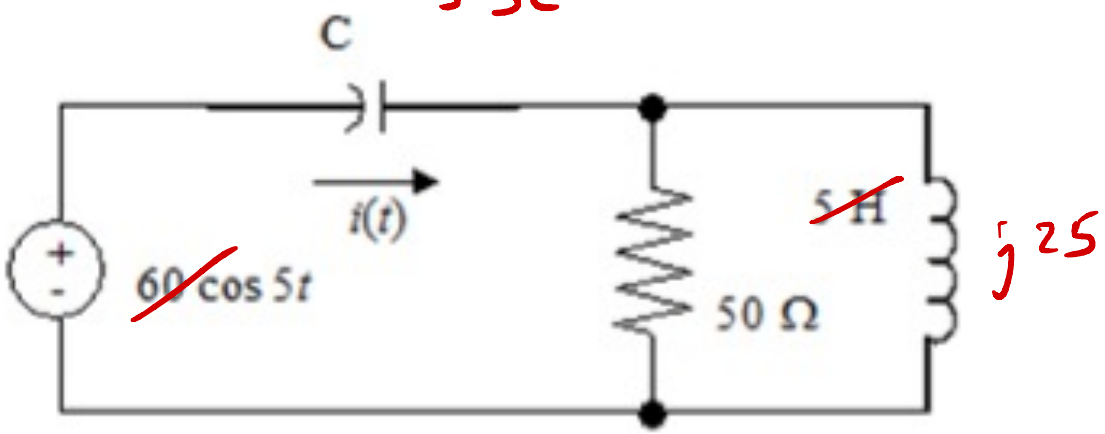
Phasors 9

design examples

Example: Find C so that the magnitude of $i(t)$ is 4 amps.



$-j\frac{1}{5C}$

60

$j25$

$\frac{(50)(j25)}{50+j25} = \frac{j50}{2+j} \cdot \frac{2-j}{2-j} = \frac{10+j20}{5}$

$Z = -j\frac{1}{5C} + 10 + j20$

$I = \frac{60}{Z} = \frac{60}{10+j\left(20-\frac{1}{5C}\right)}$

$\left|10+j\left(20-\frac{1}{5C}\right)\right| = 15$

