Theorems – 1

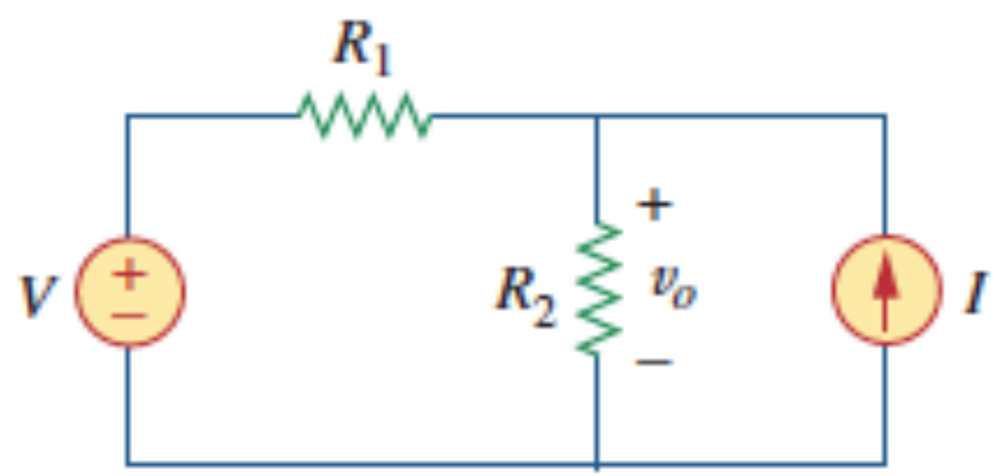
linearity & superposition;
transformations

Linearity & Superposition

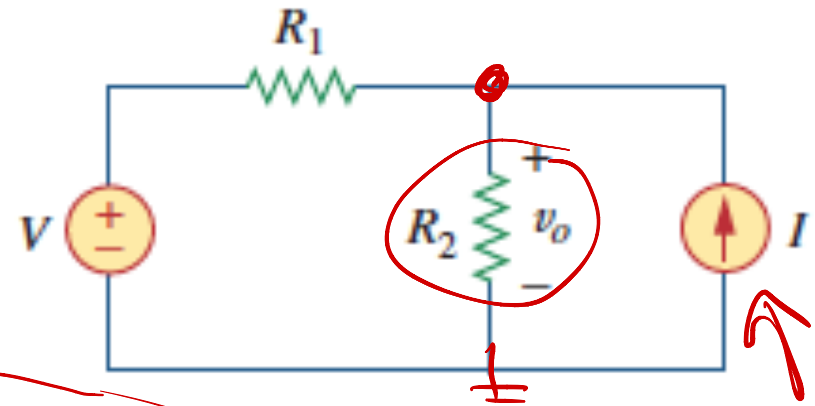
- Linearity: for a single "input" (voltage or current), the "output" (voltage or current) is proportional to that input

$$v_o = k\,v_s$$

- What about multiple "input" sources?



- Analyzing



$$\frac{v_o - V}{R_1} + \frac{v_o}{R_2} - I = 0$$

or

$$v_o = \frac{R_1 R_2}{R_1 + R_2} I + \frac{R_2}{R_1 + R_2} V$$

- And the idea extends to multiple "input" sources

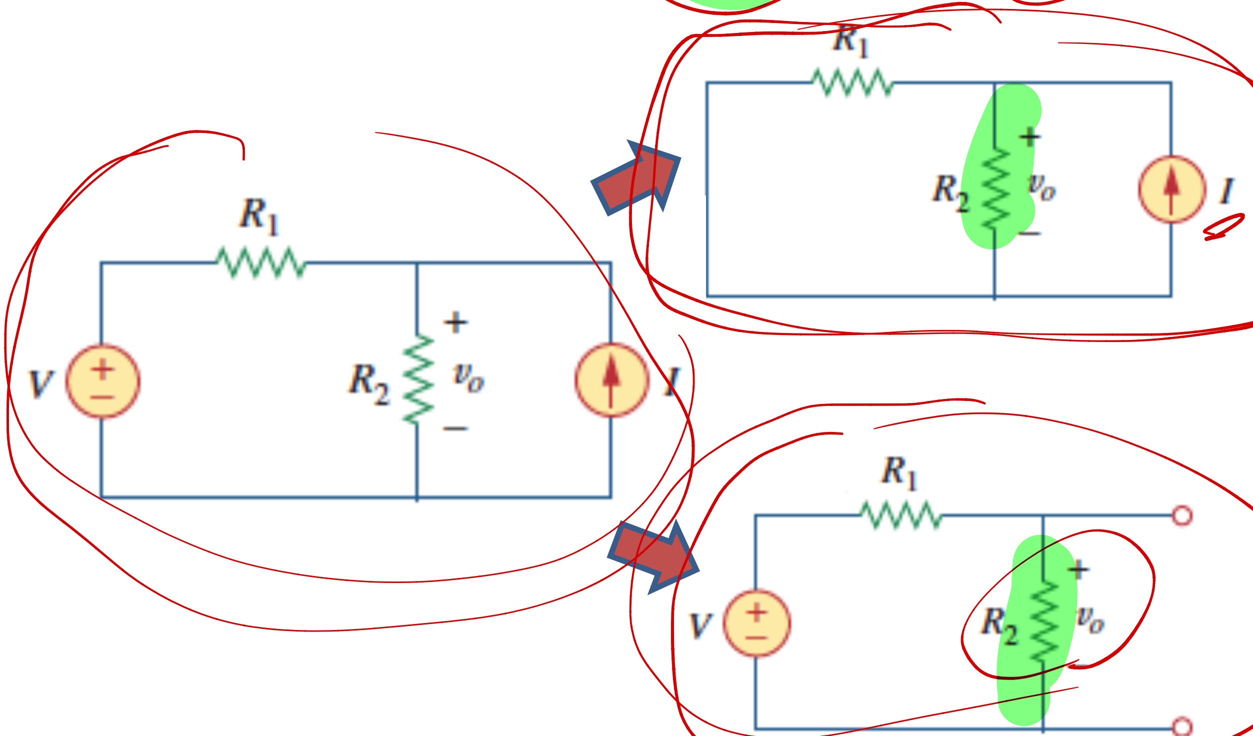
$$v_o = k_1 v_{s1} + k_2\, v_{s2}$$

"Superposition"

- We can exploit this idea to decompose problems:

  – Example:

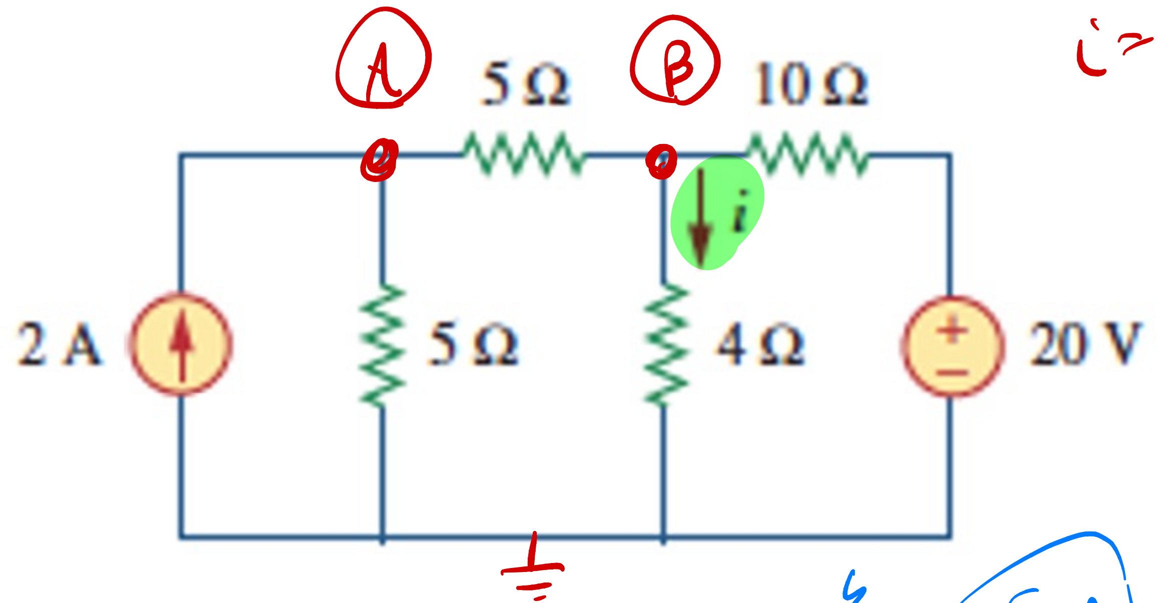
$$v_o(I,V) = \frac{R_1 R_2}{R_1+R_2} I + \frac{R_2}{R_1+R_2} V = v_o(I,0) + v_o(0,V)$$



