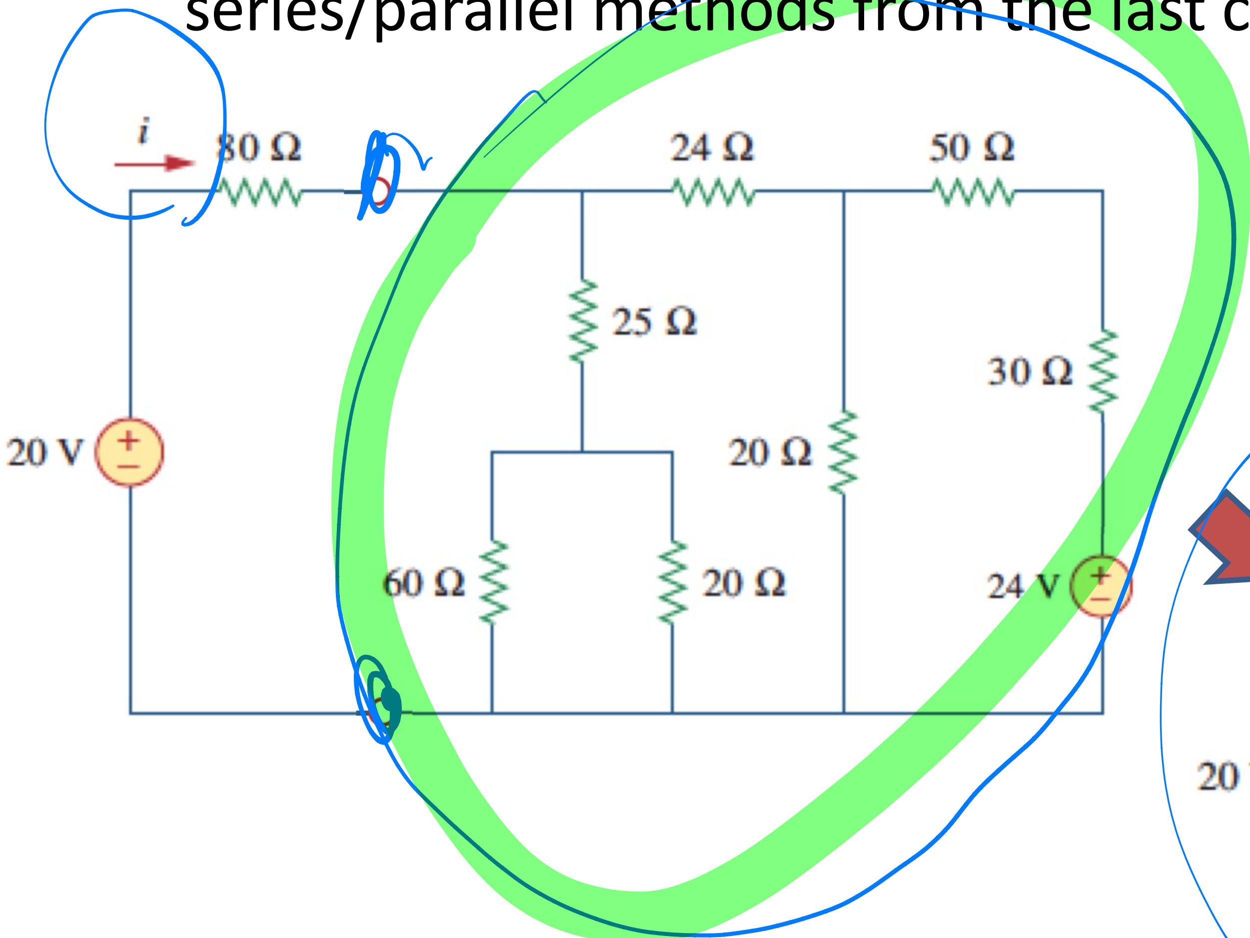
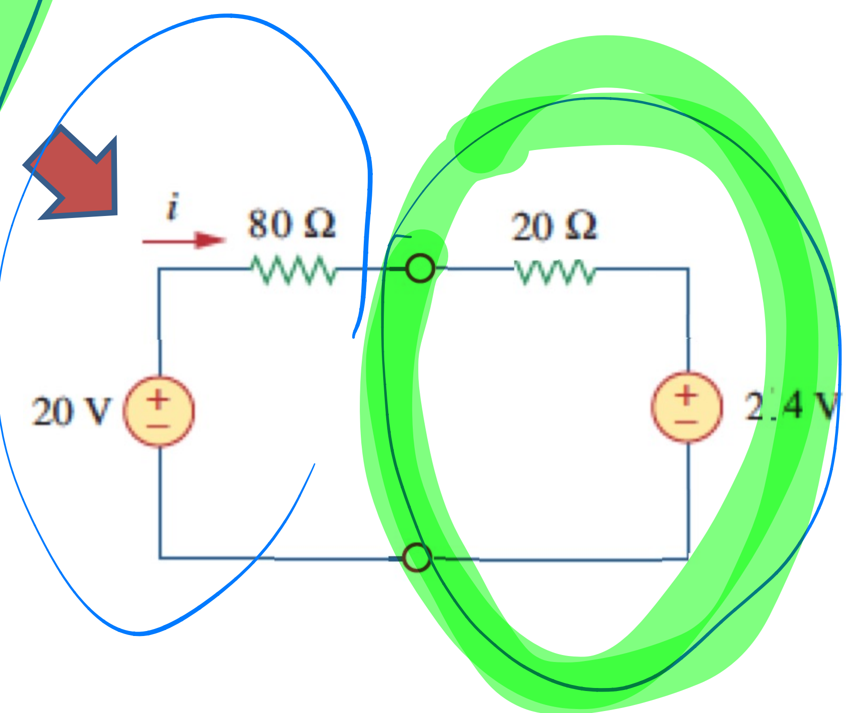
Theorems – 2

Thévenin

Thévenin Models

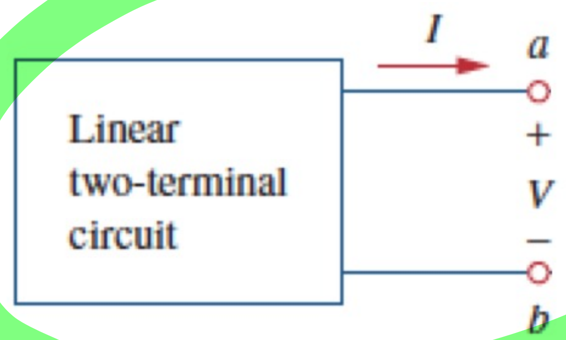
- Application – recall combining transformations and series/parallel methods from the last class:

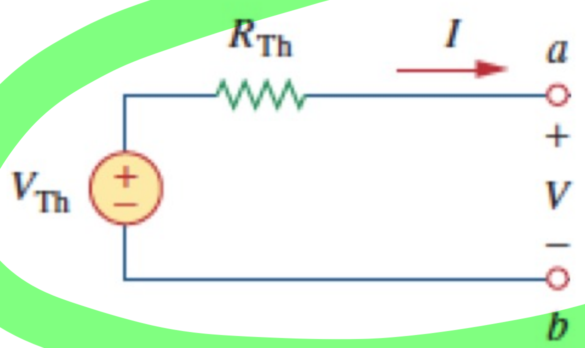




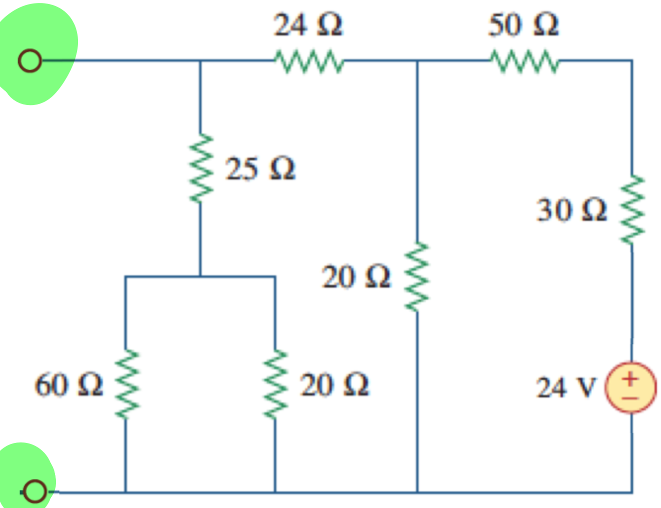
Thévenin equivalent circuit ≡ electrical equivalence at any pair of terminals

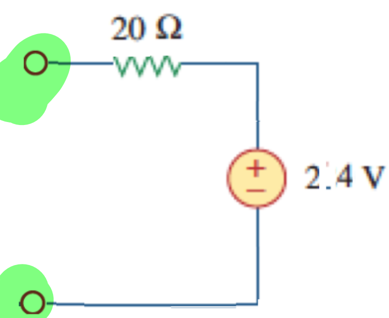
- 2 parameters:
  - V_{Th}
  - R_{Th}





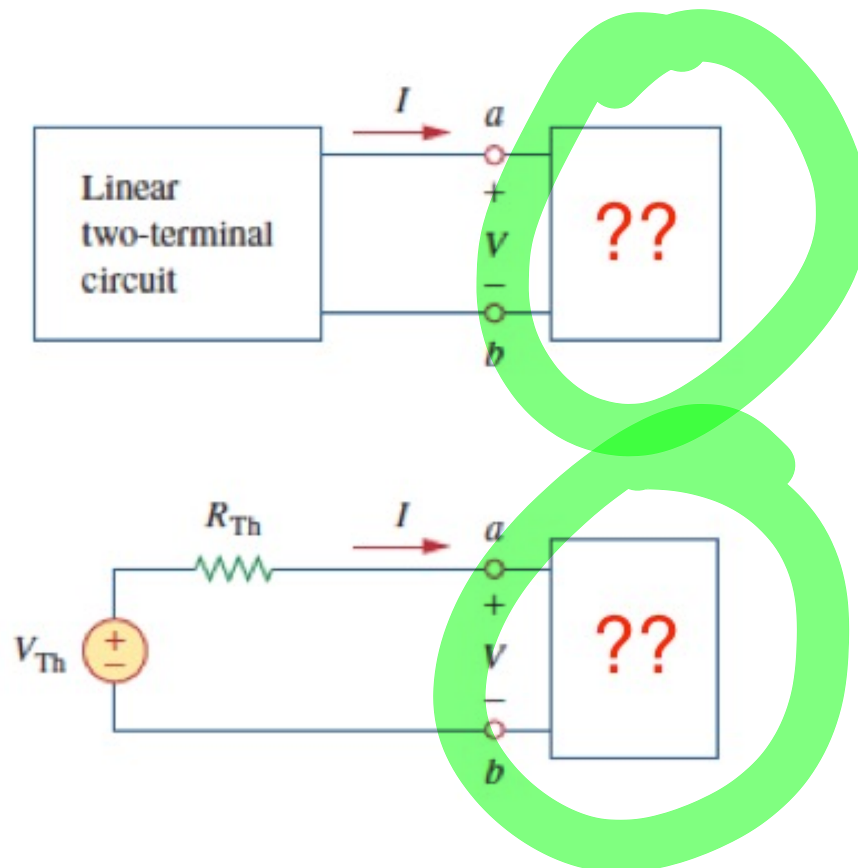
- Example:





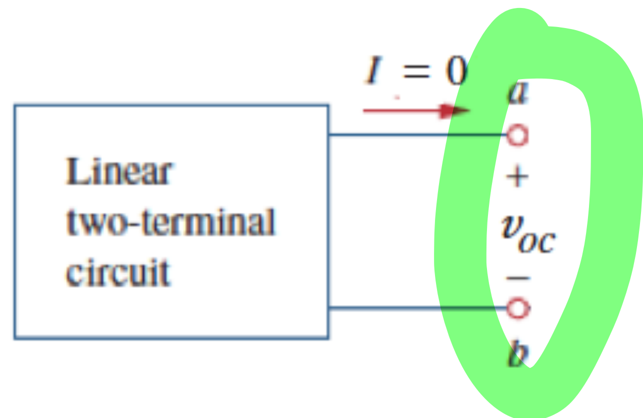
How to find V_{Th} and R_{Th} ?

- Concept: both the circuit <u>and</u> the model should behave the same way **no matter what** is connected at a-b
  - With only 2 variables in the model, we need only check **2** load situations
  - Need only match one of the V or I variables in each