```
om = 5;
R = 50;
L = 5;
ZL = 1j*om*L;
C = logspace(-9,0,1000);
ZC = 1./(1j*om*C);
Z = ZC + 50*ZL/(50+ZL);
I = 60./Z;
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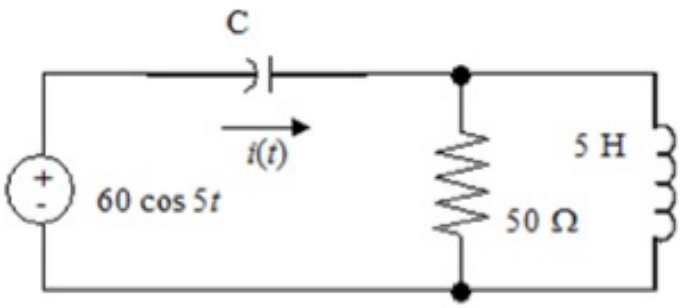
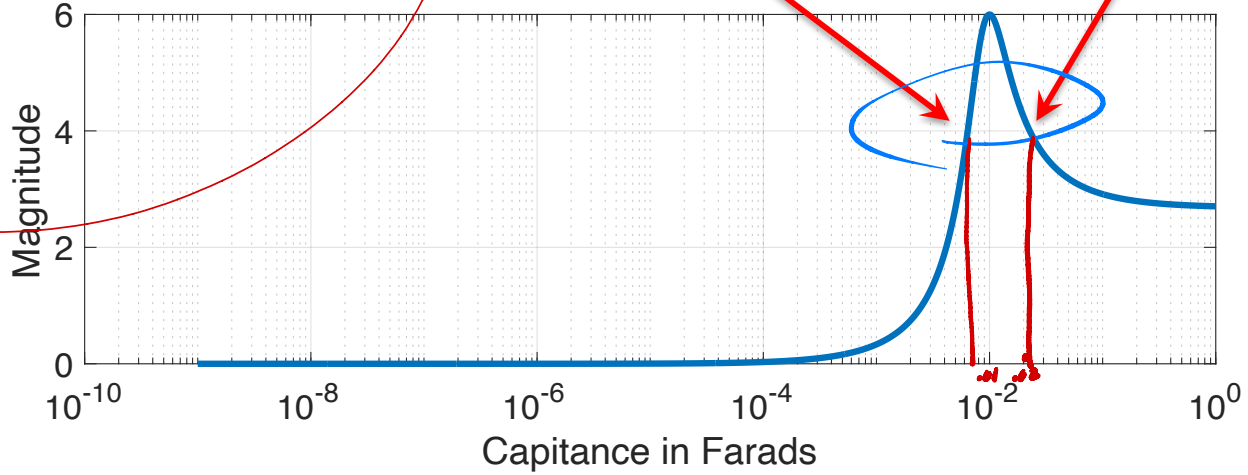
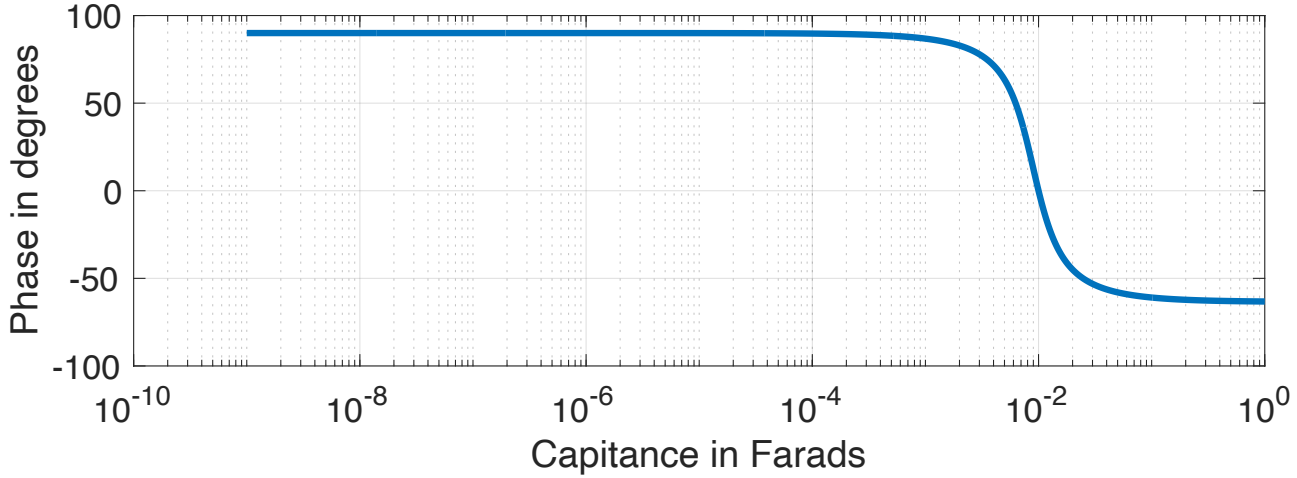
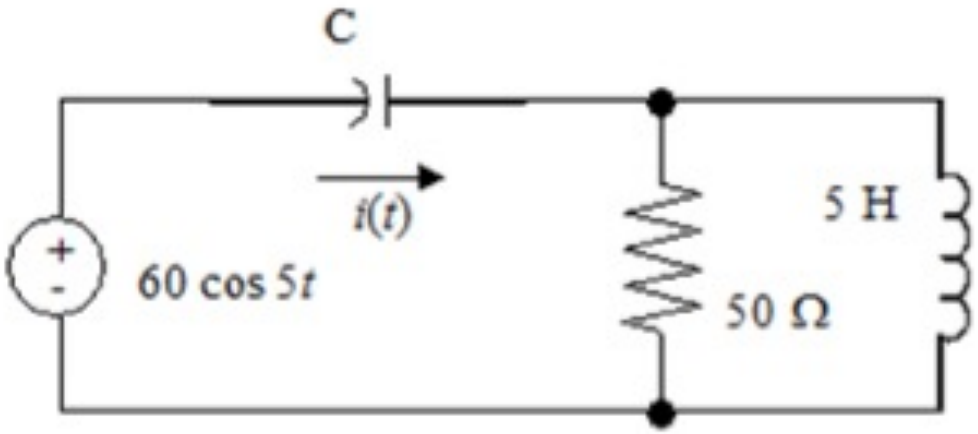


$|I| = 4$ occurs in two spots !!