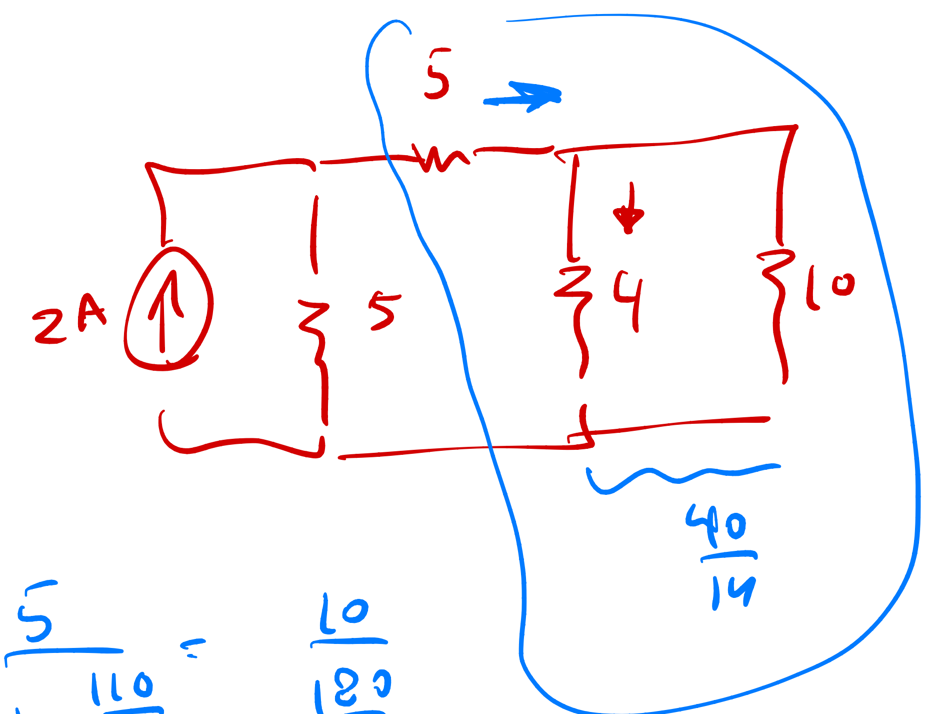
$$v_o(I,0) = \frac{R_1 R_2}{R_1 + R_2} I$$

$$v_o(0,V) = \frac{R_2}{R_1 + R_2} V$$

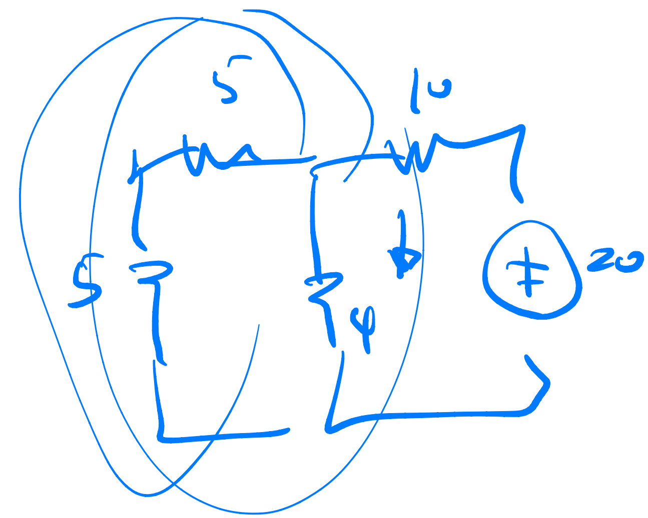
Example: find i



$i = \frac{B}{4}$



$\frac{7}{9} \cdot \frac{10}{14} = \frac{5}{9}$ A



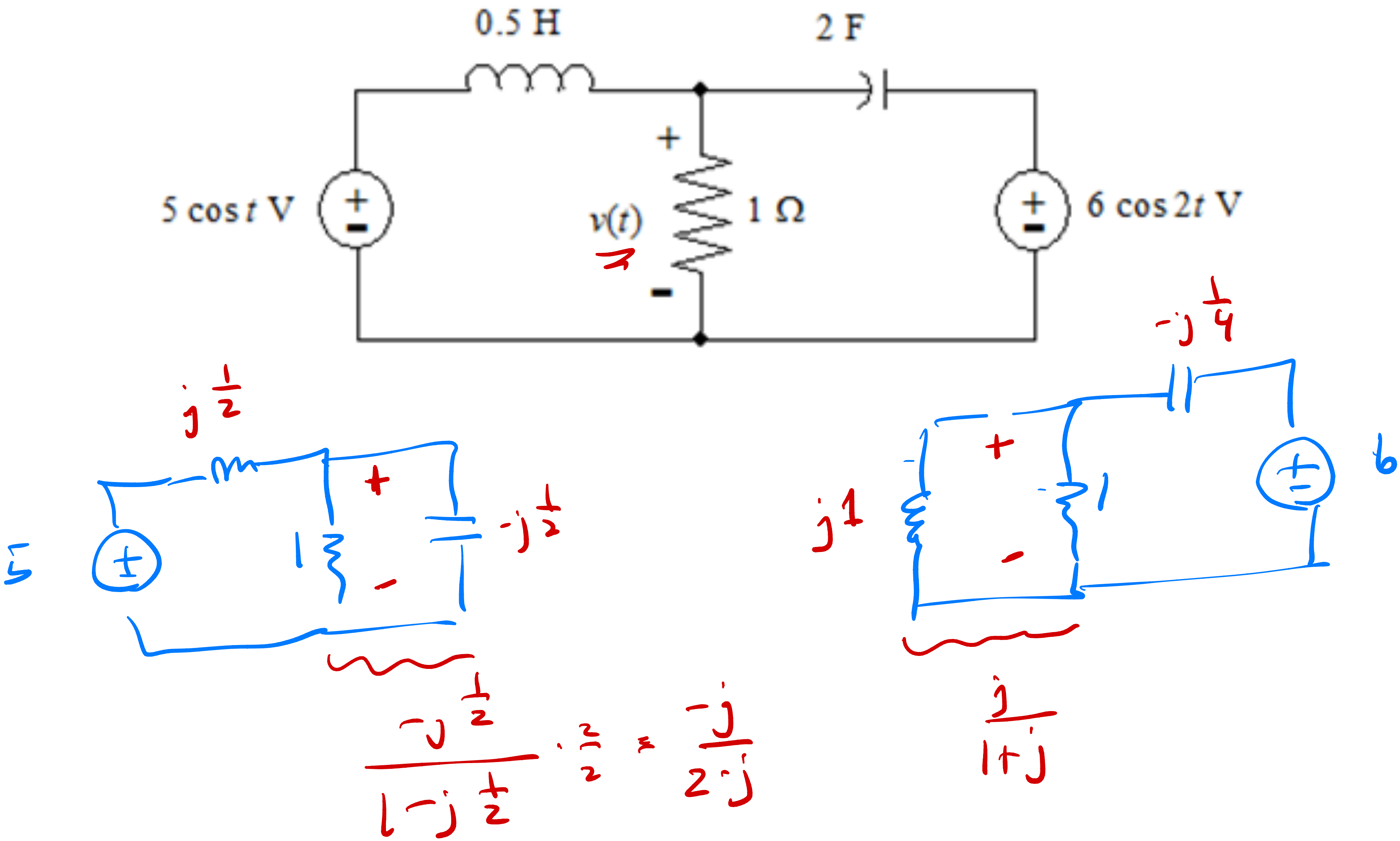
$5 + \frac{40}{14} = \frac{110}{14}$

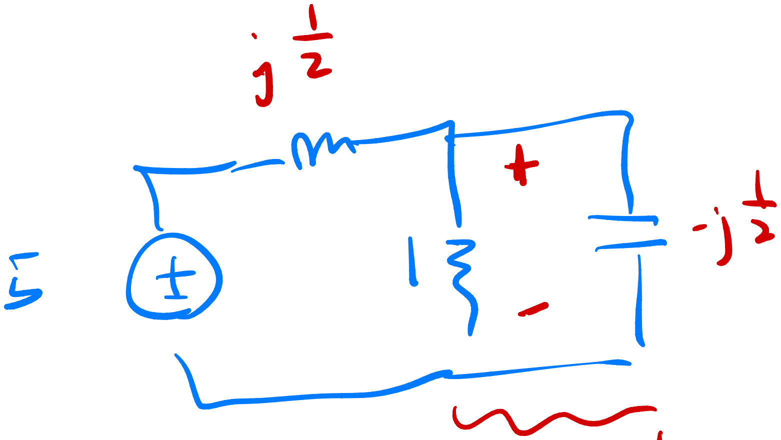
$2 \cdot \frac{5}{5 + \frac{110}{14}} = \frac{10}{\frac{180}{14}}$

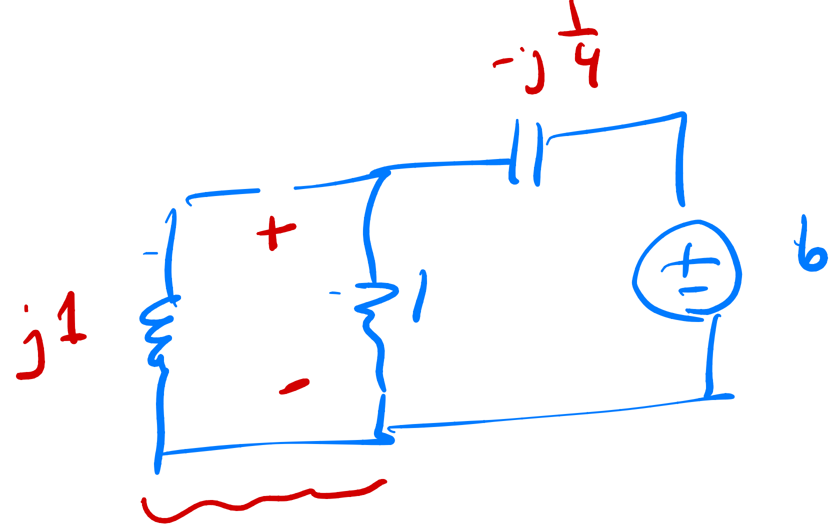
$= \frac{140}{180} = \frac{7}{9}$ A

$$i = 1.67\ A$$

Phasor example: note differet frequencies







$$\frac{-j\frac{1}{2}}{1-j\frac{1}{2}}\cdot\frac{2}{2}=\frac{-j}{2-j}$$

$$\frac{j}{1+j}$$

$$v=5\cdot\frac{\frac{-j}{2-j}}{\frac{j}{2}-\frac{j}{2-j}}\quad\frac{2(2-j)}{2(2-j)}$$

