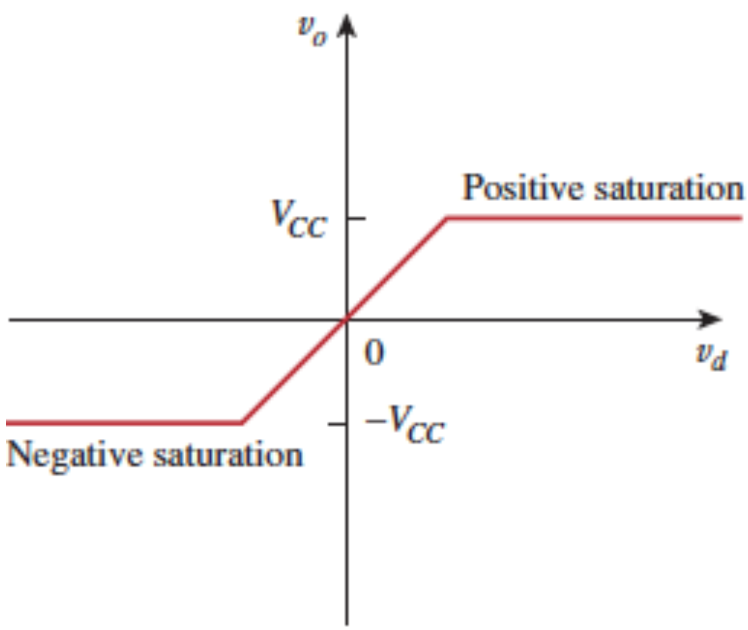

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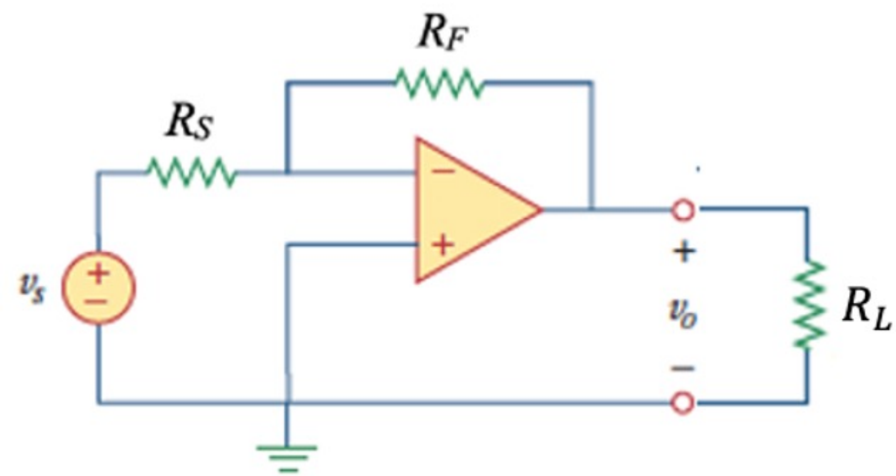
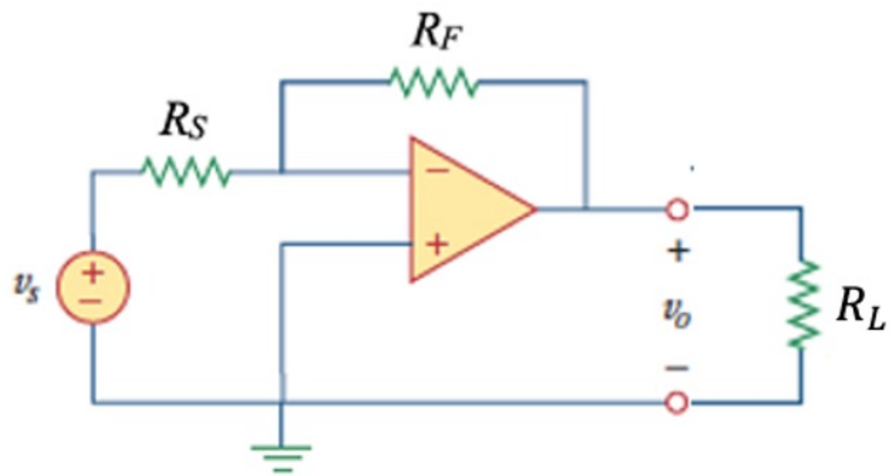
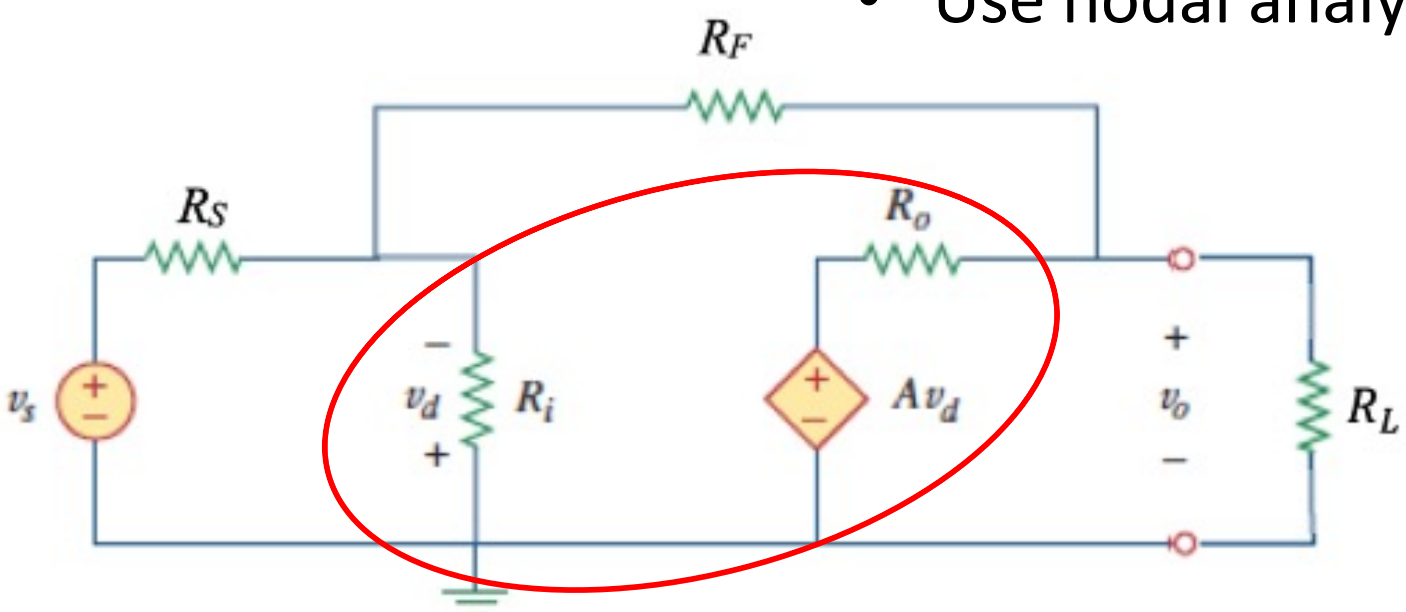