Let's actually solve for C



$$\left| 10 + j\left(20 - \frac{1}{5C}\right) \right| = 15$$

$$\sqrt{100 + \left(20 - \frac{1}{5C}\right)^2} = 15$$

$$100 + \left(20 - \frac{1}{5C}\right)^2 = 225$$

$$\left(20 - \frac{1}{5C}\right)^2 = 125$$

$$20 - \frac{1}{5C} = \pm \underbrace{\sqrt{125}}_{11.1}$$

$$20 \mp 11.1 = \frac{1}{5C}$$

$$\frac{1}{5C} = 8.9 \quad \text{or} \quad 31$$

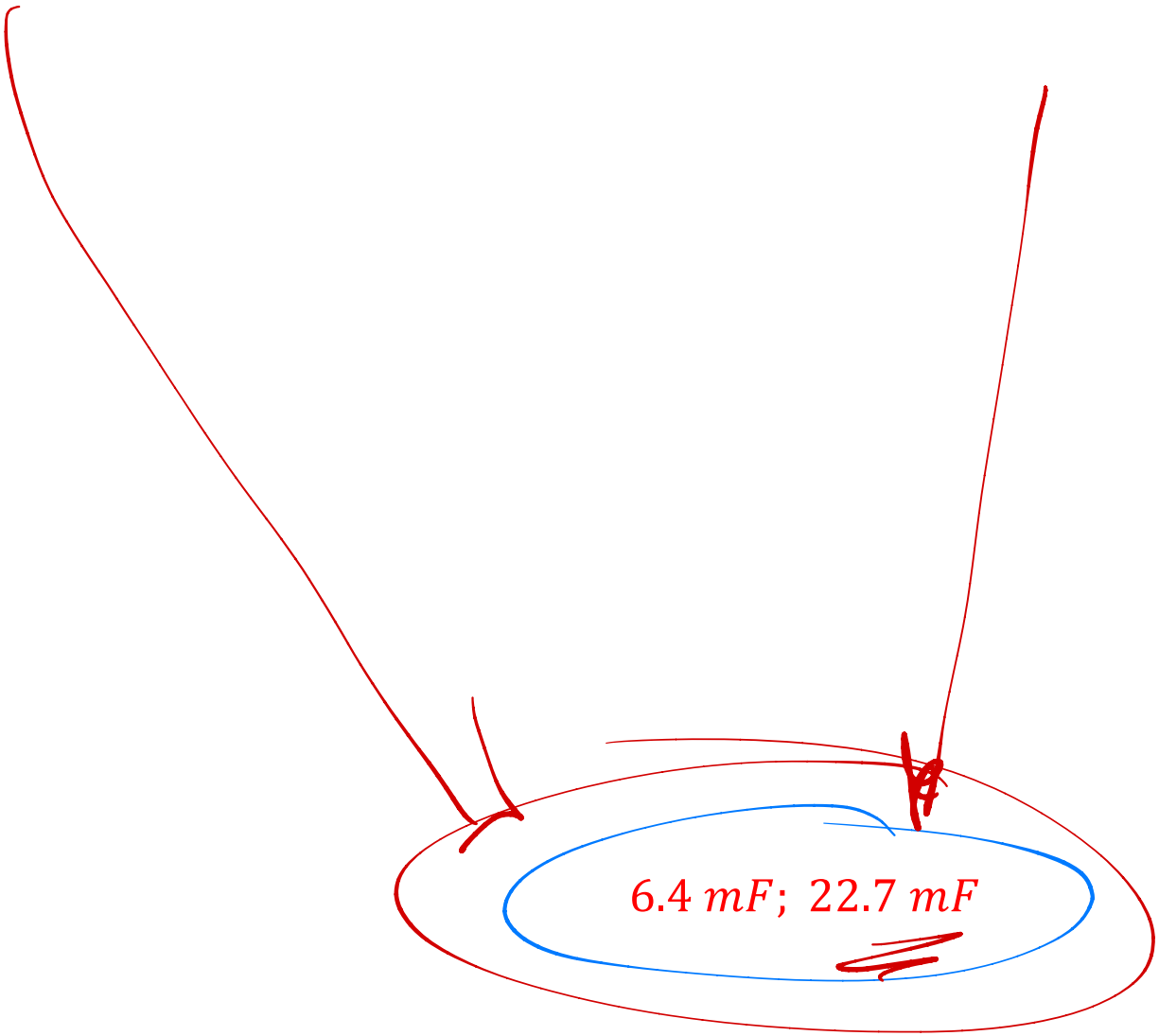
$$|a + bj| = \sqrt{a^2 + b^2}$$

$\frac{1}{5C} = 31$

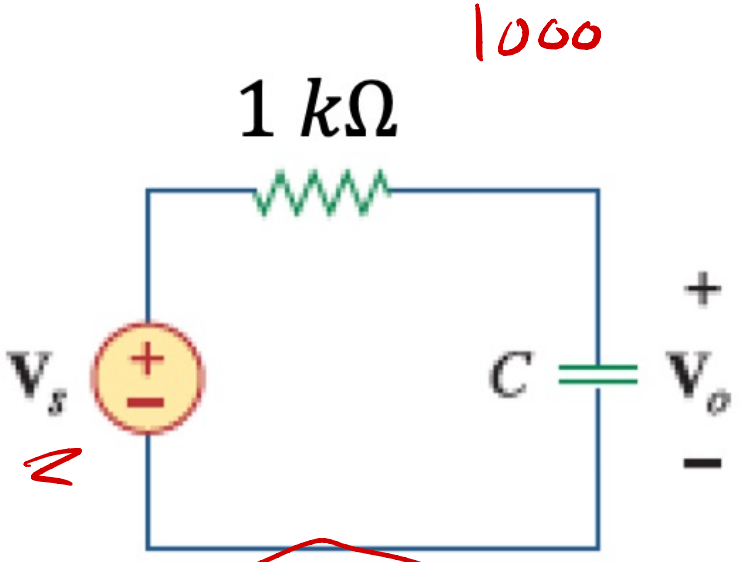
$C = \frac{1}{5.31} \approx \frac{1}{150}$

$\frac{1}{5C}$ 1 [illegible] 9

$C \approx \frac{1}{5.[illegible]5} = \frac{1}{4[illegible]}$



Example: The circuit shown is a low pass filter meaning that it passes lower frequency sinusoids and attenuates higher frequency sinusoids. Find a