$$=\frac{-10j}{j(2-j)-2j}=\frac{-10j}{1}$$

$$j2+1-j2=1$$

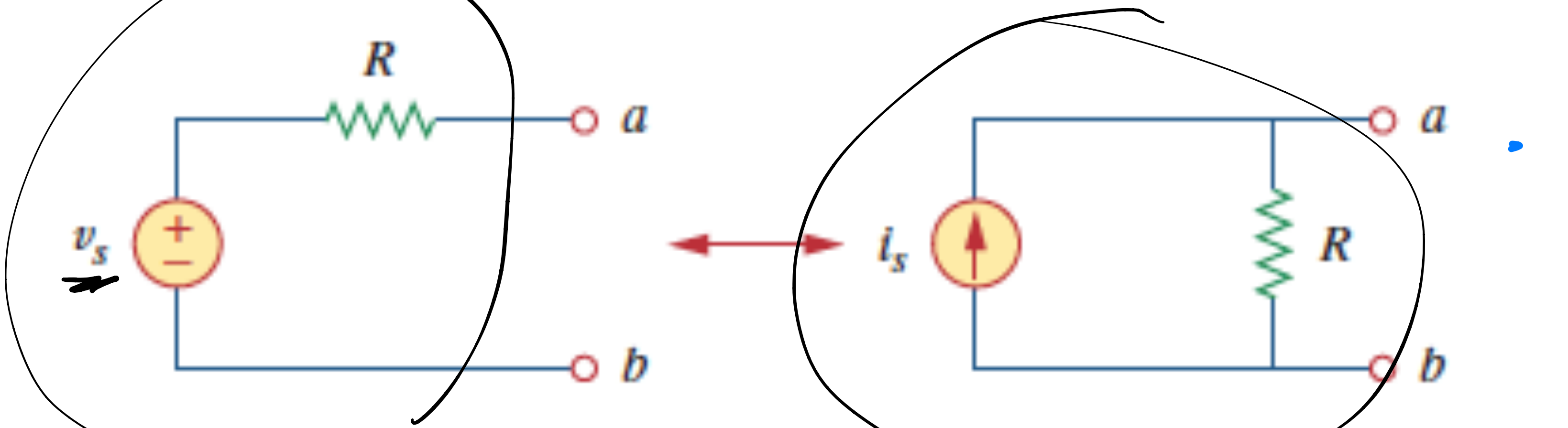
$$V=6\cdot\frac{\frac{j}{1+j}}{-\frac{j}{4}+\frac{j}{1+j}}\cdot\frac{4(1+j)}{4(1+j)}$$

$$=\frac{j24}{-j(1+j)+4j}=\frac{j24}{-j+1+4j}$$

$$1+3j$$

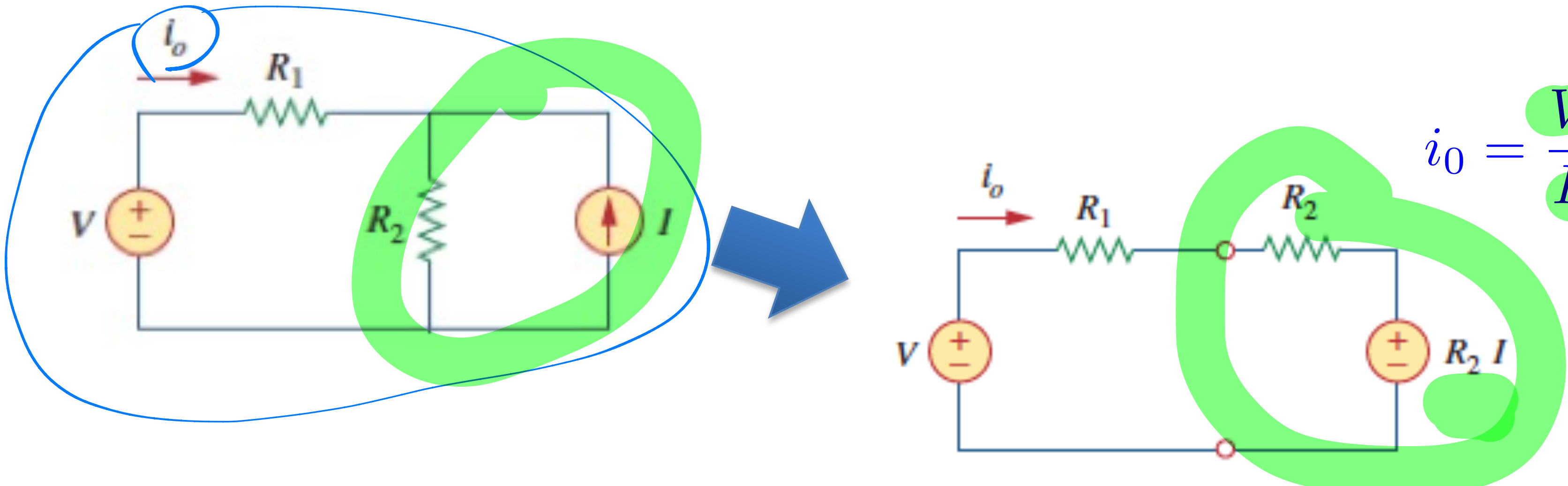
$v(t) = 10\cos(t-90°) + 1.2\cos(2t+127°)\ V$

Source Transformations



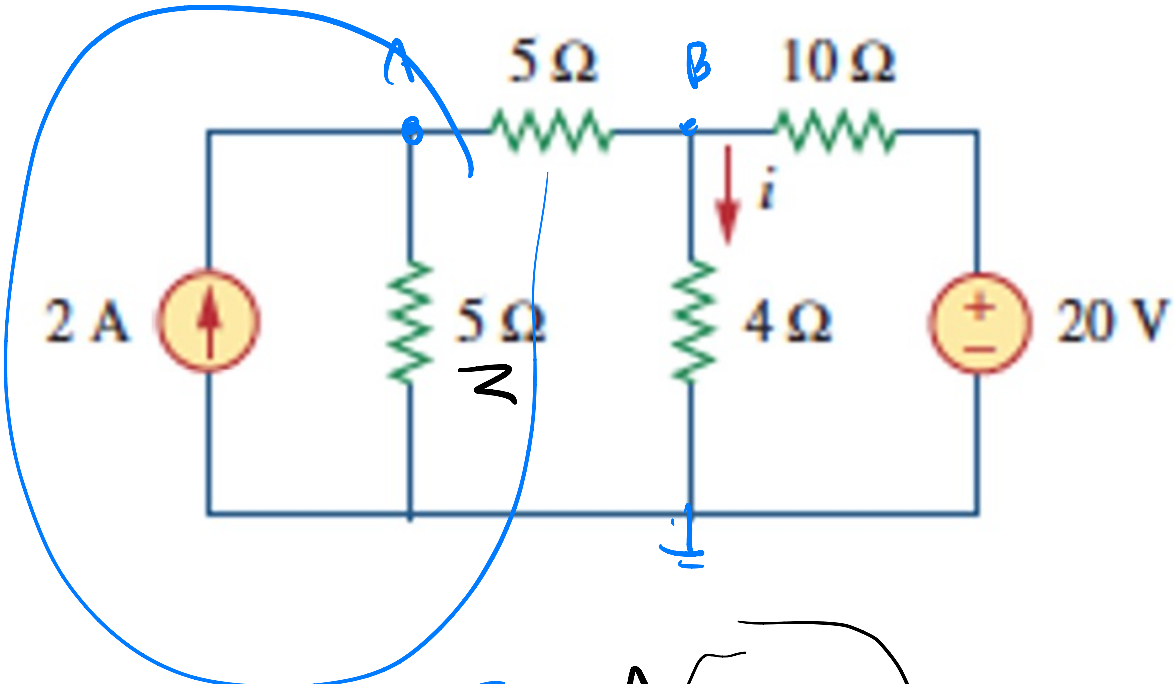
– These two sub-circuits are equivalent *at the terminals* a, b
  iff $v_s = R\, I_s$

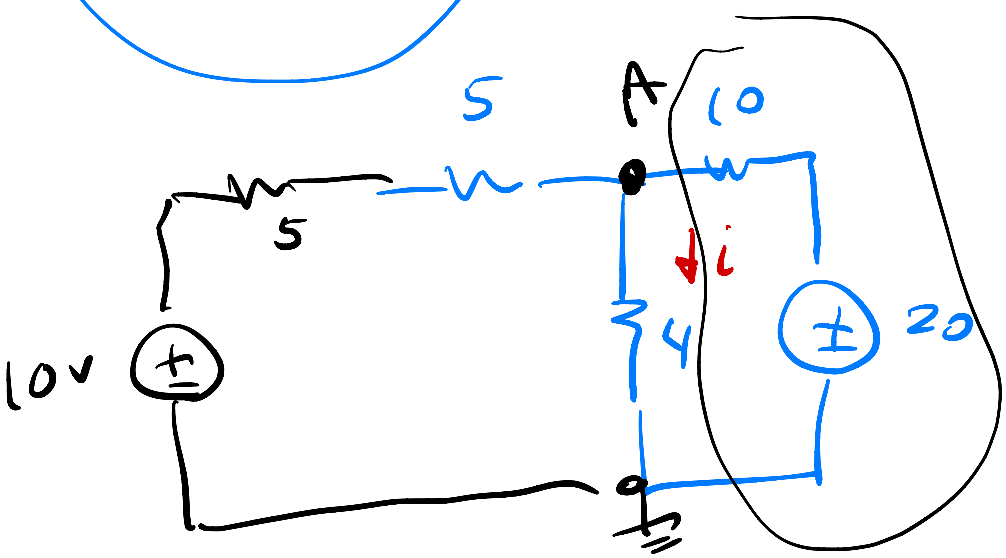
– Utility: simplify circuit for quick analysis



$$i_0 = \frac{V - R_2 I}{R_1 + R_2}$$

Example: find i (convert to just one node)





