Phasors – 8

more AC power; start design

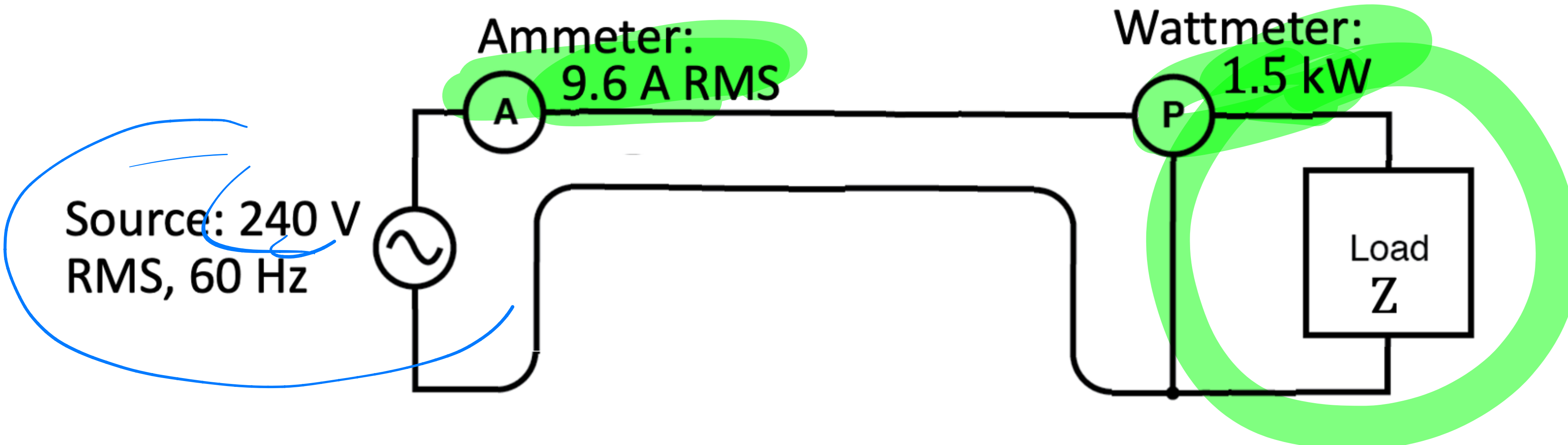
Improving AC Power Delivery

- Real average power is

$$P = \frac{\mathrm{V\,I}}{2}\cos\ \theta = V_{RMS}\, I_{RMS} \cos \theta$$

- The goal is to get θ close to 0 (equivalently, S close to real
  - Load close to resistive