value for the capacitor C so that <u>all</u> sinusoids above 1000 Hz in frequency are attenuated by at least 90% (i.e. their amplitude scaling, |Vo/Vs|, is at most 0.1). Using your result what happens to a 60 Hz signal? How is its amplitude changed? How is its phase angle changed?

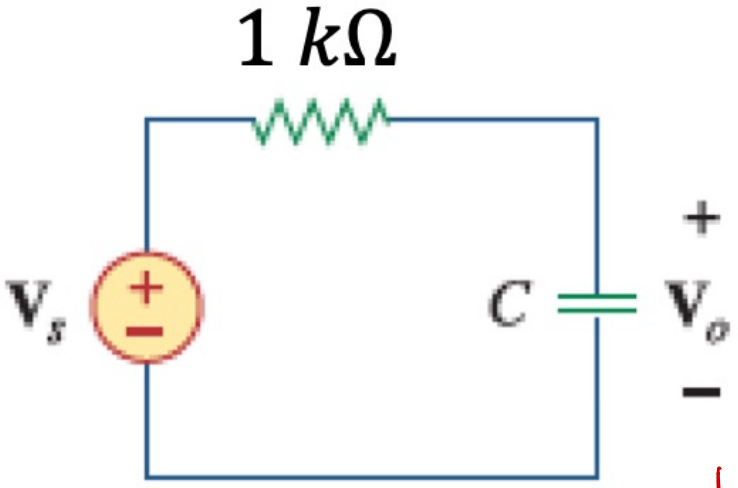


1000

$-j\frac{1}{\omega C}$

$\frac{V_o}{V_s} = \frac{-j}{1000\omega C - j} = \frac{1}{1 - j1000\omega C}$

$\frac{V_o}{V_s} = \frac{-j\frac{1}{\omega C}}{1000 - j\frac{1}{\omega C}}$



$$\frac{V_o}{V_s} = \frac{1}{1 + j\,1000\,\omega C}$$

$\omega = 1000$ $\quad \left|\frac{V_o}{V_s}\right| < .1$

$$\left|\frac{1}{1 + j\,10^6 \cdot C}\right| = .1$$

$$\frac{1}{\sqrt{1 + 10^6 C^2}} = .1$$

$$\frac{1}{1 + 10^6 C^2} = 10^{-2}$$

$$1 = 10^{-2} + 10^{10} C^2$$

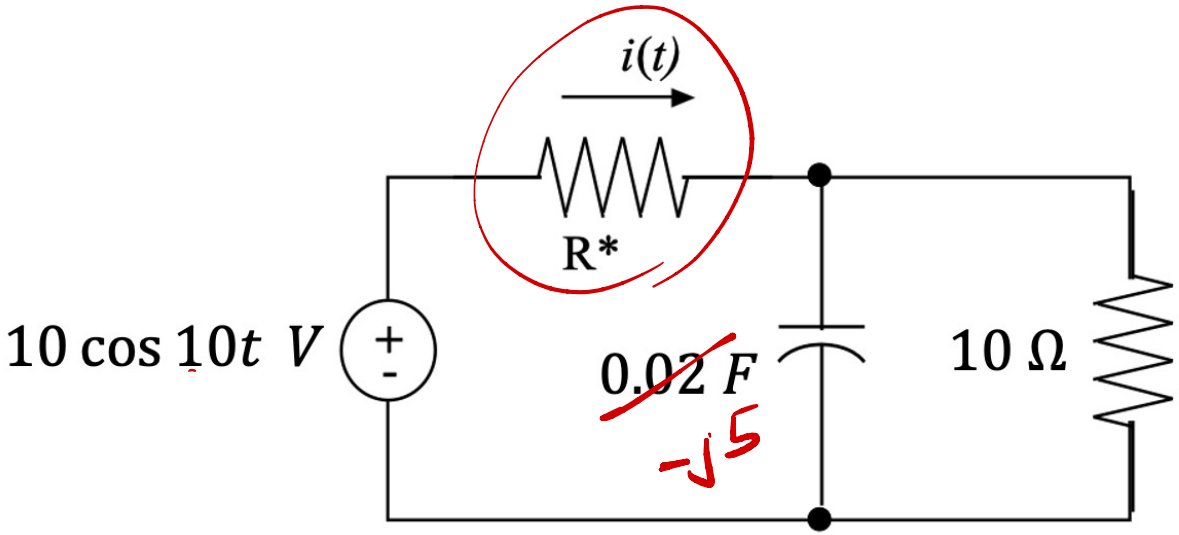