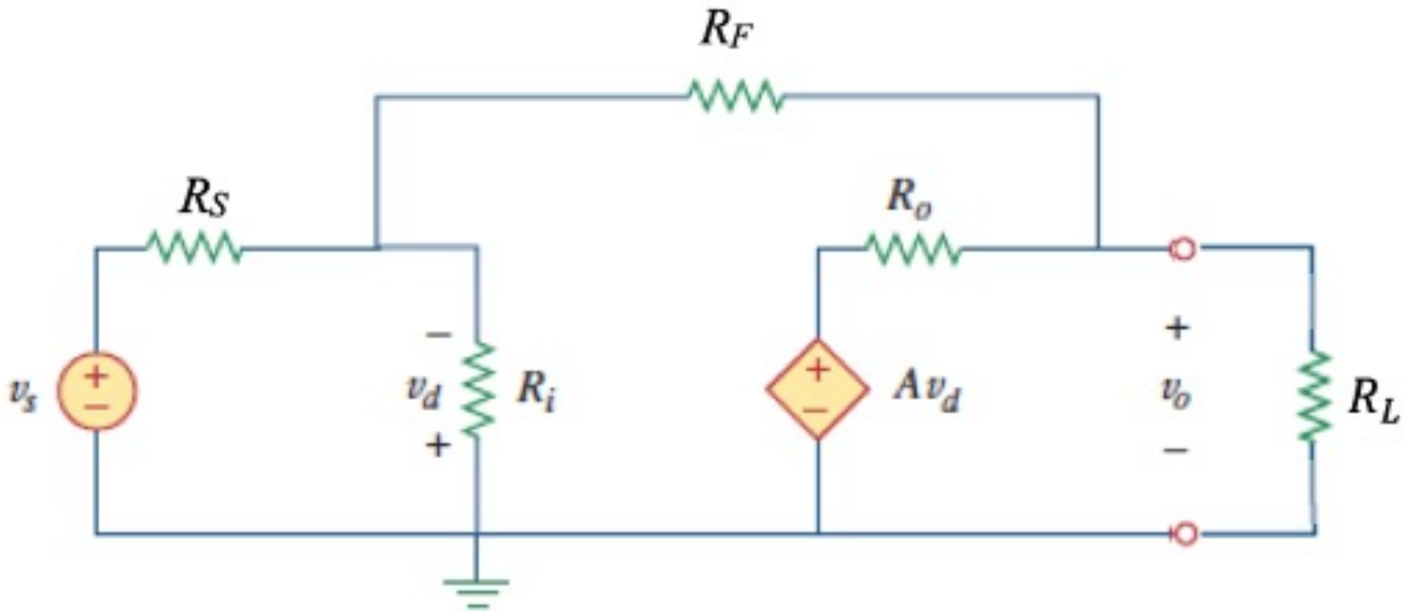


Sample Analysis



- Circuit with "source", "feedback", and "load" resistors
- Substitute the simple model for the op amp
- Use nodal analysis