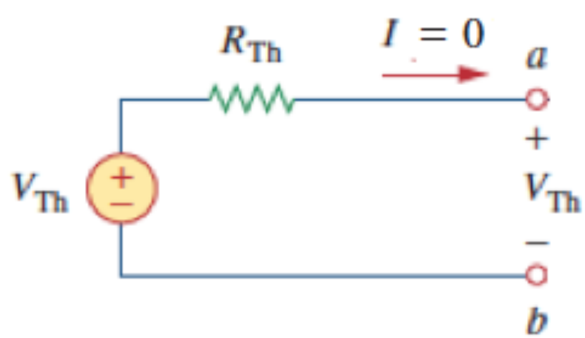
Thevenin: Most Common Approach

- Connect nothing – “open circuit” test
  - Just need a voltmeter



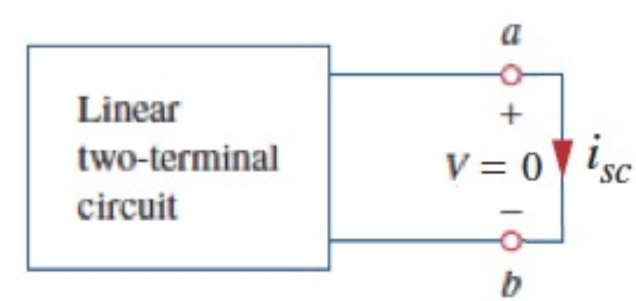
Clearly

$$V_{Th} = V_{OC}$$



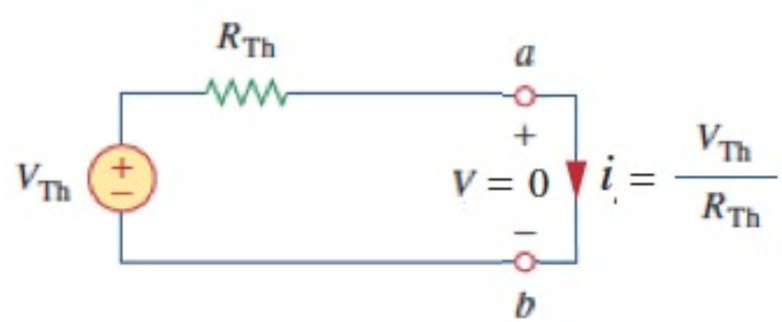
Another popular approach

- Connect a wire – “short circuit” test
  - Popular for paper analysis



$$i_{SC} = \frac{V_{Th}}{R_{Th}}$$

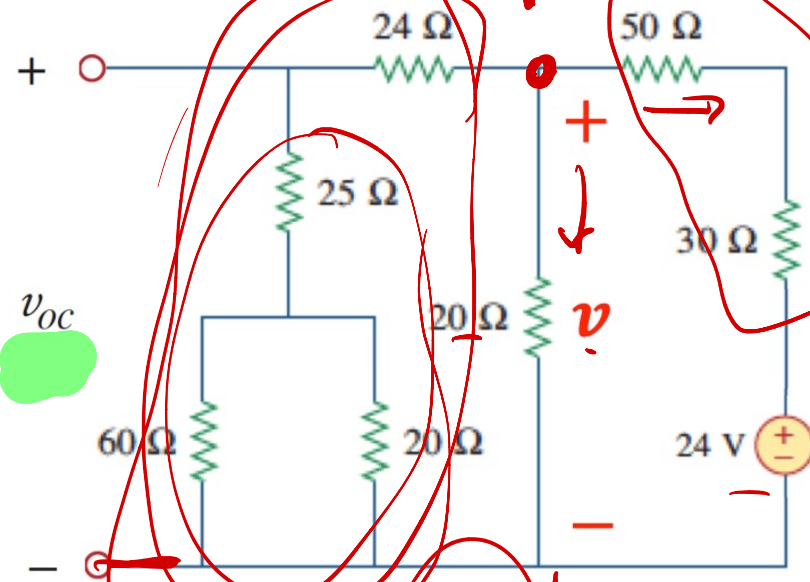
So



$$R_{Th} = \frac{V_{Th}}{i_{SC}} = \frac{V_{OC}}{i_{SC}}$$

Example:

- Use node analysis:

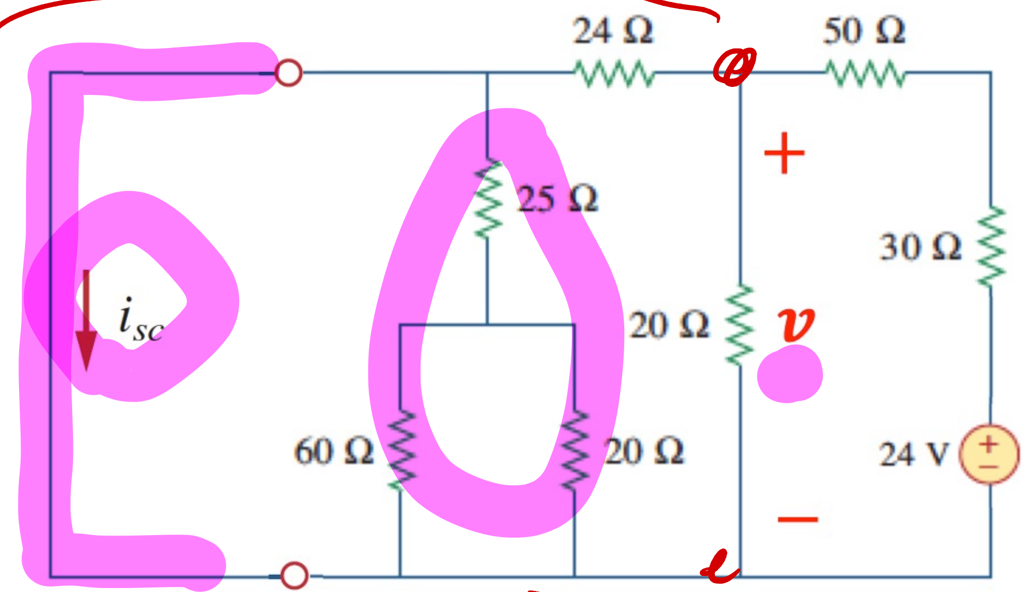


$$\frac{v}{20} + \frac{v - 24}{80} + \frac{v}{64} = 0 \quad \Rightarrow \quad v = \frac{96}{25}$$

- Then voltage division:

$$v_{OC} = \frac{40}{64} v \quad \Rightarrow \quad V_{Th} = v_{OC} = 2.4 \text{ volts}$$

Note how the short changes the circuit



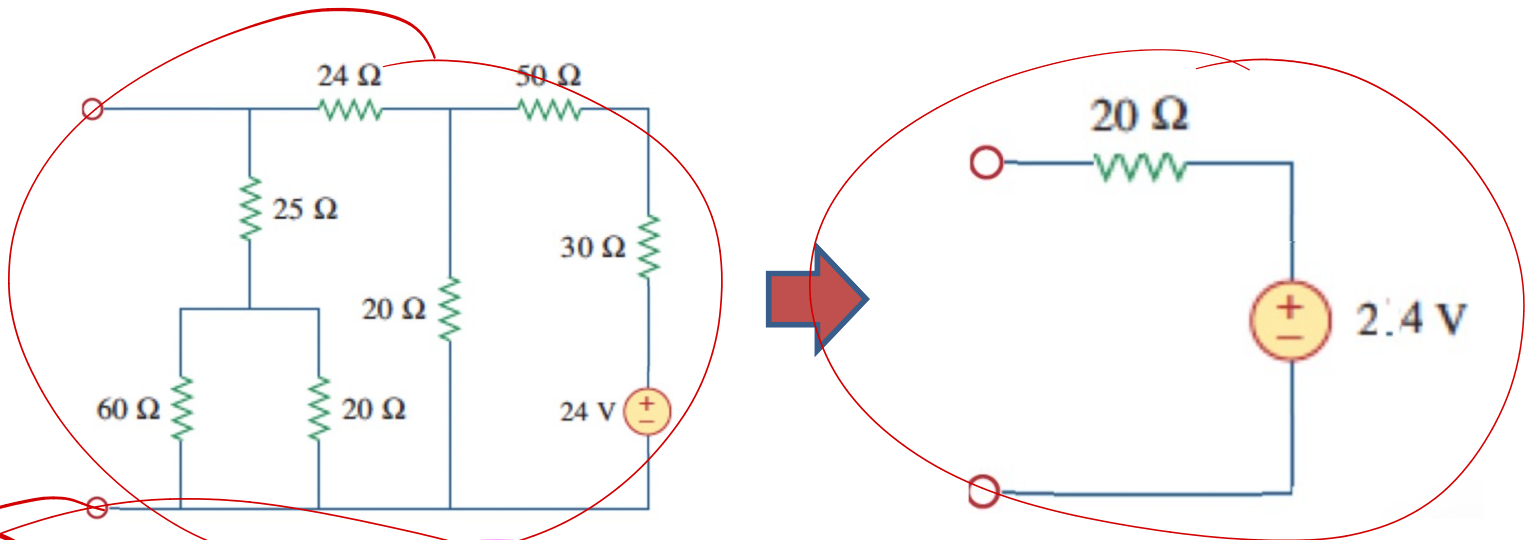
- Use node analysis:

$$\frac{v}{20} + \frac{v-24}{80} + \frac{v}{\mathbf{24}} = 0 \quad \Rightarrow \quad v = \frac{72}{25}$$

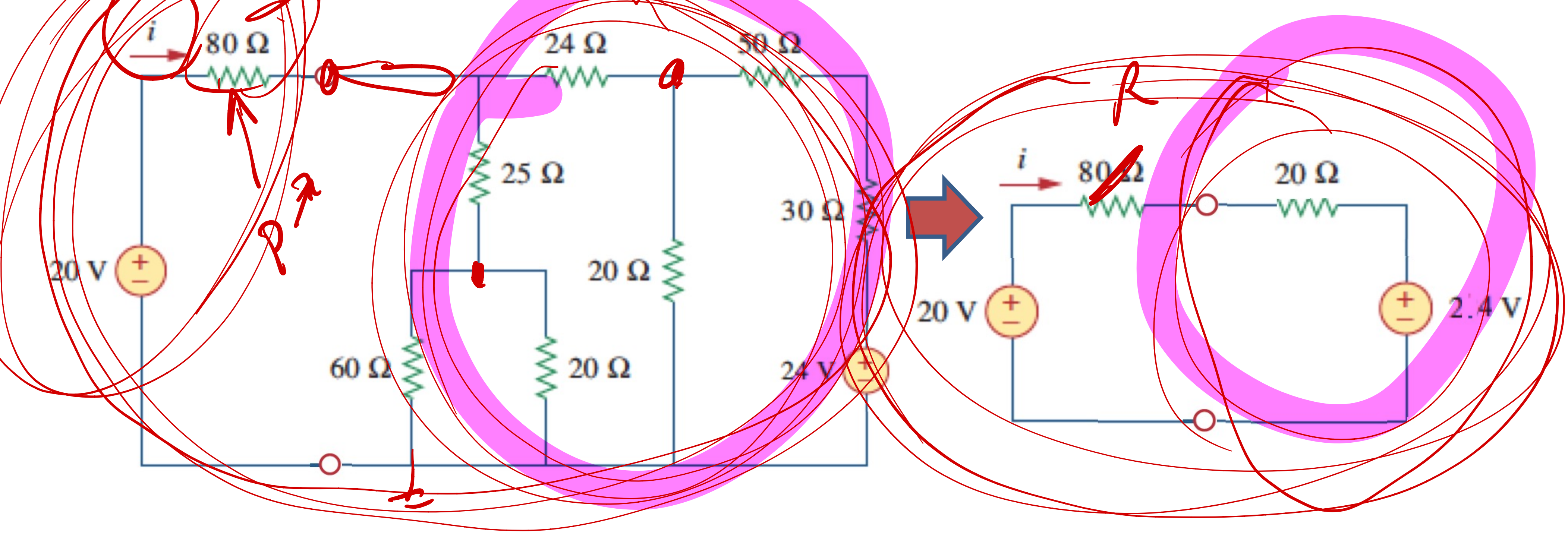
- Then Ohm's Law:

$$i_{SC} = \frac{v}{24} = 0.12 \quad \Rightarrow \quad R_{Th} = \frac{v_{OC}}{i_{SC}} = \frac{2.4}{0.12} = 20\ \Omega$$