$$10^{10} C^2 = 1 - 10^{-2} = .99$$

$$10^{10} C^2 + 10^{-2} - 1 = 0$$

$$C^2 = .99\,10^{-10}$$

1.58 μF; 85.9 %; −20.8°

Example: Find the resistor R* so that the magnitude of the current $i(t)$ is 1 A.



$-j5$

$$\frac{(10)(-j5)}{10-j5} = \frac{-j10}{2-j} \cdot \frac{2+j}{2+j} = 2-j4$$

$$Z = R^* + 2 - j4$$

$$I = \frac{10}{R^* + 2 - j4} \quad \text{if} \quad \frac{10}{9-j4}$$

$$I = \frac{10}{R^* + 2 - j4}$$

97.9 Ω

$$|I| = \frac{10}{\sqrt{(R^* + 2)^2 + 16}} = 1$$

$$\sqrt{\quad} = 10$$

$$(R^* + 2)^2 + 16 = 100$$

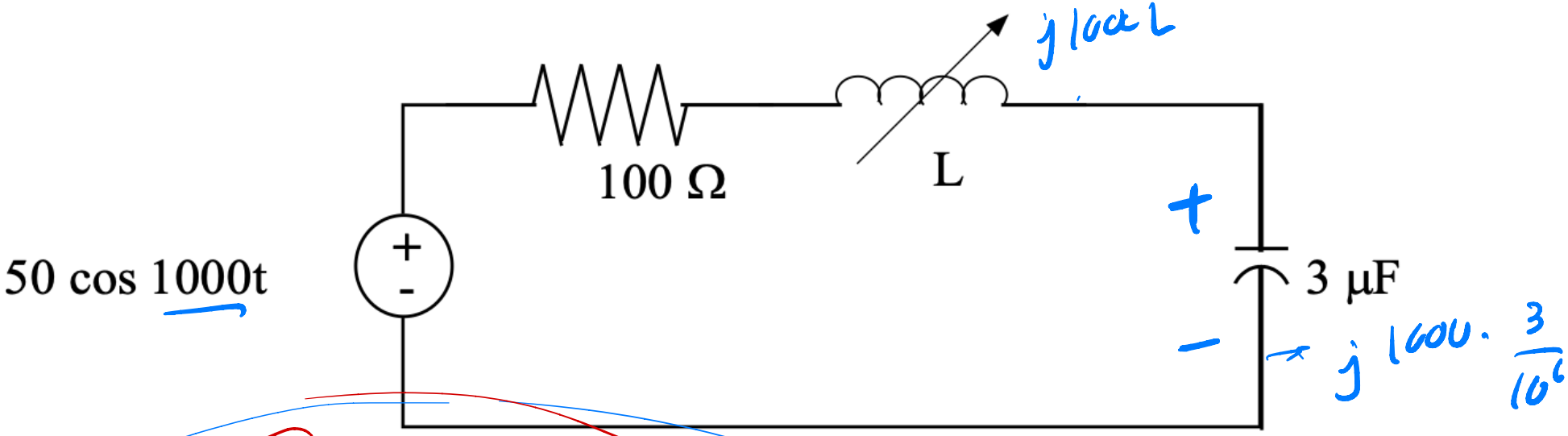
$$(R^* + 2)^2 = 84$$

$$R^* + 2 = \pm\sqrt{84} = \pm 9.1$$

$$R^* = -2 \pm 9.1$$

$R^* = 7\ \Omega$

Example: Find the inductor value to maximize the magnitude of the voltage appearing across the capacitor. What is that magnitude?