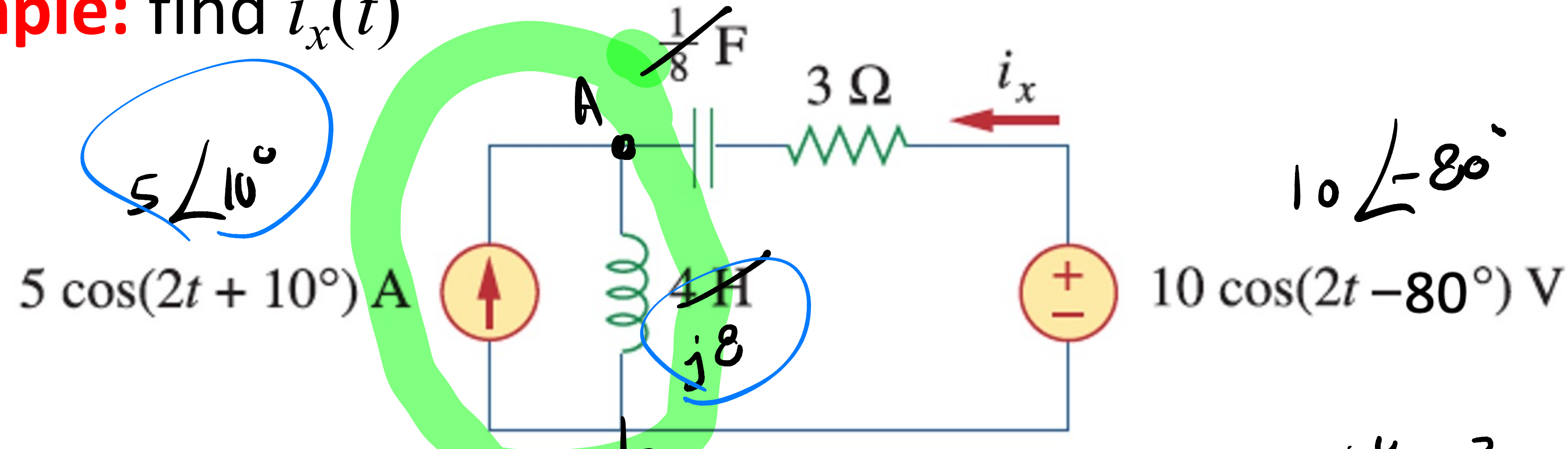
$$\frac{A}{4} + \frac{A-20}{10} + \frac{A-10}{10} = 0$$