So $V_{Th} = 2.4$ volts and $R_{Th} = 20\ \Omega$

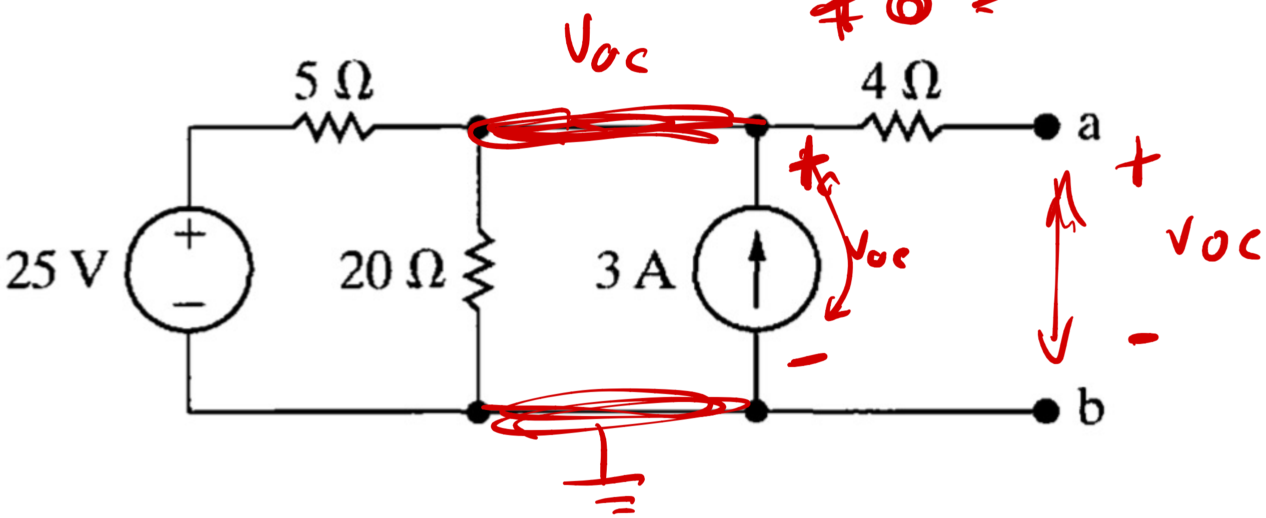


and



Example: find V_{th} and R_{Th}

$$V_{Th} = v_{OC}$$
$$R_{Th} = \frac{v_{OC}}{i_{SC}}$$



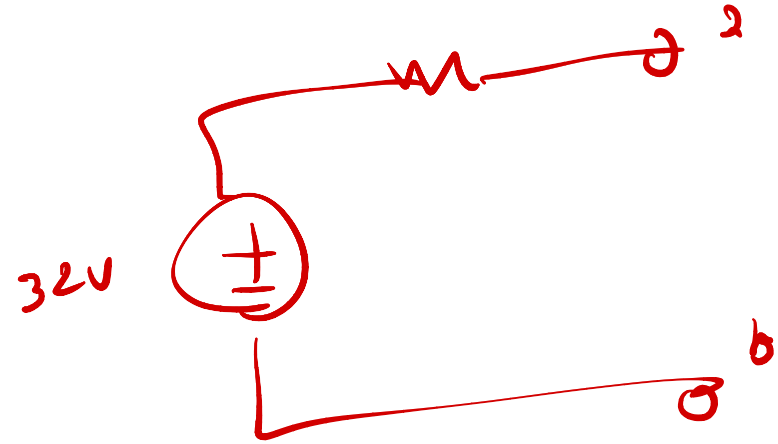
$V_{TH} = 32\,V$

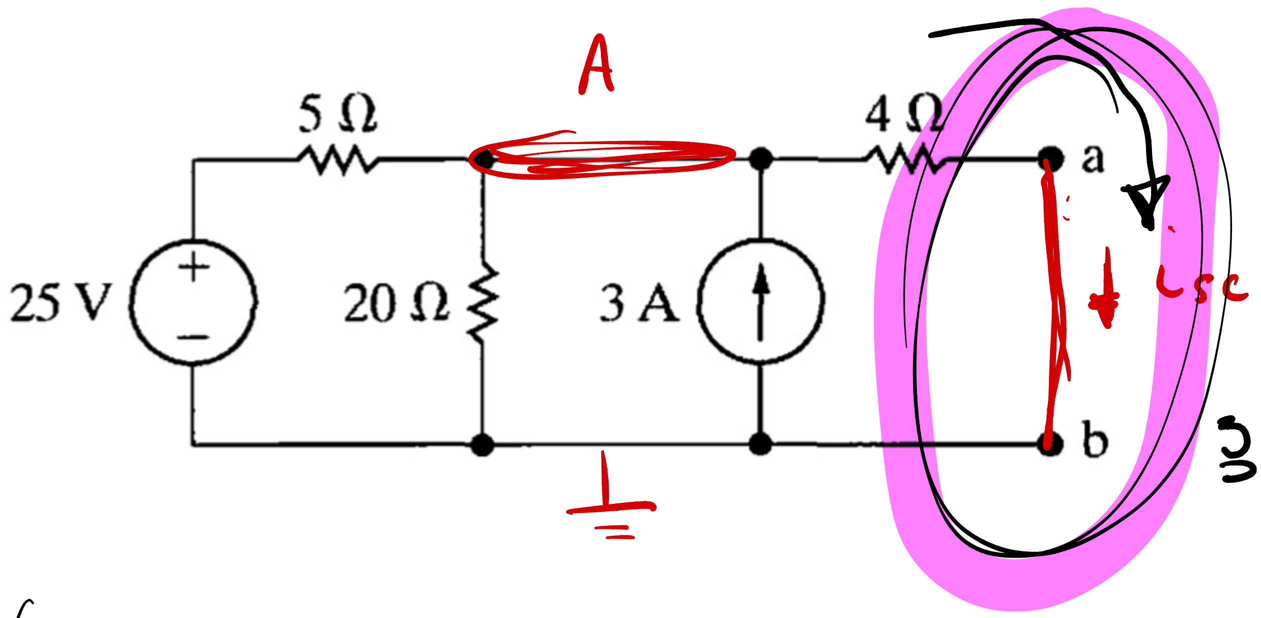
$$20\left(\frac{V_{oc}}{20} + \frac{V_{oc} - 25}{5} - 3 = 0\right)$$

$$V_{oc} + 4V_{oc} - 100 - 60 = 0$$

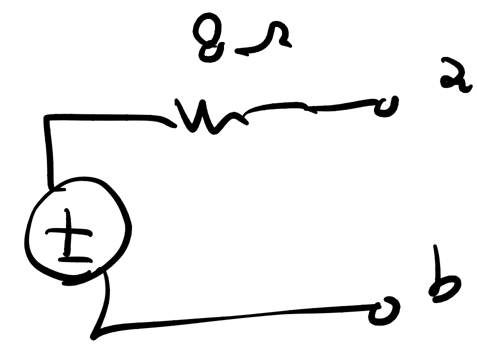
$$5V_{oc} = 160$$

$$V_o = 32\,V$$





32 V, 8 Ω



$$20\left(\frac{A-25}{5} + \frac{A}{20} - 3 + \frac{A}{4} = 0\right.$$

$$4A - 100 + A - 60 + 5A = 0$$

$$10A = 160$$

$$A = 16 \text{ V.}$$

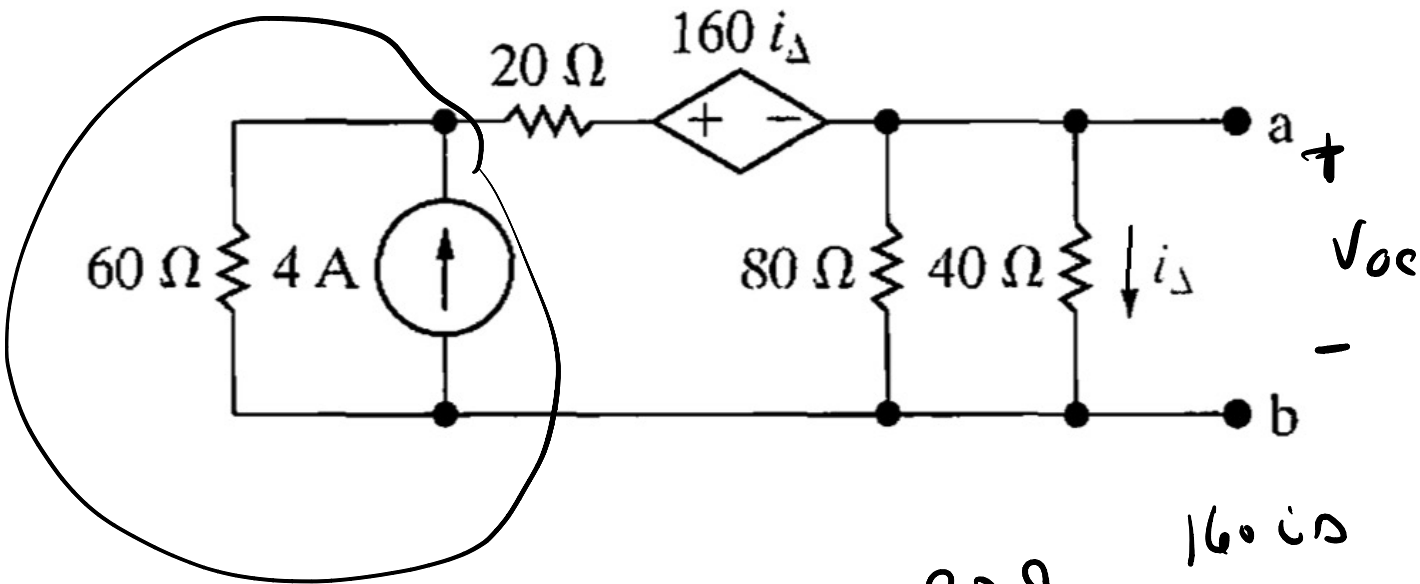
$$i_{sc} = \frac{16\text{ V}}{4\,\Omega} = 4\text{ A}$$