$j 1000 L$

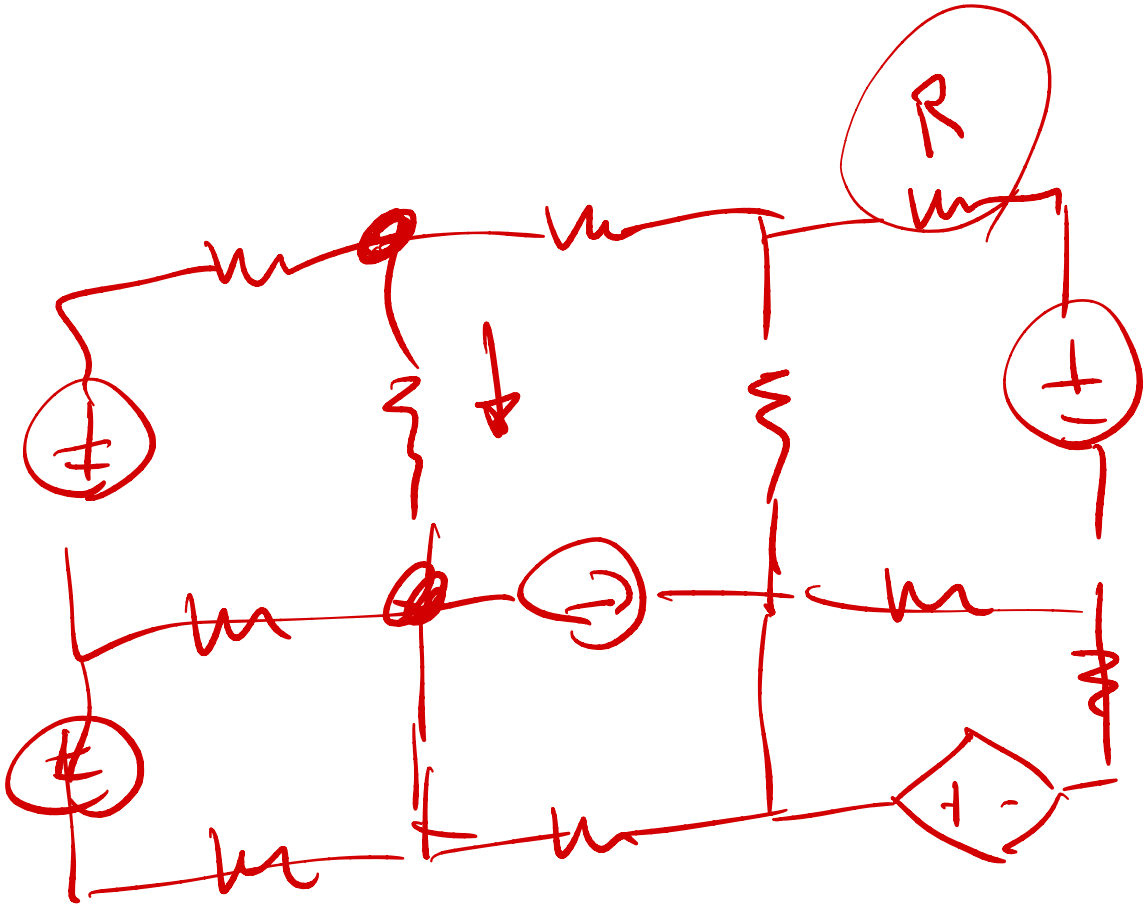
$-j \frac{3}{10^6} = -j \frac{3}{1000}$

$V_c = \dots$

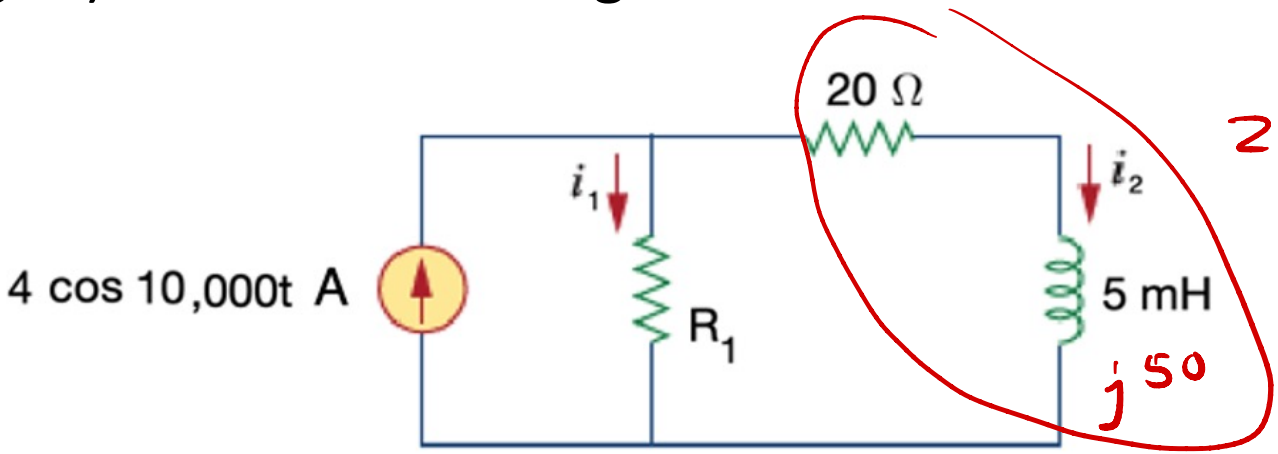
$50 \cdot \left(-j \frac{3}{1000}\right)$

$1000 + j 1000 L - j \frac{3}{1000}$

$\frac{1}{3}$ A, $\frac{10}{3}$ V



Example: what value for resistor R_1 results in the two currents, i_1 and i_2, having the same magnitude (but might be different phase angles)? What is that magnitude?