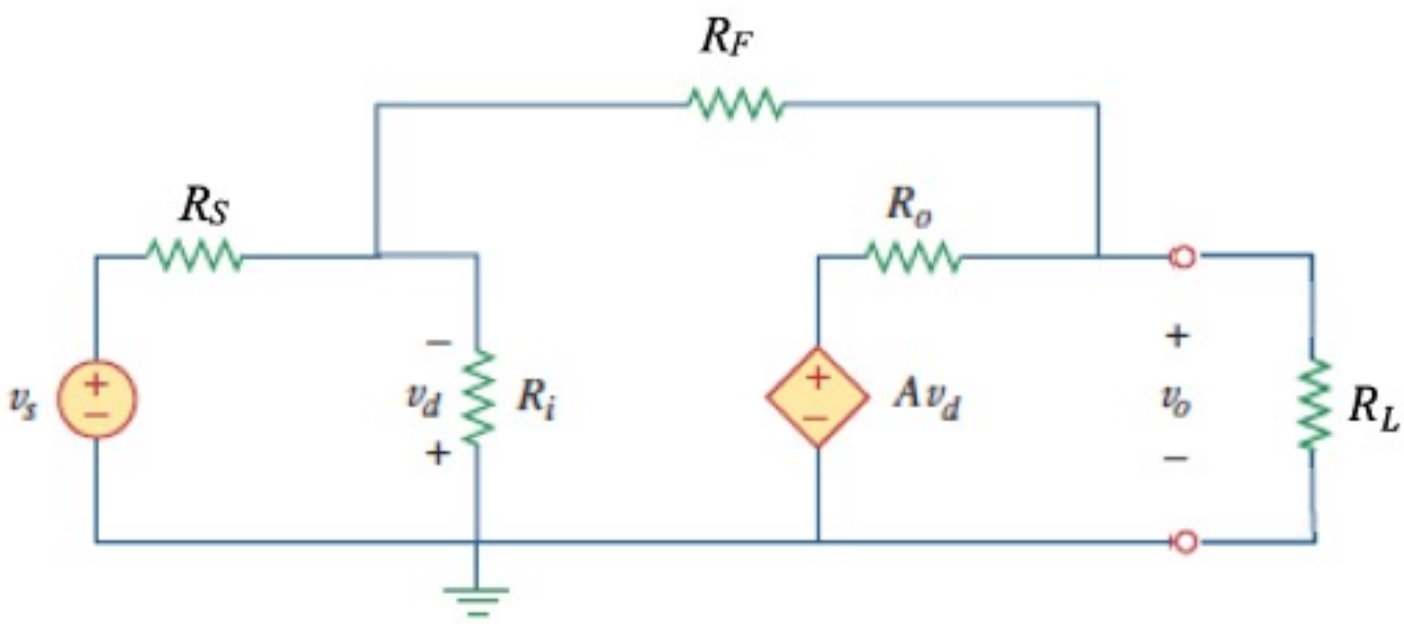




- **Nodes:** ground, v_o , and $v_1 = -v_d$

$$\frac{v_1}{R_i} + \frac{v_1 - v_S}{R_s} + \frac{v_1 - v_o}{R_F} = 0$$

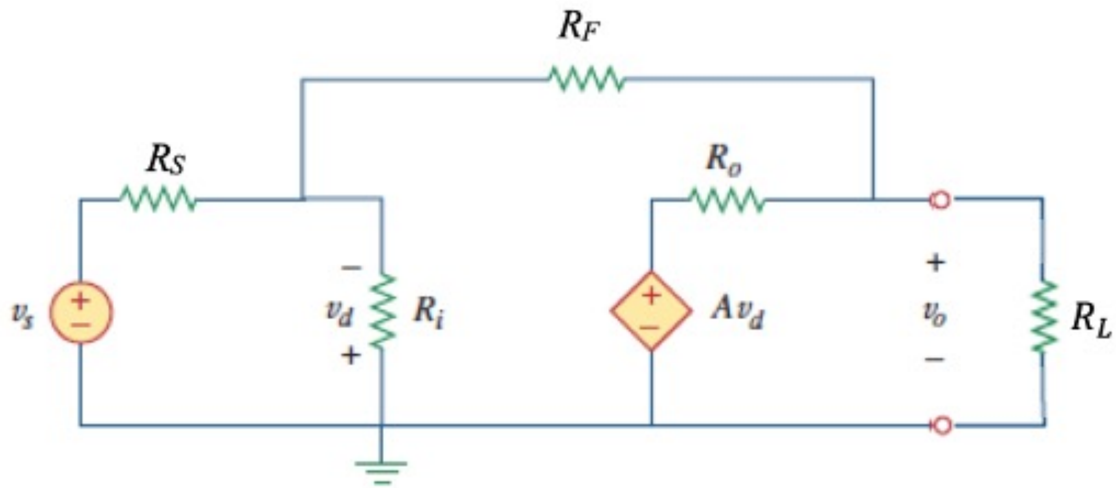
$$\frac{v_o + Av_1}{R_o} + \frac{v_o - v_1}{R_F} + \frac{v_o}{R_L} = 0$$