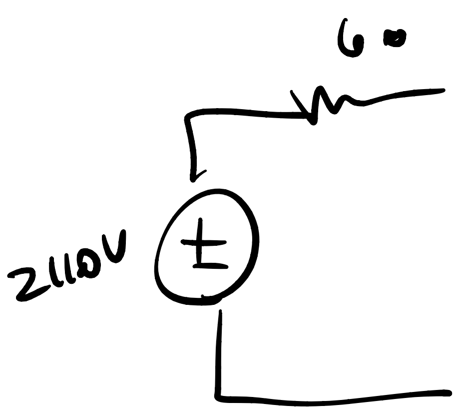
$$4 = \frac{32}{R_{TH}}$$

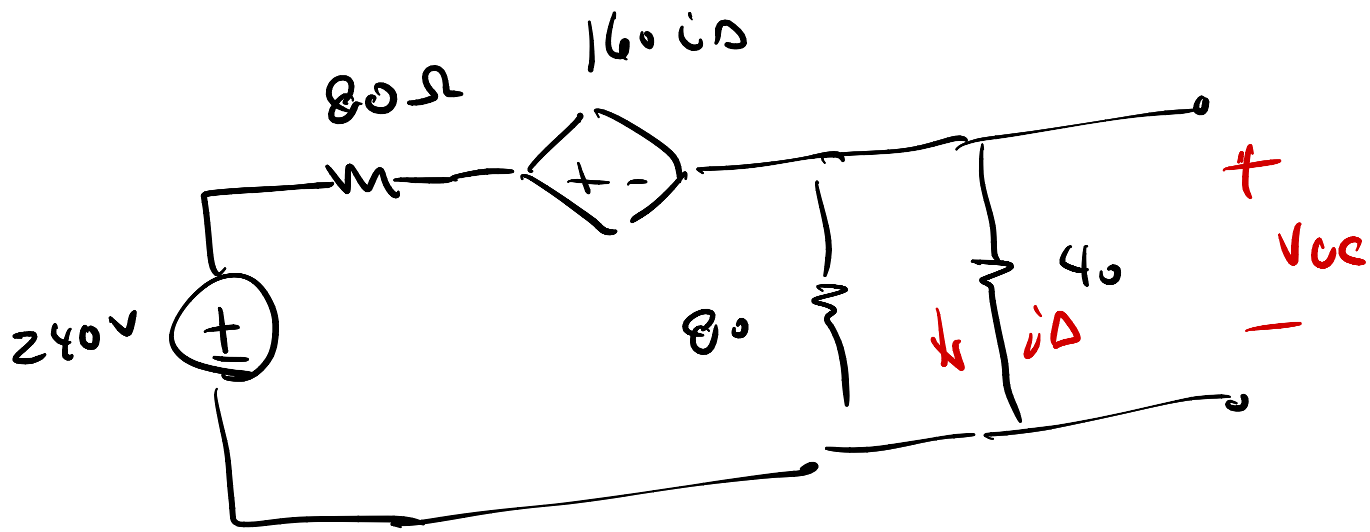
$$R_{TH} = 8\,\Omega$$

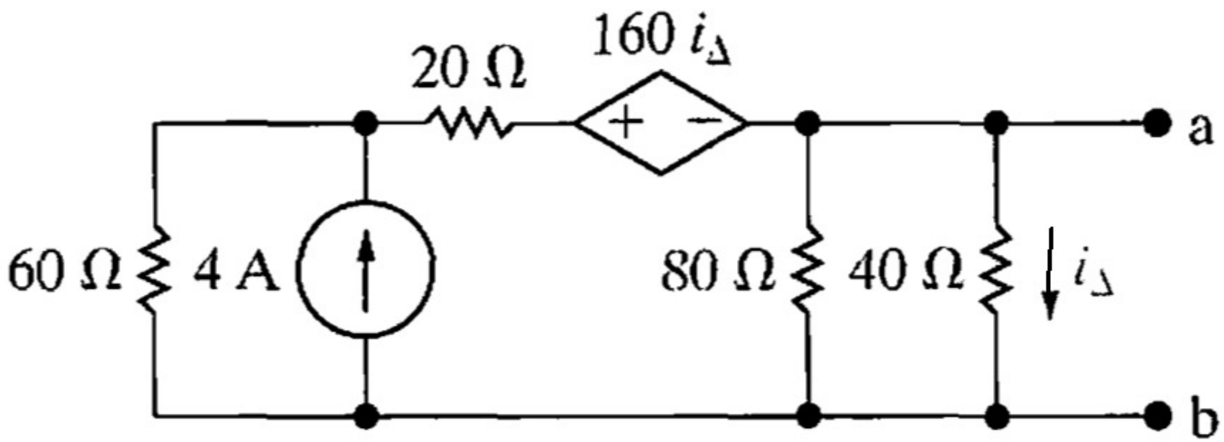
$$V_{Th} = v_{OC}$$
$$R_{Th} = \frac{v_{OC}}{i_{SC}}$$