$1.67\ A$

Example: find $i_x(t)$

$5\angle 10^\circ \cdot j8$

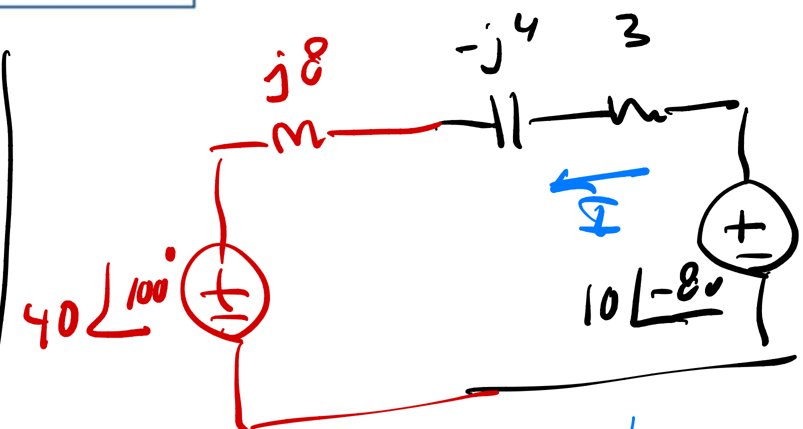
$8\angle 90^\circ$



$$\frac{A}{j8} + \frac{A - 10\angle 80}{3 - j4} - 5\angle 10 = 0$$

$A = \ldots$

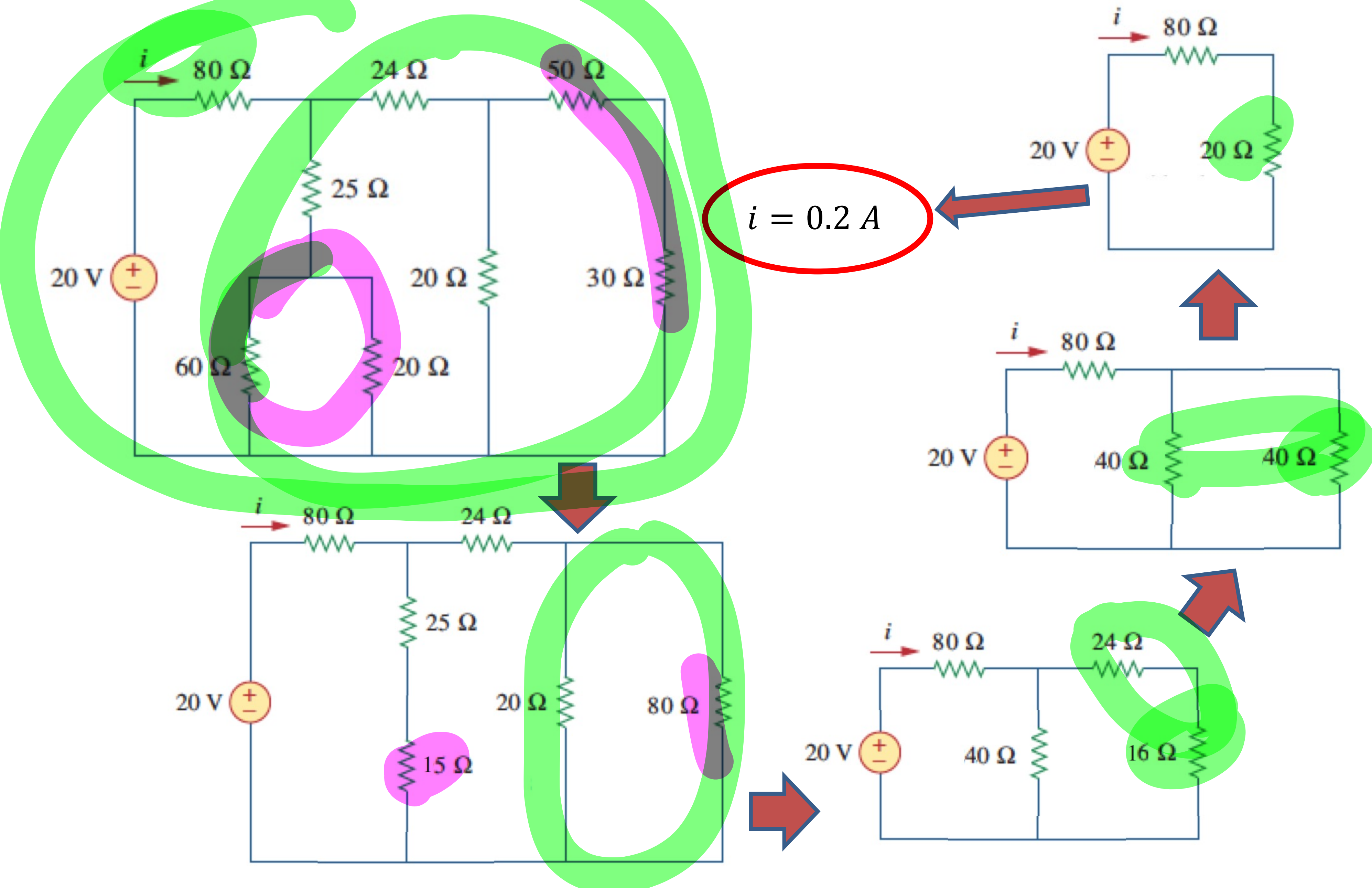
$$I = \frac{10\angle -80 - A}{3 - j4}$$



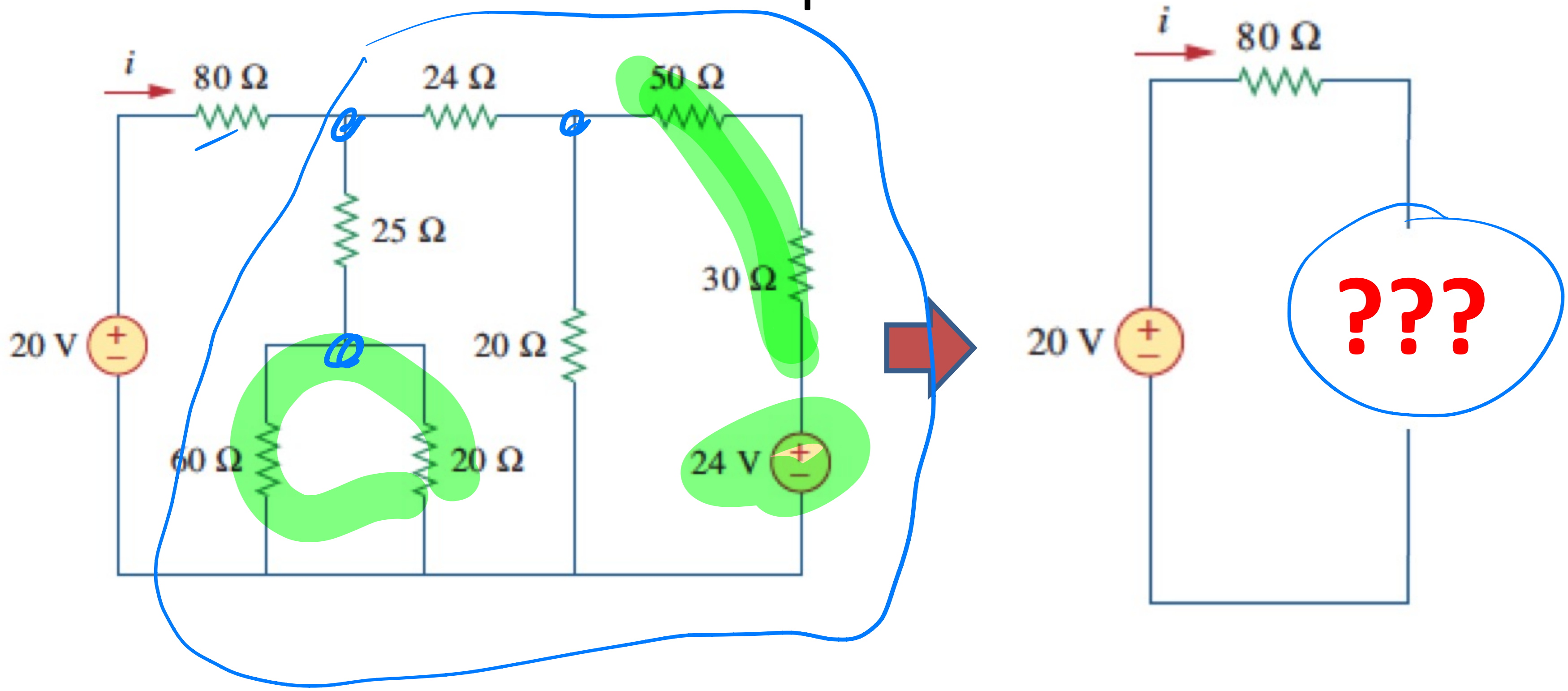
$$I = \frac{10\angle -80 - 40\angle 100}{3 + j4}$$

$$i_x(t) = 10\cos(2t + 174°)\ V$$

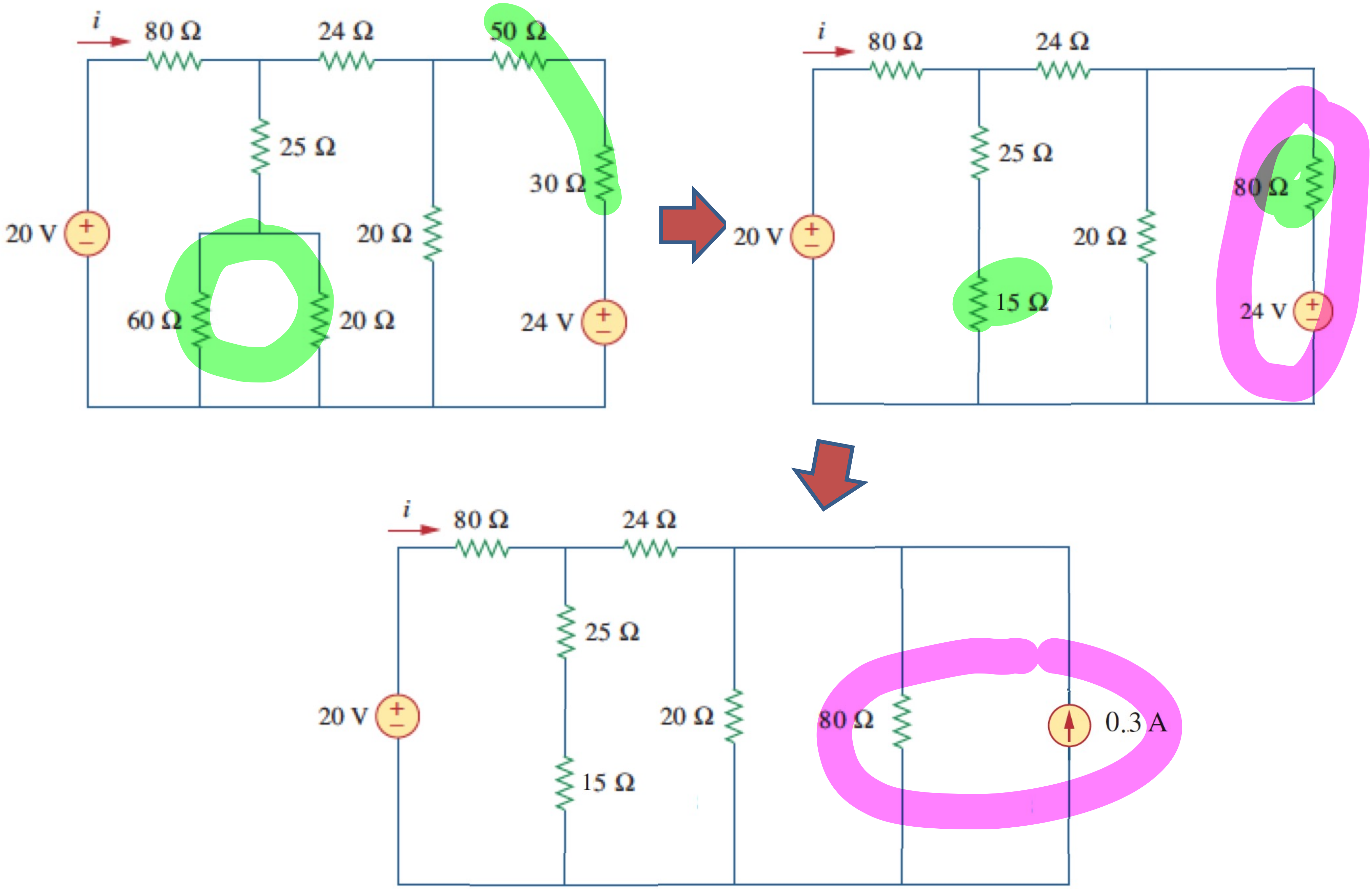
- To find i, recall series/parallel combining:



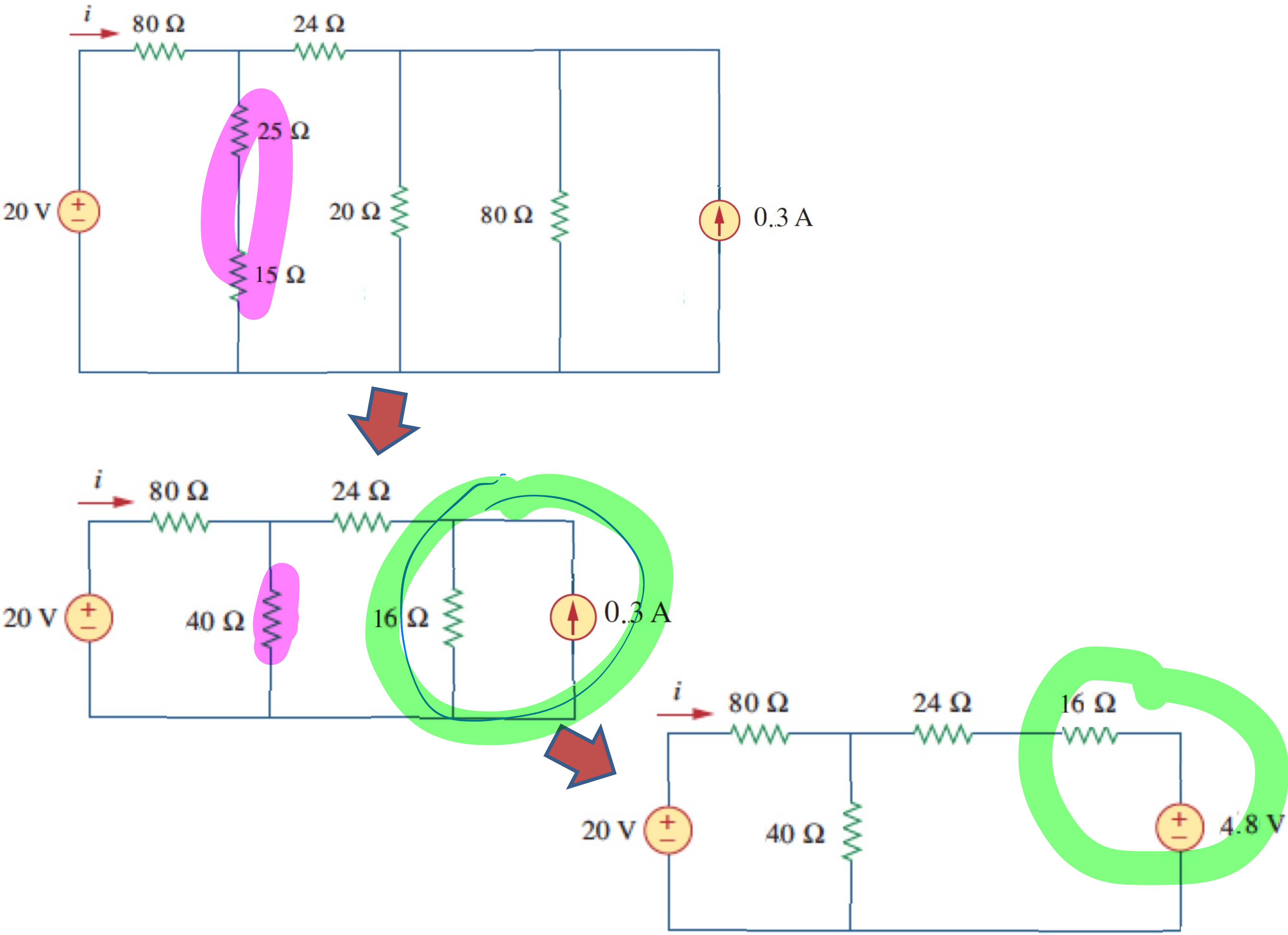
- What if there was a source present?