- After some tedious, symbolic algebra:

$$v_o = -\frac{R_i R_L (R_F A - R_o)}{R_L(R_S + R_i)(R_F + R_o) + (A+1)R_S R_i R_L + R_o(R_S R_i + R_F R_i + R_S R_F)} v_s$$

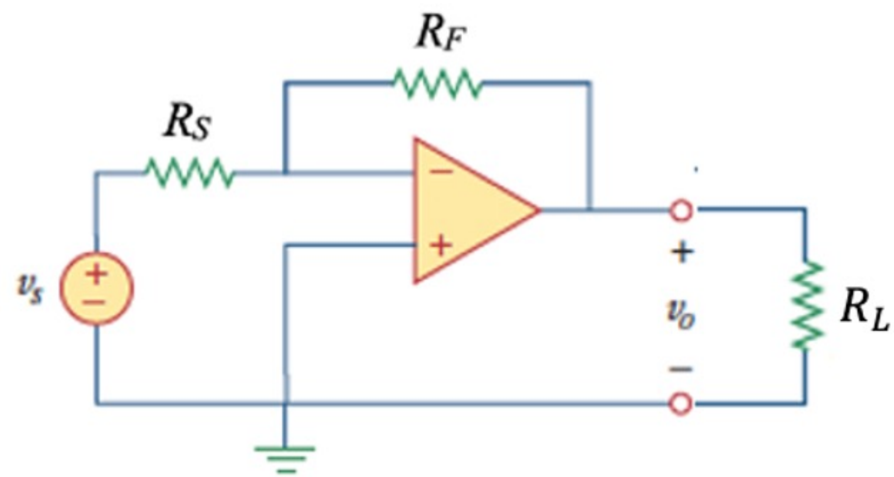
- Note linearity, gain term is negative (“inverting amplifier”)



- External components:
  - $R_S = 10\ k\Omega$
  - $R_F = 20\ k\Omega$
  - $R_L = 40\ k\Omega$
- Nominal model parameters:
  - $R_i = 2\ \mathrm{M}\Omega$
  - $R_o = 50\ \Omega$
  - $A = 200{,}000$

$$v_o = -1.99997\ v_s = -2.00\ v_s$$

Sensitivity Analysis



For $R_S = 10\ k\Omega$,
$R_F = 20\ k\Omega$, and
$R_L = 40\ k\Omega$
we have $\mathbf{v_0 = -2.00\ v_s}$

- How do the op amp's internal parameters ($R_i, R_o,$ and A) impact this result?
- What happens when we change the circuit's components (R_S, R_F, and R_L)?

Sensitivity to internal values:

R_S	R_F	R_L	R_i	R_o	A	gain
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \mathrm{M}\Omega$	$50\ \Omega$	$200{,}000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	**$1\ \mathrm{M}\Omega$**	$50\ \Omega$	$200{,}000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	**$4\ \mathrm{M}\Omega$**	$50\ \Omega$	$200{,}000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \mathrm{M}\Omega$	**$25\ \Omega$**	$200{,}000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \mathrm{M}\Omega$	**$100\ \Omega$**	$200{,}000$	-1.99997
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \mathrm{M}\Omega$	$50\ \Omega$	**$100{,}000$**	**-1.99994**
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \mathrm{M}\Omega$	$50\ \Omega$	**$400{,}000$**	**-1.99998**
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \mathrm{M}\Omega$	$50\ \Omega$	**$50{,}000$**	**-1.99988**
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \mathrm{M}\Omega$	$50\ \Omega$	**$20{,}000$**	**-1.99970**

Seems insensitive!

Sensitivity to circuit component values:

R_S	R_F	R_L	R_i	R_o	A	gain
$10\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	-1.99997
$10\ k\Omega$	$20\ k\Omega$	**$20\ k\Omega$**	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	-1.99997
$10\ k\Omega$	$20\ k\Omega$	**$80\ k\Omega$**	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	-1.99997
$10\ k\Omega$	**$10\ k\Omega$**	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-0.99999**
$10\ k\Omega$	**$40\ k\Omega$**	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-3.99990**
$10\ k\Omega$	**$100\ k\Omega$**	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-9.99945**
$5\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-3.99990**
$20\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-0.99999**
$40\ k\Omega$	$20\ k\Omega$	$40\ k\Omega$	$2\ \text{M}\Omega$	$50\ \Omega$	200,000	**-0.499996**

Only depends upon source and feedback resistors!