Example: find V_{th} and R_{Th}



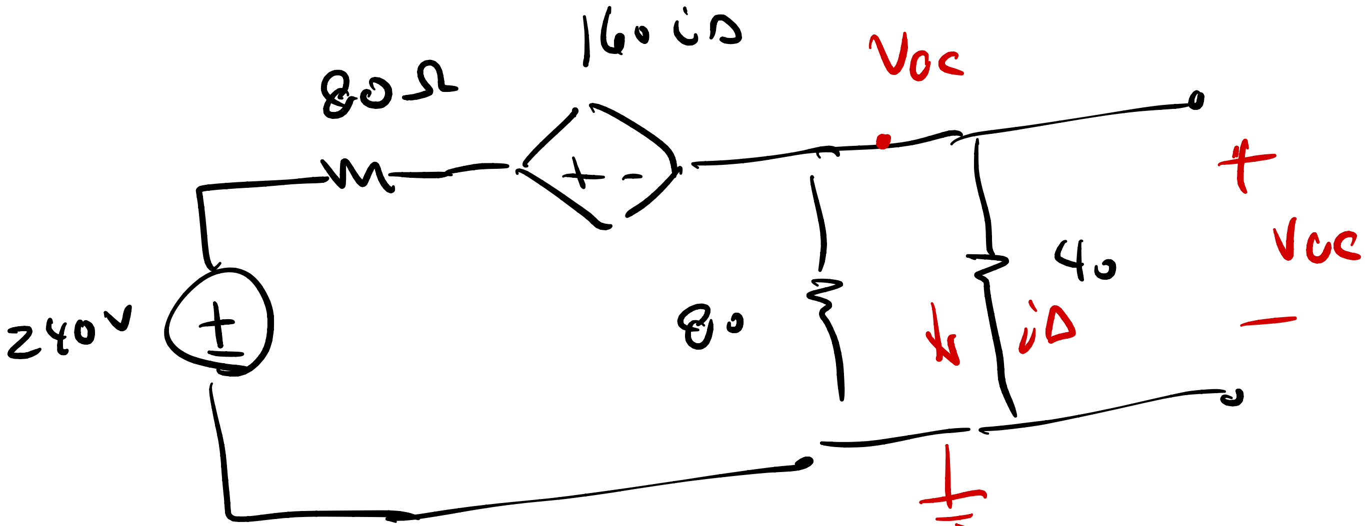






30 V, 10 Ω

$V_{oc} = V_{th}$



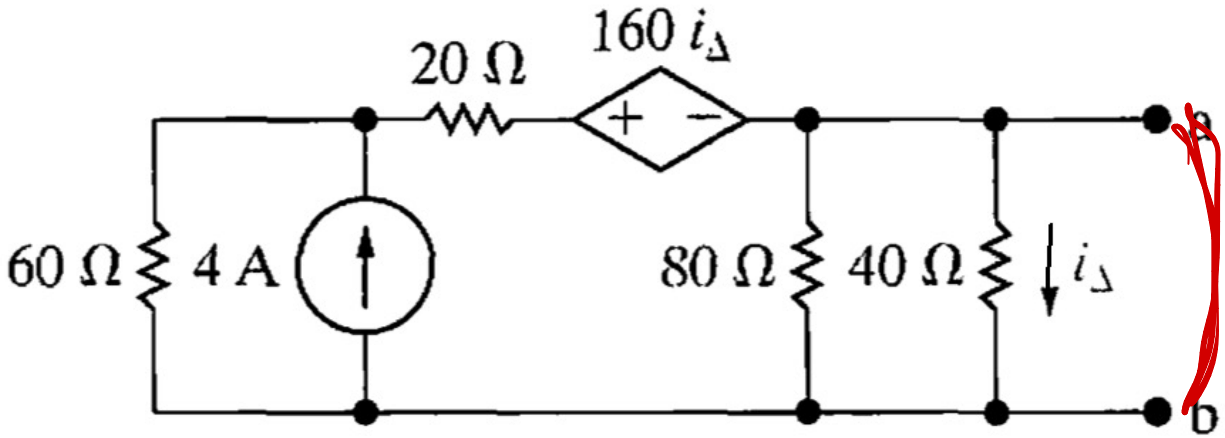
$$\frac{V_{oc}}{40} + \frac{V_{oc}}{80} + \frac{V_{oc} - 240 + 160\, i_\Delta}{80} = 0$$