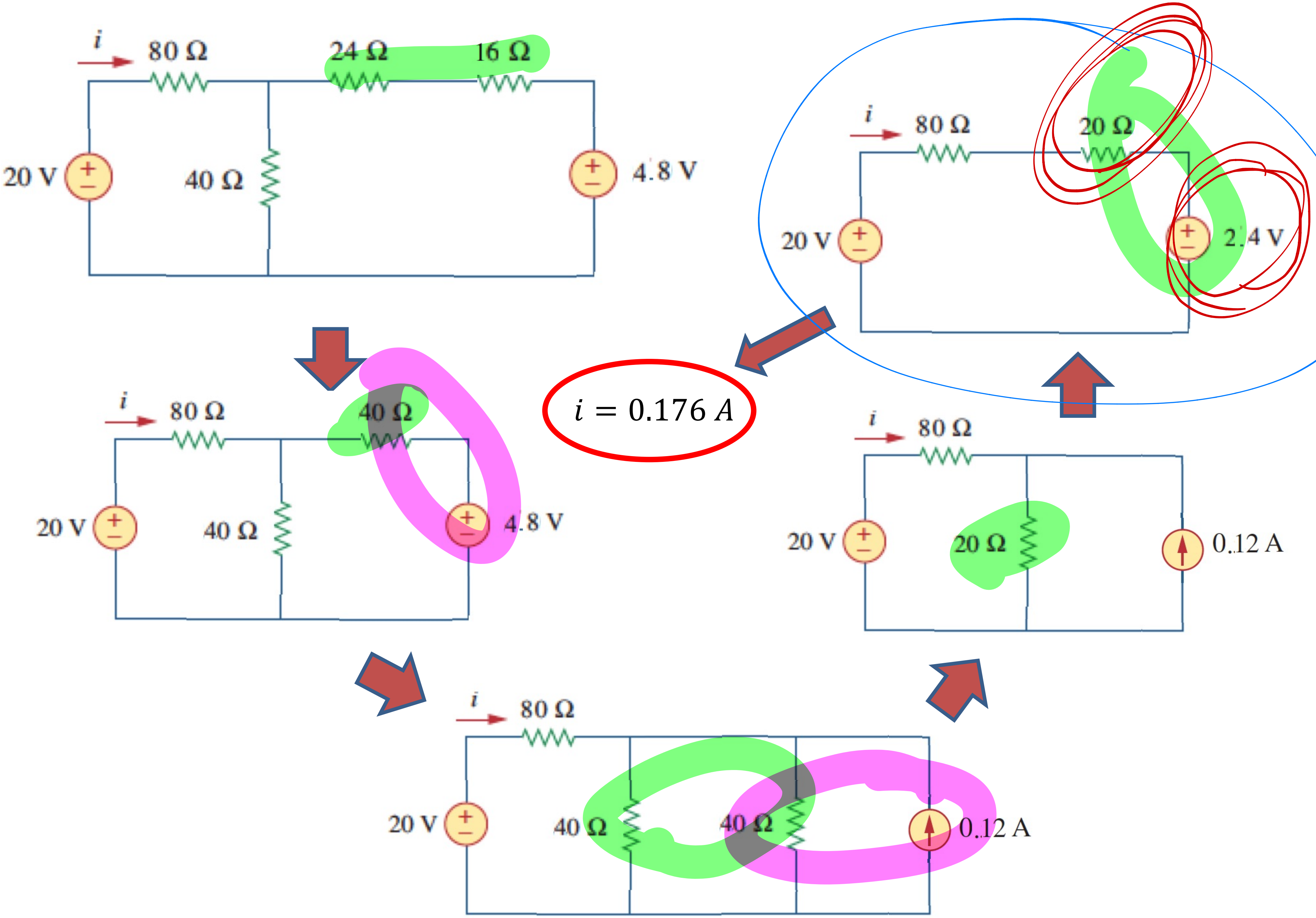
- Can combine transformations with series/parallel combining



i
80 Ω
24 Ω
50 Ω
25 Ω
30 Ω
20 V
20 Ω
60 Ω
20 Ω
24 V
i
80 Ω
24 Ω
25 Ω
80 Ω
20 V
20 Ω
15 Ω
24 V
i
80 Ω
24 Ω
25 Ω
20 V
20 Ω
80 Ω
0.3 A
15 Ω

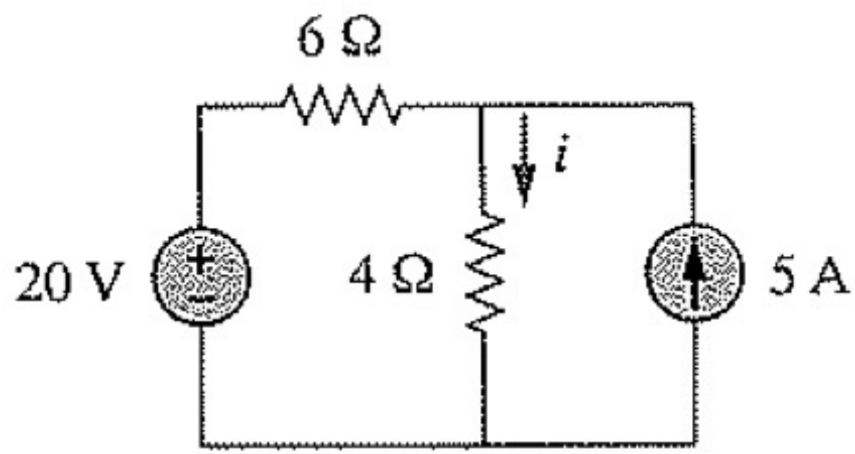


i
80 Ω
24 Ω
25 Ω
15 Ω
20 V
20 Ω
80 Ω
0.3 A
i
80 Ω
24 Ω
20 V
40 Ω
16 Ω
0.3 A
i
80 Ω
24 Ω
16 Ω
20 V
40 Ω
4.8 V



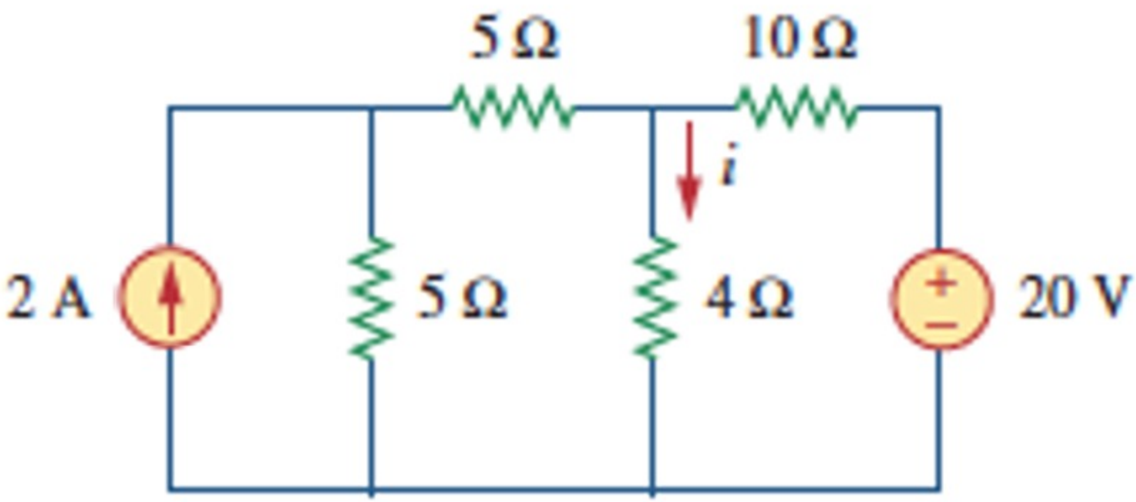
$$i = 0.176\ A$$

Example: find i (convert to parallel current sources and then current division)



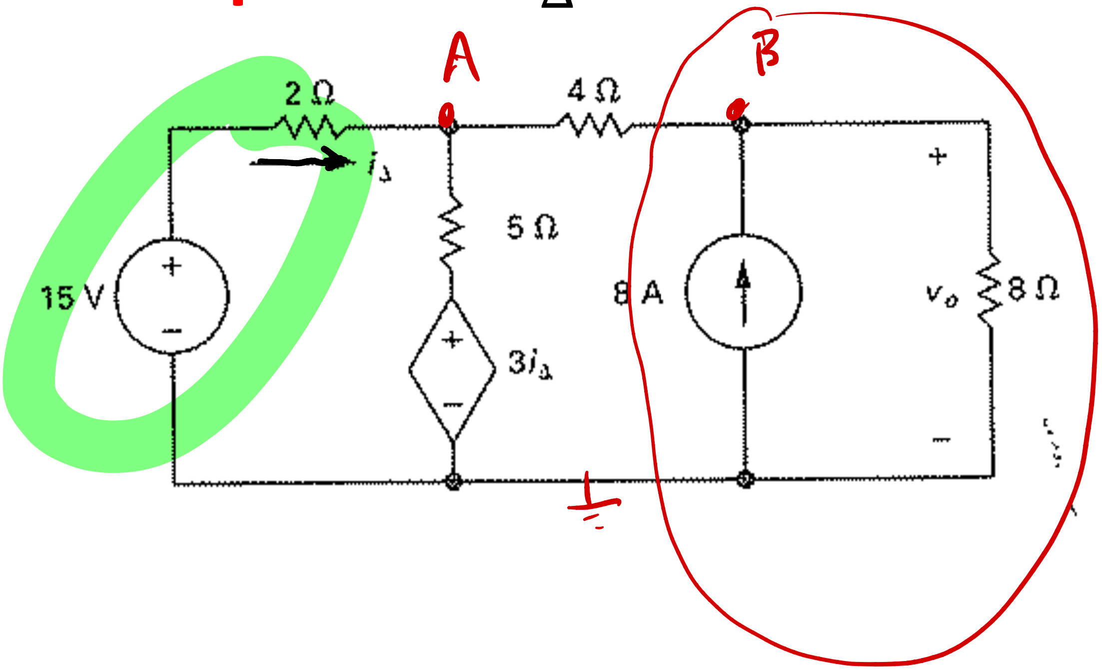
$i = 5\ A$

Example from above: find i (use current division)