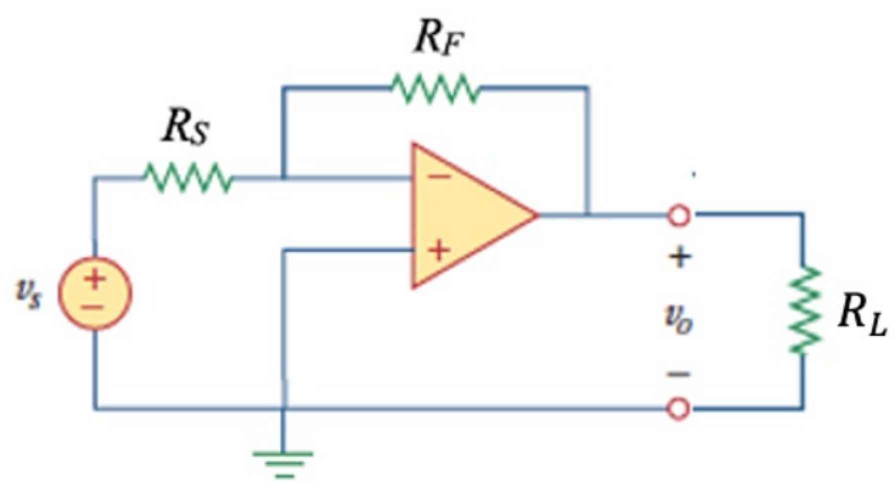
Asymptotics

- Consider the gain term when A is large, R_i is large, and R_o is small

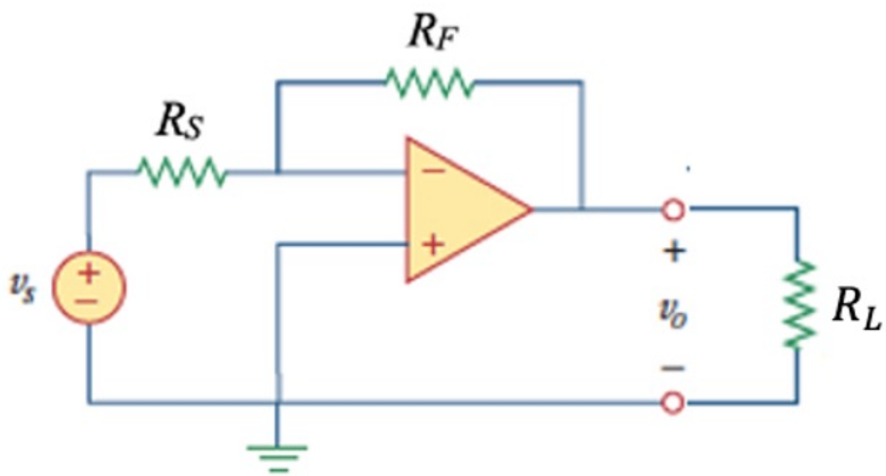
$$-\frac{R_i R_L (R_F\, A - R_o)}{R_L (R_S + R_i)(R_F\ + R_o) + (A+1) R_S\, R_i\ R_L + R_o (R_S R_i + R_F R_i + R_S R_F)}$$

- Then

$$\frac{v_o}{v_s} \approx -\frac{R_i R_L R_F\, A}{(A+1) R_S\, R_i\, R_L} \approx -\frac{R_F}{R_S}$$



- Further, consider the op amp′s input side:



$$v_d = -\frac{R_i(R_L R_o + R_L R_F + R_o R_F)}{R_L(R_S + R_i)(R_F + R_o) + (A+1)R_S R_i R_L + R_o(R_S R_i + R_F R_i + R_S R_F)} v_s$$

- For the nominal values

$$v_d = -\frac{v_s}{99{,}628} \approx 0 \qquad\qquad i_d = \frac{v_d}{2{,}000{,}000} \approx 0$$

More general model:

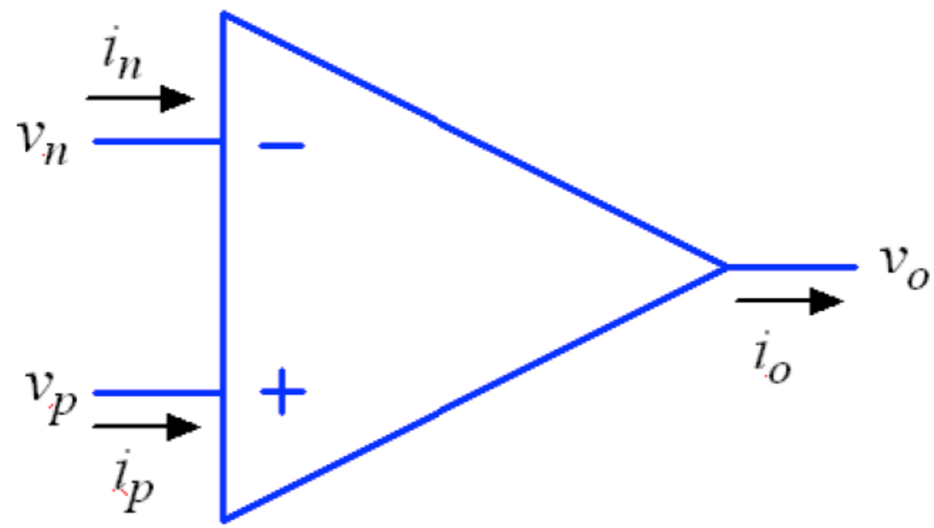
- $v_d \approx 0$
  - we have a "virtual short" across the inputs