$20 + j50$

$$I_1 = \frac{4 \cdot (20 + j50)}{20 + R_1 + j50}$$

$$I_2 = \frac{4 \cdot R_1}{20 + R_1 + j50}$$

$$|I_1| = \frac{4 \cdot |20 + j50|}{|20 + R_1 + j50|}$$

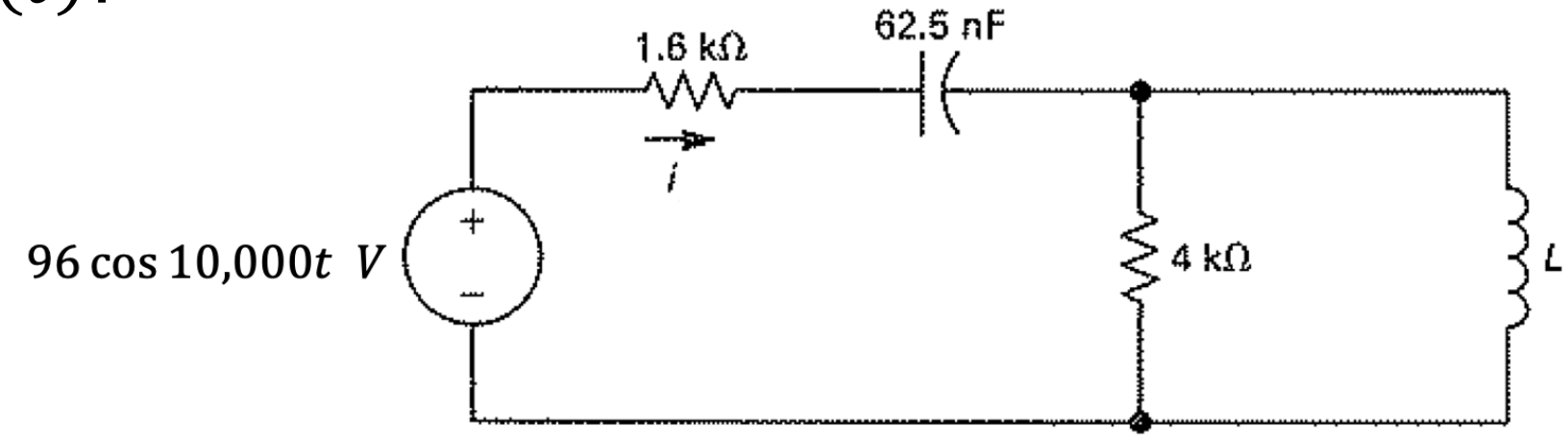
$$|I_2| = \frac{4 R_1}{|20 + R_1 + j50|}$$

$\sqrt{20^2}$

53.9 Ω; 2.42 A

$53.9\ \Omega; 2.42\ A$

Practice problem: find the inductor value so that the current i is "in phase" with the voltage source. What is $i(t)$?