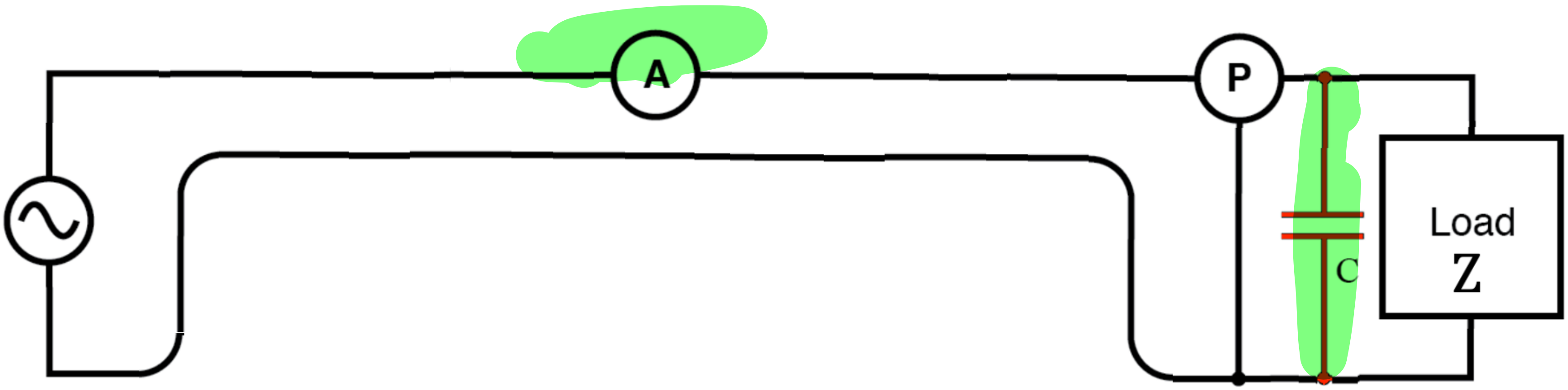
A Typical Example – Power Distribution



- Load Apparent power is

$$P = V_{RMS}\ I_{RMS} = 240 * 9.6 = 2.3\ kVA$$

- Real power = 1.5 kW, so power factor $= \frac{1.5}{2.3} = 0.65$
- Can we improve this? e.g. get current below 7 amps?



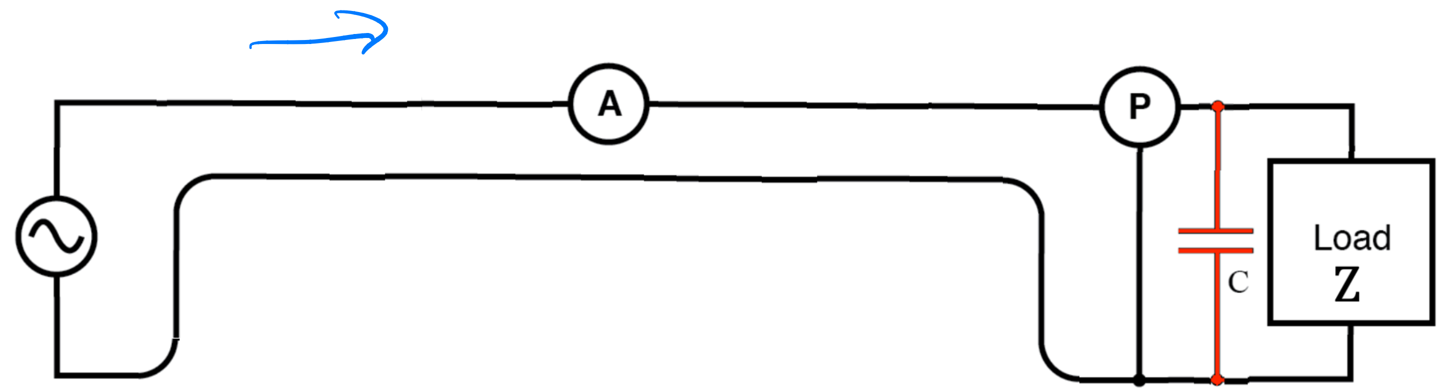
- Since a typical load is inductive, using the power factor value of 0.65 then the model is likely to be:

$$|Z| = \frac{240}{9.6} = 25\ \Omega$$

$$\mathrm{Z} = R + jX$$
$$= 16.25 + j19$$

- Try to improve matters with a shunt capacitor, C, so

$$Z_{new} = Z || \frac{1}{j\omega C}$$

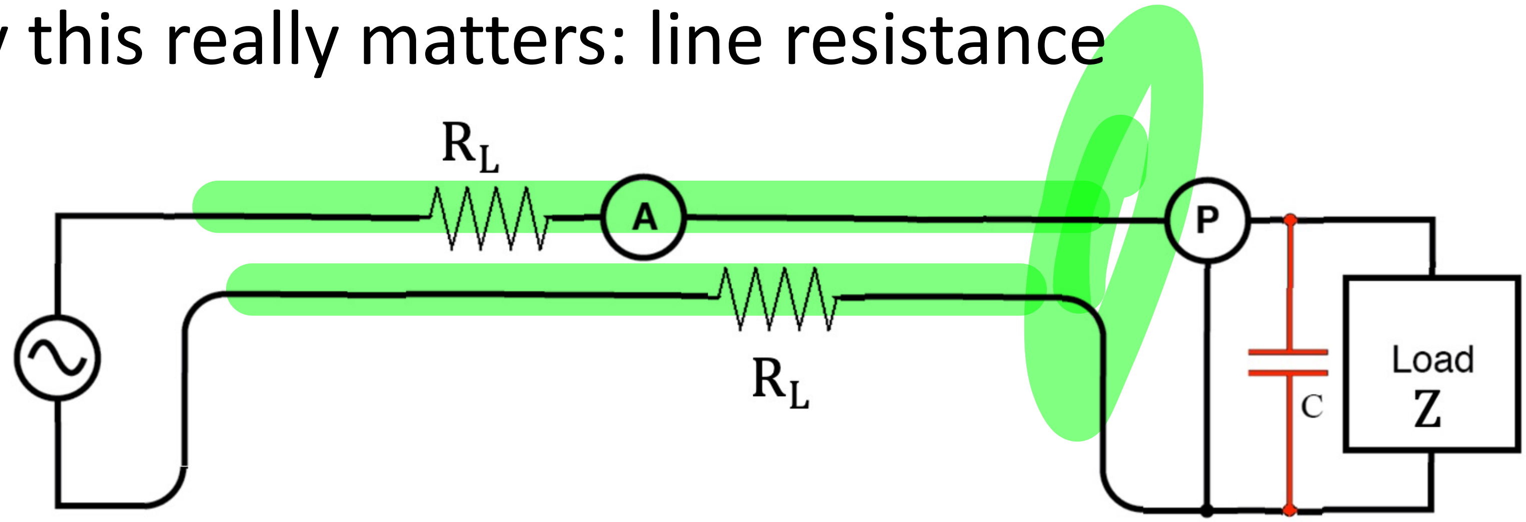


$$Z_{new} = Z || \frac{1}{j\omega C} = \frac{Z\frac{1}{j\omega C}}{Z + \frac{1}{j\omega C}} = \cdots$$

$$= \frac{R(1-\omega CX) + \omega RCX}{(1-\omega CX)^2 + \omega^2 R^2 C^2} + j\frac{X(1-\omega CX) - \omega R^2 C}{(1-\omega CX)^2 + \omega^2 R^2 C^2}$$

- Want Z_{new} to be real for best performance → $C = 80.6\ \mu F$
  - $Z_{new} = 38.46\ \Omega$ and now ammeter reads 6.24 A