$$v_p = v_n$$

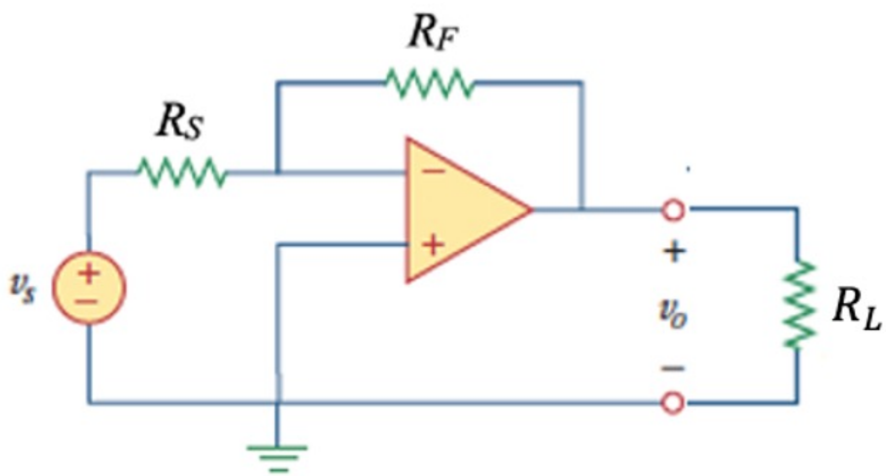
- $i_d \approx 0$
  - the inputs are open circuits