$1.67\ A$

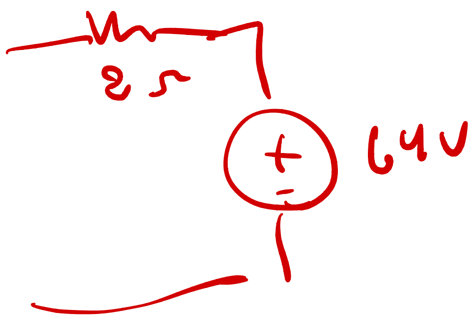
Example: find i_Δ

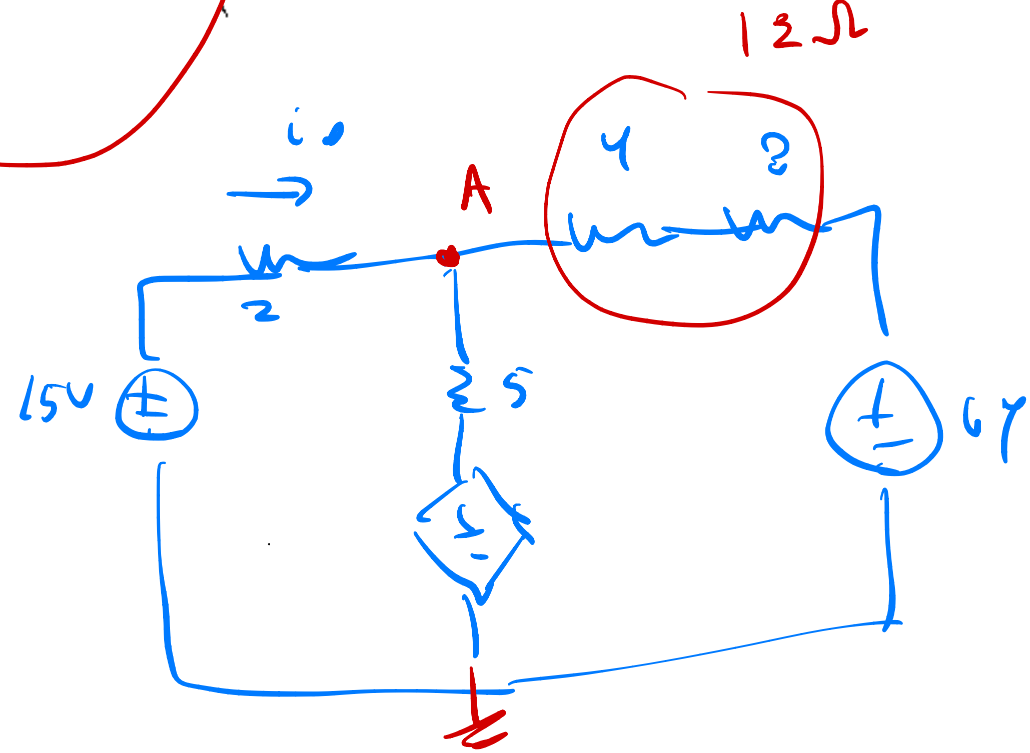


$$\frac{A-15}{2}+\frac{A-B}{4}+\frac{A-3i_\Delta}{5}=0$$

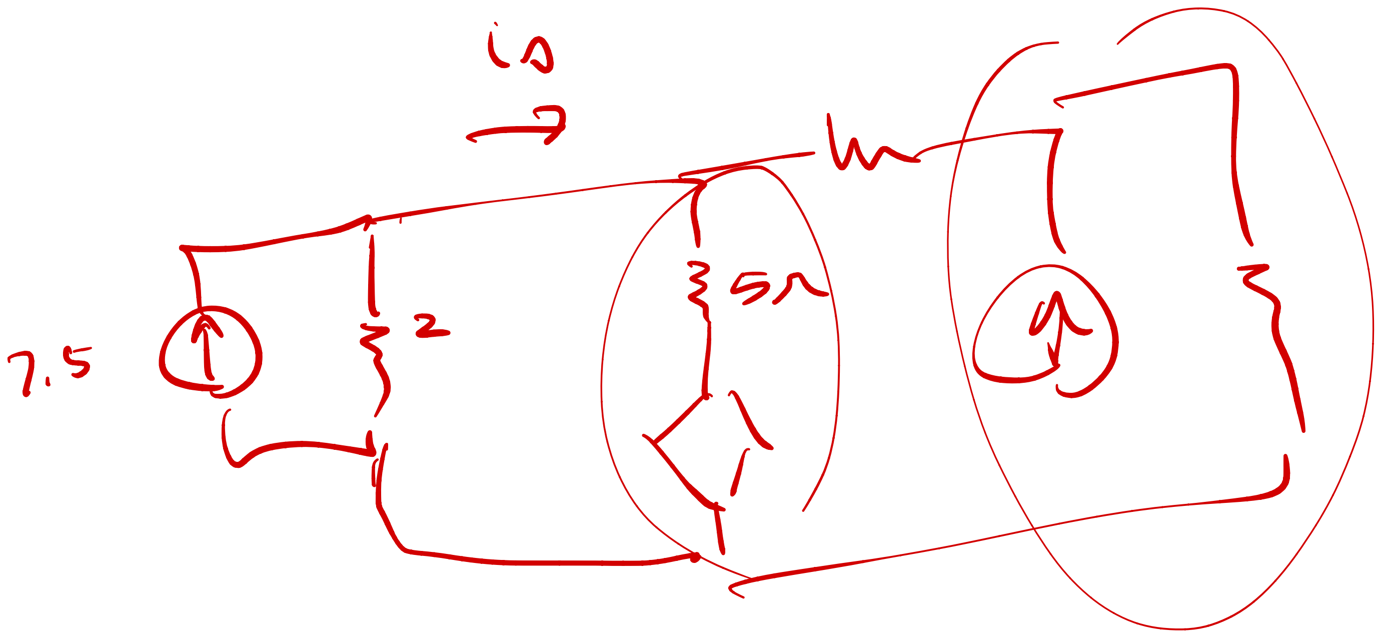
$$\frac{B-A}{4}+\frac{B}{8}-8=0$$

$$i_\Delta=\frac{15-A}{2}$$



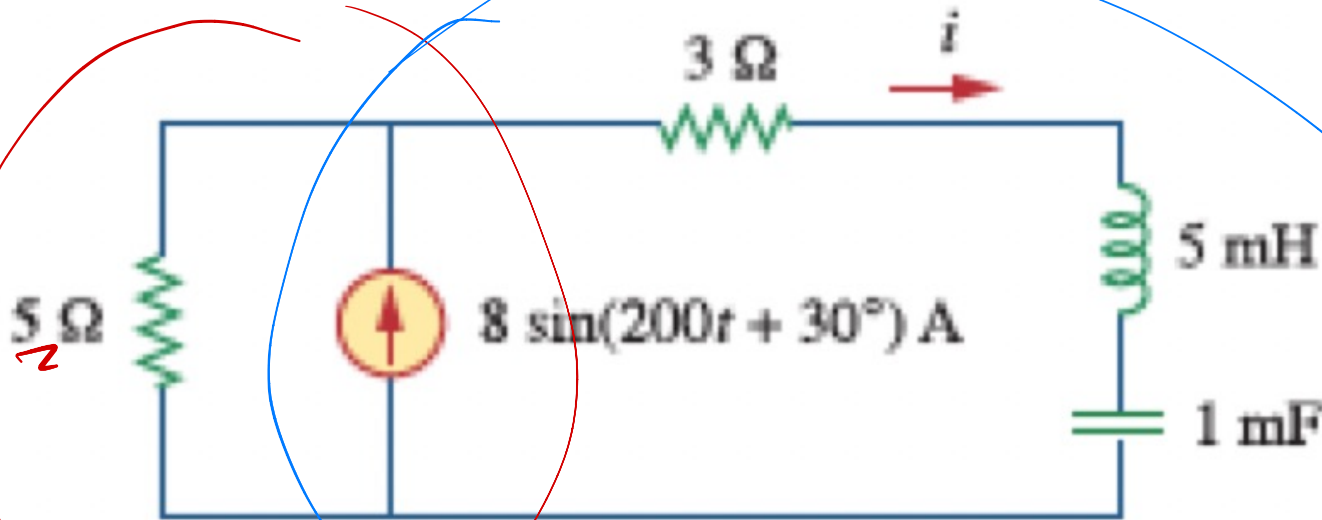


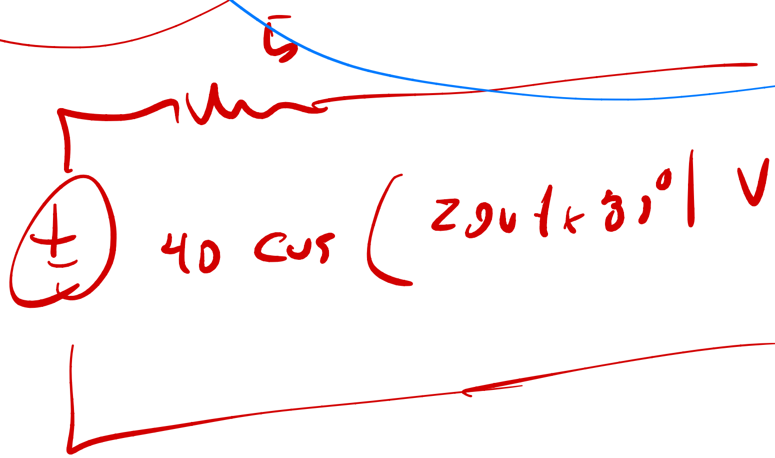
$$i_\Delta = \frac{223}{130}\ A$$



$$I = \frac{40\angle 30°}{5+3+j\omega L+\frac{1}{j\omega C}}$$

Example: find i

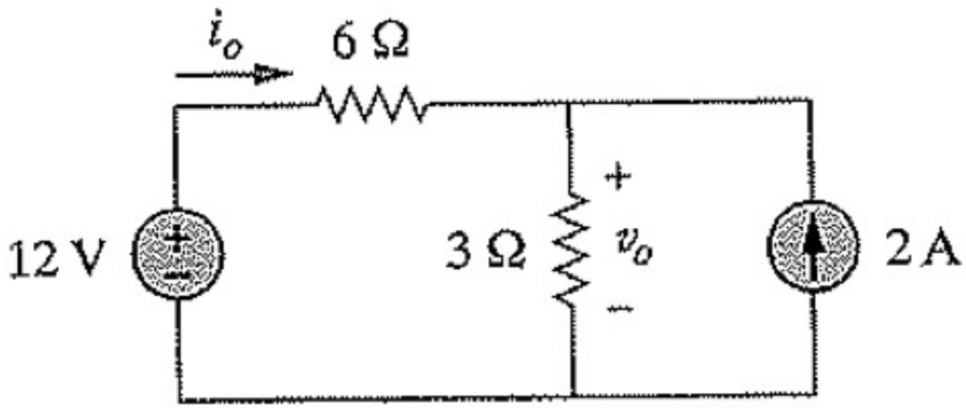