Why this really matters: line resistance

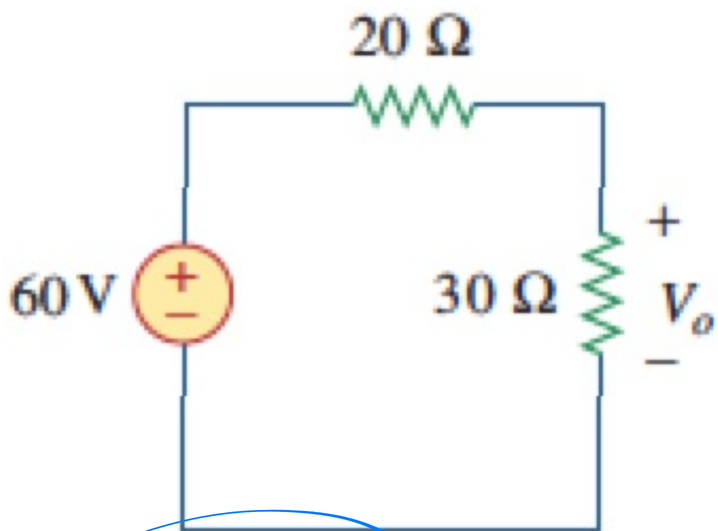


Shunt C yields higher voltage at the load and less heat in the wires

	$R_L = 0$	$R_L = 0.1\ \Omega$	$R_L = 0.1\ \Omega\ \ C = 80.6\ \mu F$
Z_L	$16.25 + j19\ \Omega$	$16.25 + j19\ \Omega$	$38.46\ \Omega$
I_L	9.6 A	9.55 A	6.24 A
V_L	$240\ V$	$238\ V$	$239\ V$
P_{Load}	1500 W	1480 W	1500 W
P_{Line}	0	18.2 W	7.79 W

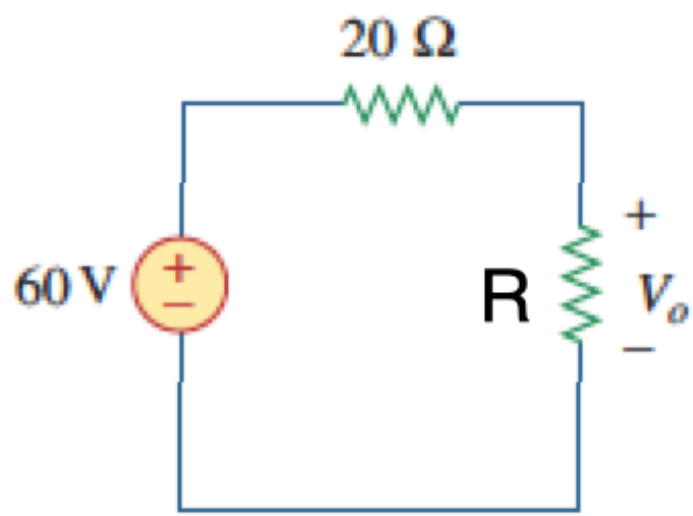
Analysis vs Design

- Voltage division **analysis** yields



$$V_o = \frac{30}{20+30} 60 = 36 \text{ volts}$$

- **Design**: How do we choose R for $V_o = 10$ volts? And is this even possible?

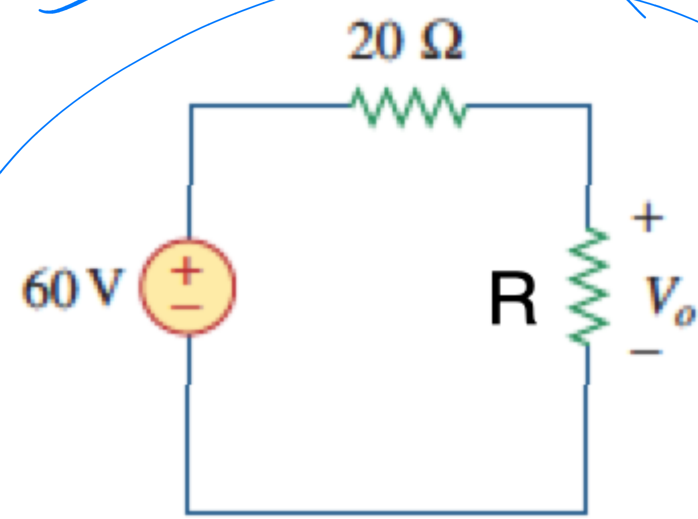


$$60 \cdot \frac{R}{R+20} = 10$$

- Solving :

$$V_o = \frac{\boldsymbol{R}}{20 + \boldsymbol{R}} 60 \quad \Longrightarrow \quad \boxed{R = \frac{20V_o}{60 - V_o}}$$