$$i_p = i_n = 0$$

- And $i_o = ?$
  - The device generates whatever current is needed



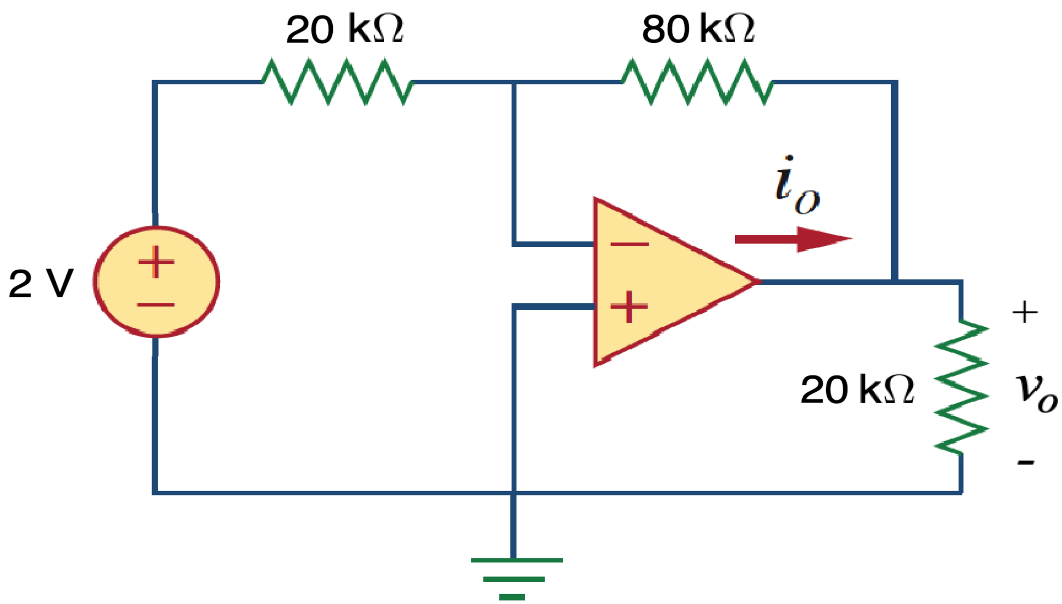
Using the Simple Model



Input voltages equal
Input currents zero

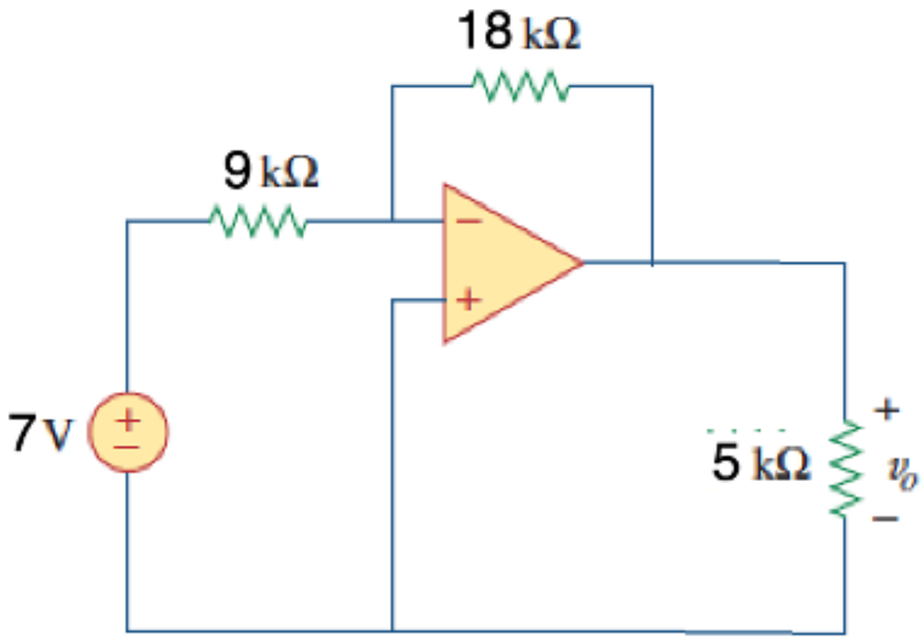
Example: find v_o and i_o

$v_o = -8\ V, i_0 = -0.5\ mA$



Input voltages equal
Input currents zero

Example: find v_o

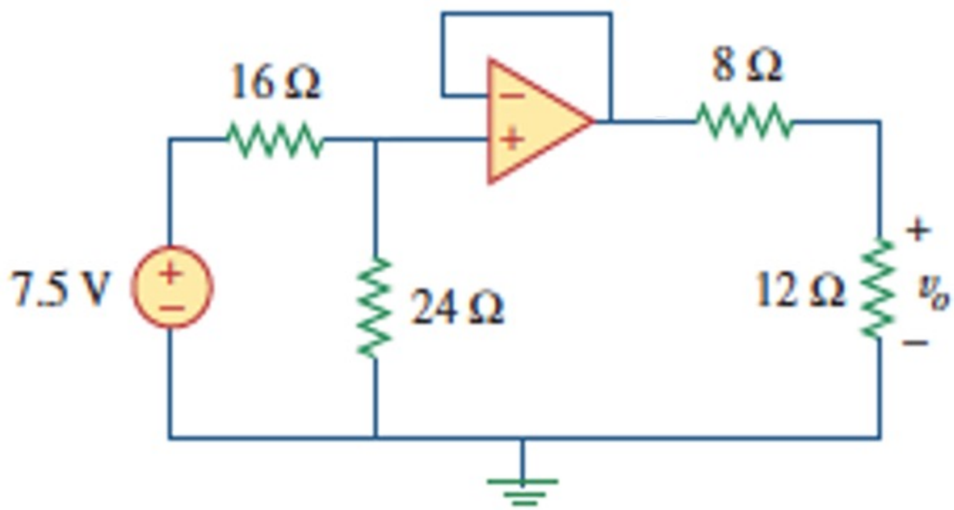


$v_o = -14\ V$

Input voltages equal
Input currents zero

Example: find v_o

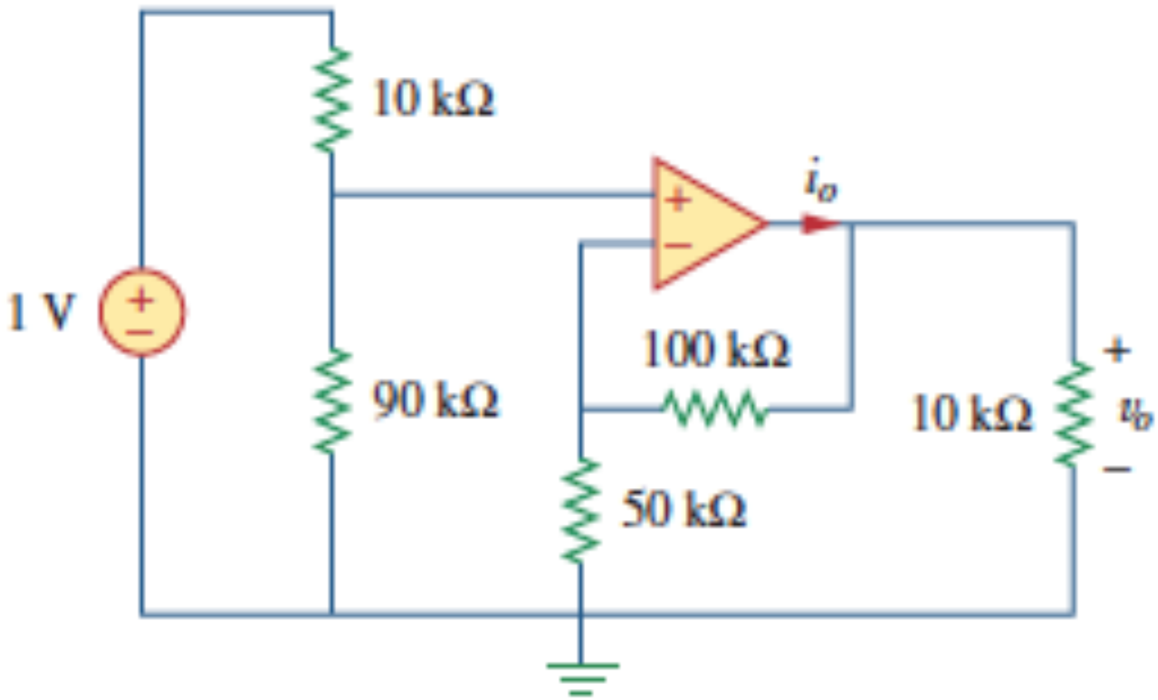
$v_o = 2.7\ V$



Input voltages equal
Input currents zero

Example: find v_o and i_o

$v_o = 2.7\ V, i_o = 0.2718\ mA$



Practice problem:

Input voltages equal
Input currents zero

$$i_0 = -0.5625\ mA$$

5.19 Determine i_o in the circuit of Fig. 5.58.

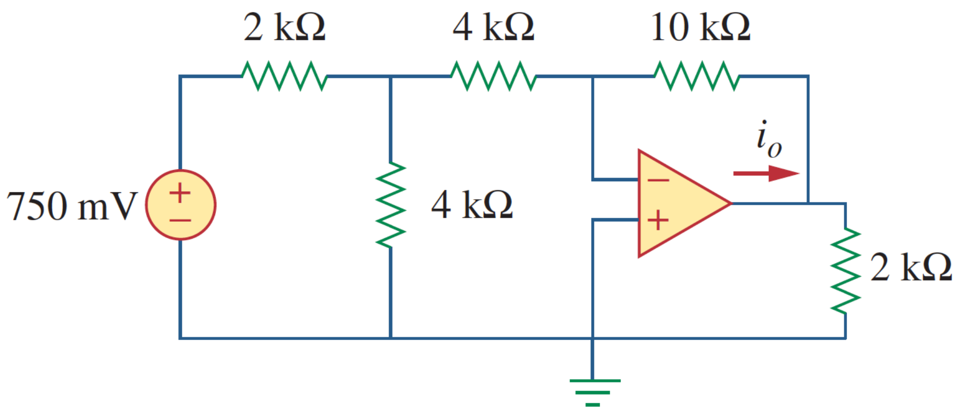


Figure 5.58
For Prob. 5.19.

Practice problem:

Input voltages equal
Input currents zero

$i_x = 0.727\ mA$

5.31 For the circuit in Fig. 5.69, find i_x.

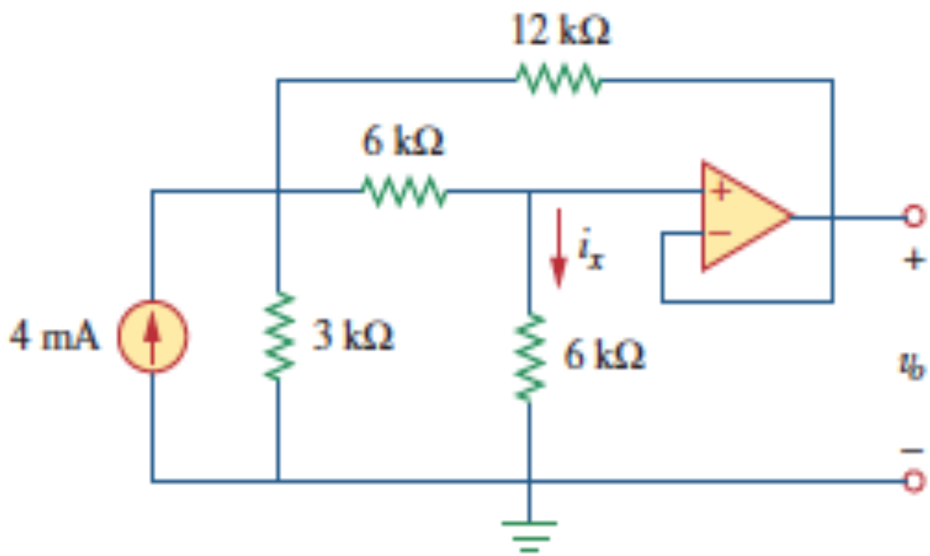
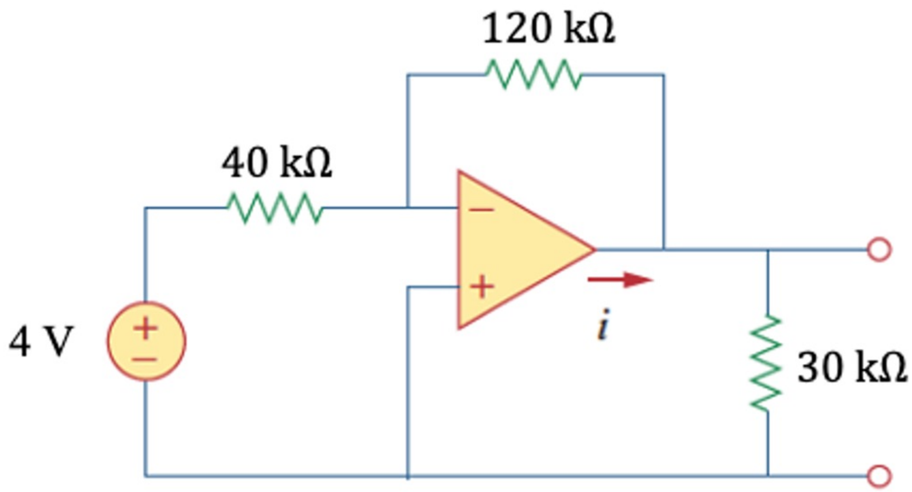


Figure 5.69
For Prob. 5.31.

$i = -0.51\ mA$

Practice problem: find i



$i_o = 0.42\ mA$

Practice problem: find i_o