$$i(t) = 8.94\cos(200t + 56.6°)\ A$$

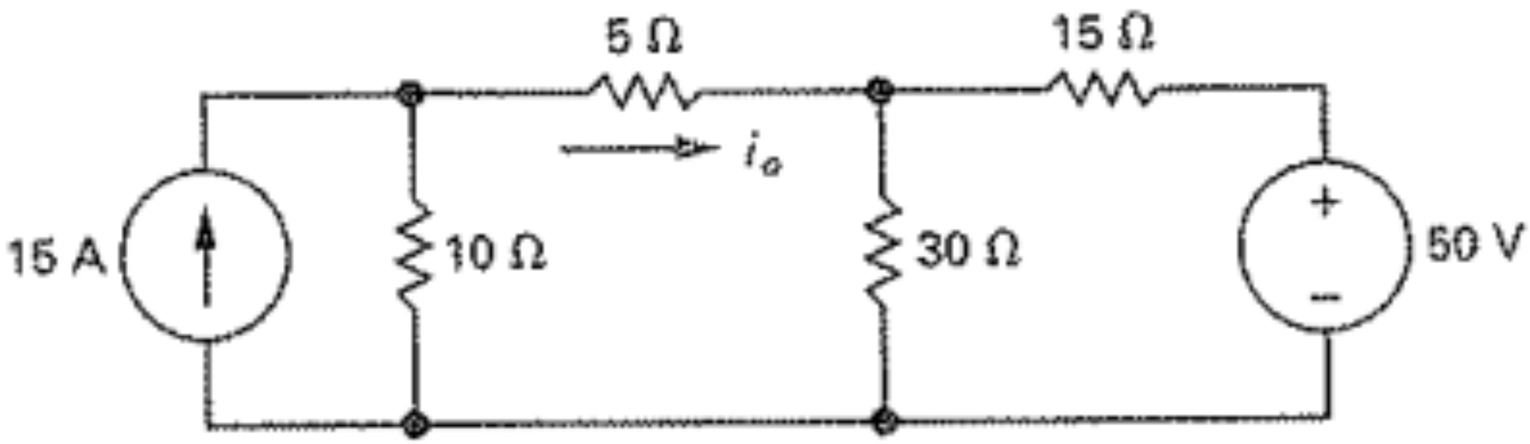
Practice problem: find v_o and i_o

$v_o = 8\ V, \qquad i_o = \frac{2}{3}\ A$



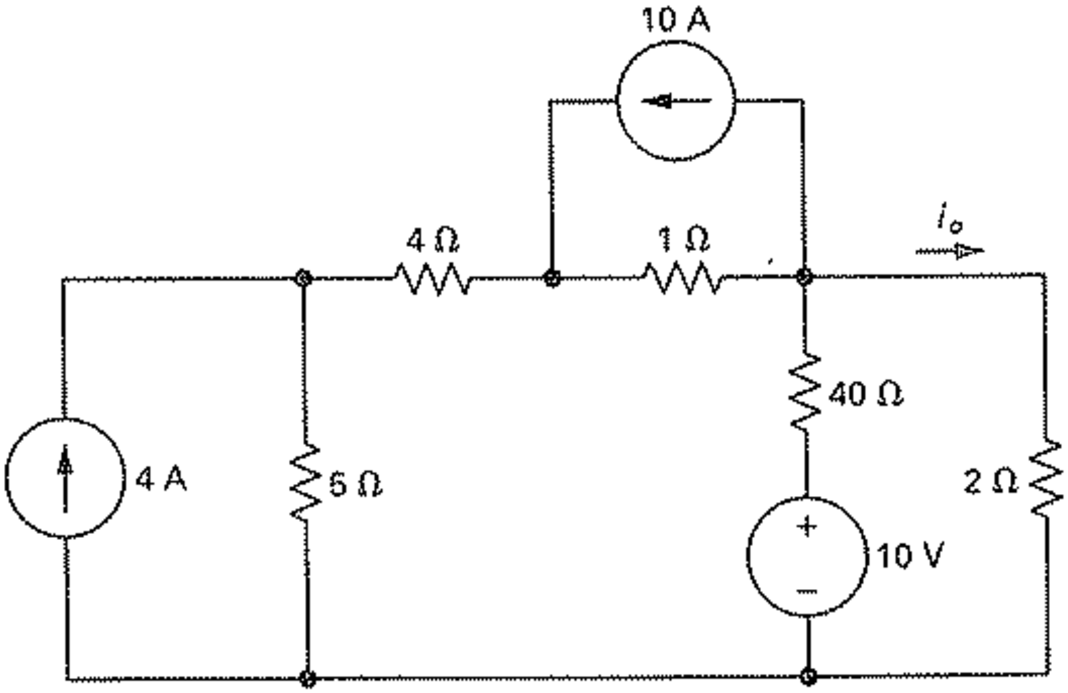
$i_o = \frac{14}{3} A$

Practice problem: find i_o



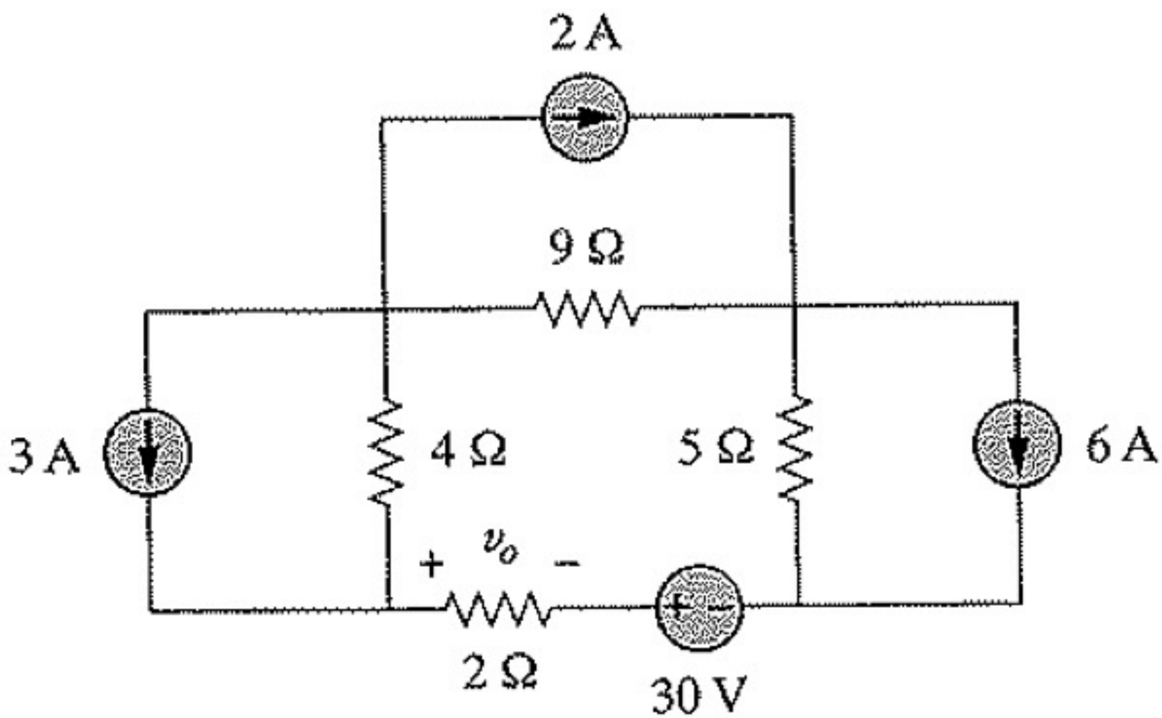
$i_o = 1.79\ A$

Practice problem: find i_o



Practice problem: find v_o

$v_o = -11\ V$



Practice problem: find v_x if $= v_s\,(t) = 50\cos(2t + 90°)$ V and $i_s(t) = 12\,cos(2t + 10°)$ A

$v_x(t) = 129\cos(2t + 28.76°)\,V$