$0.8\ H, 20\cos(10{,}000t)\ mA;\ 0.2\ H, 405\cos(10{,}000t)\ mA$

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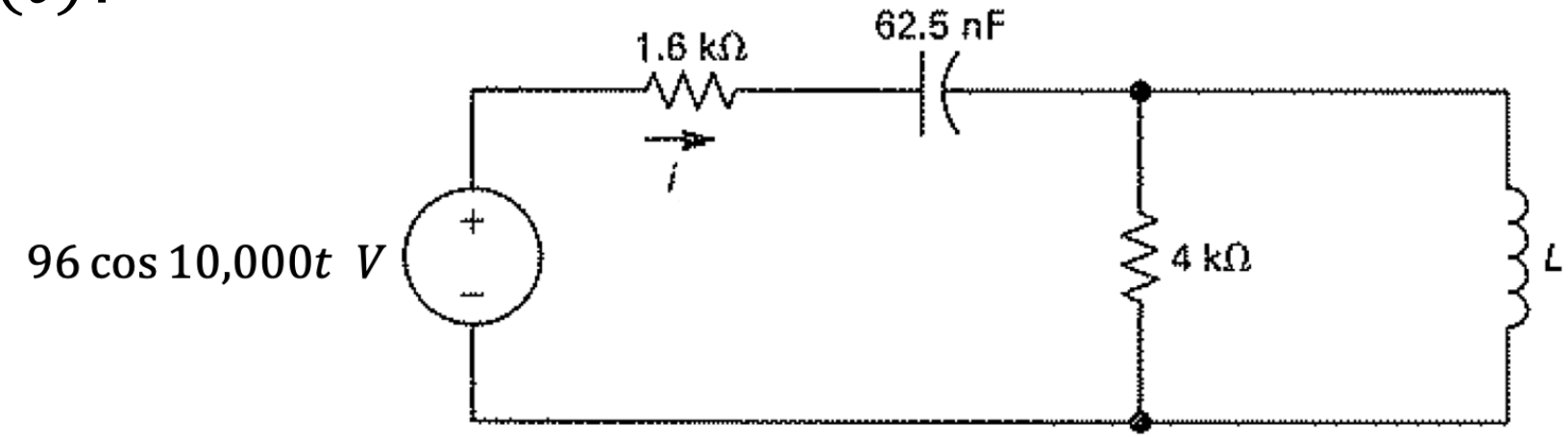

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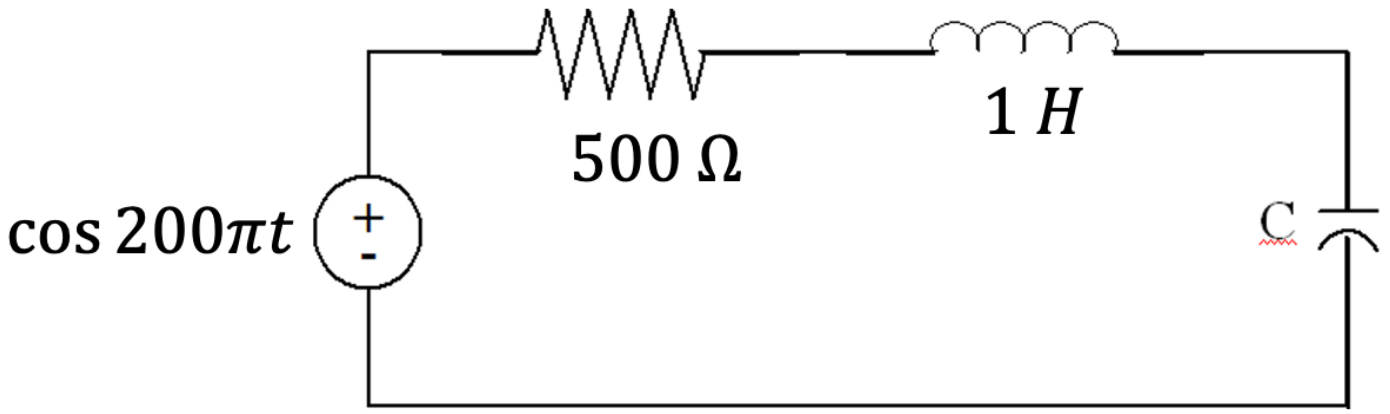
Practice problem: find the capacitor value to maximize the magnitude of the voltage appearing across the capacitor. What is that magnitude?



$1.55\ \mu F, 1.61\ V$