$$i_\Delta = \frac{V_{oc}}{40}$$

Example: find V_{th} and R_{Th}

$$V_{Th} = v_{OC}$$

$$R_{Th} = \frac{v_{OC}}{i_{SC}}$$



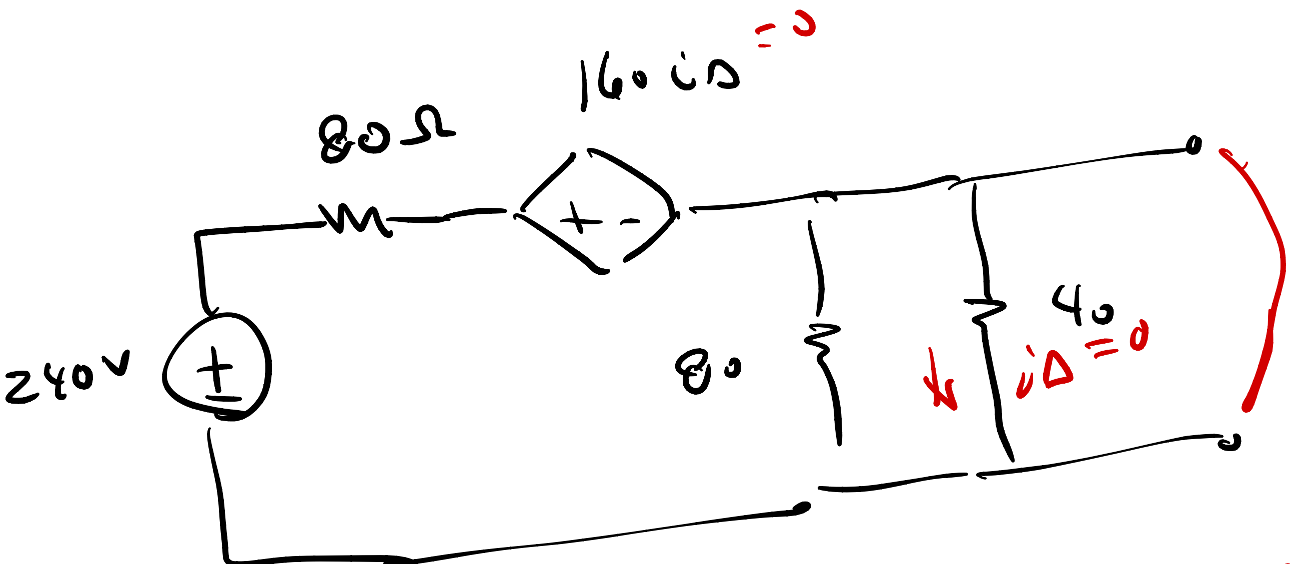
$V_{oc} = 30$ ✓

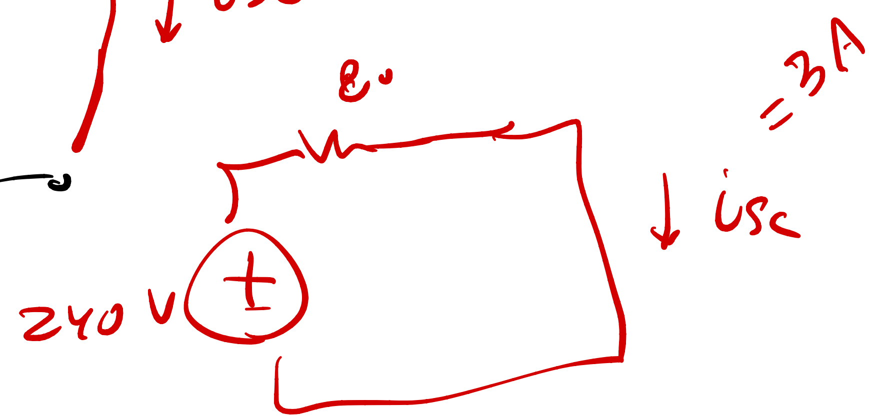
$V_{th} = 30$ ✓

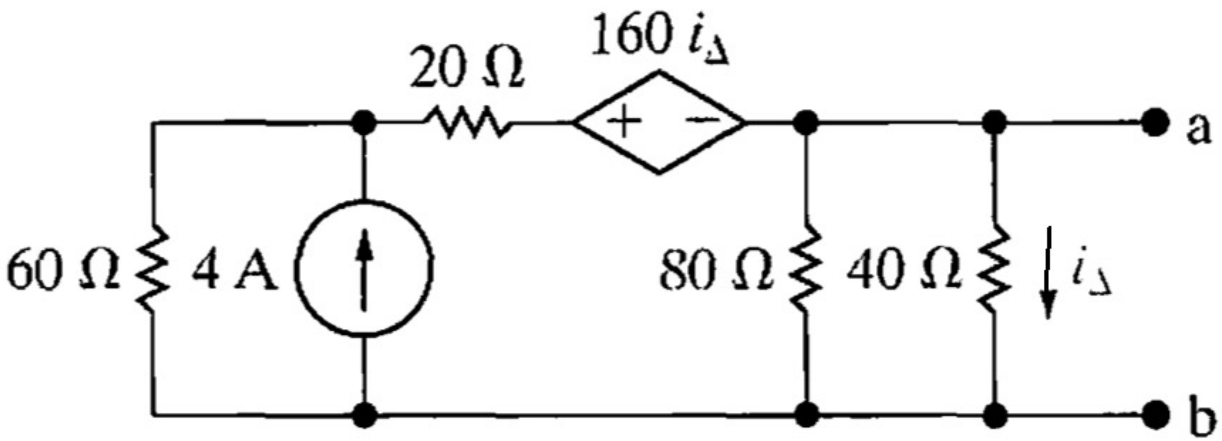
$R_{th} = 10\Omega$

$i_{sc} = 3$ A

$3 = \frac{30}{R_{th}}$



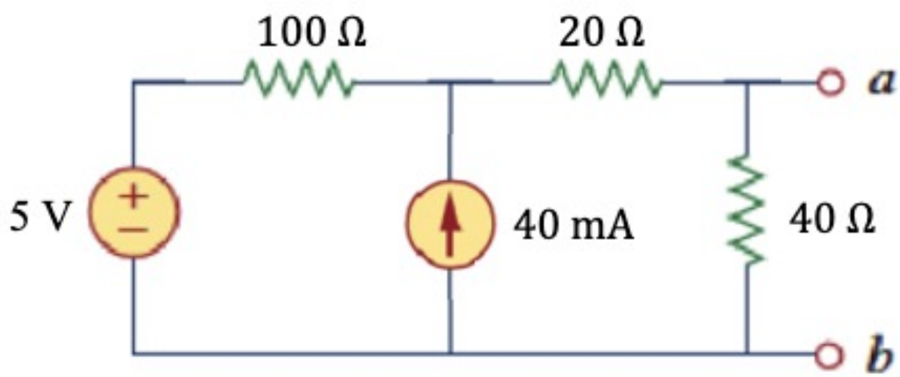




$30\ V, 10\ \Omega$

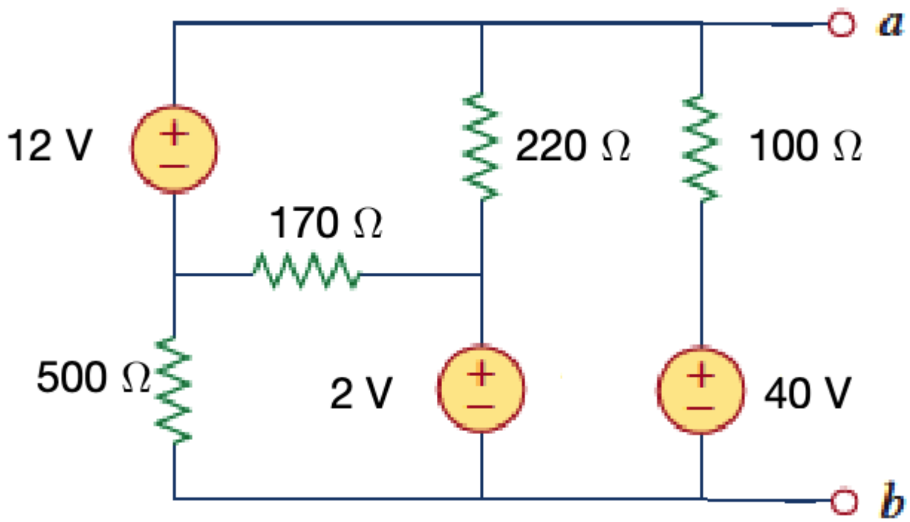
$\frac{9}{4}\,V, 30\,\Omega$

Practice problem: find V_{th} and R_{Th}



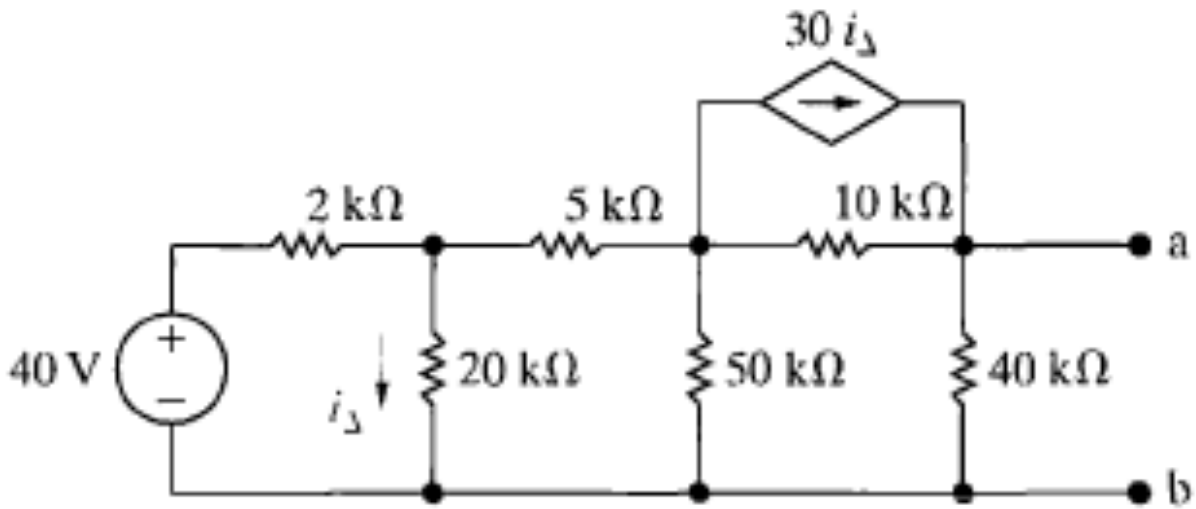
Practice problem: find V_{th} and R_{Th}

$23.0\ V, 44.6\ \Omega$



Practice problem: find V_{th} and R_{Th}

$200\,V, 14{,}3\ k\,\Omega$



$\frac{500}{3}\,V, 10\,\Omega$

Practice problem: find V_{th} and R_{Th}