  - One solution if $0 < V_o < 60$
  - None otherwise

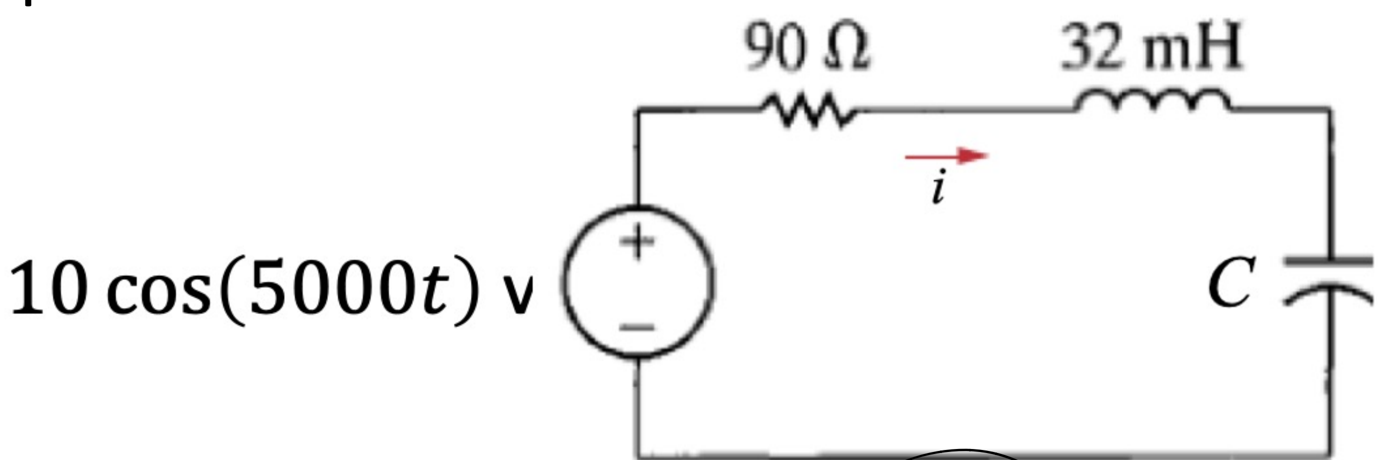


$-45°$

Phasor Circuit Design

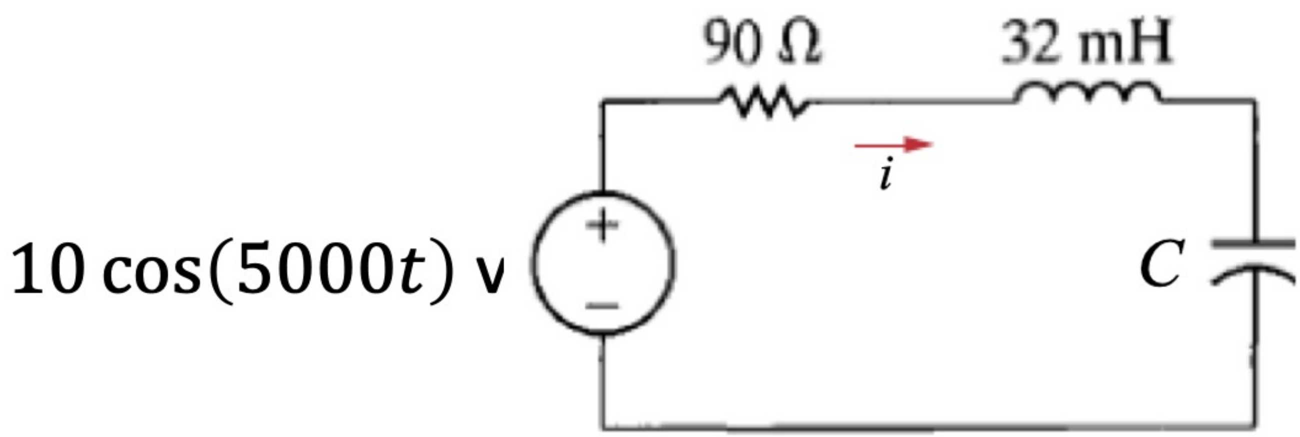
$i(t) = I \cos(5000t + \phi)$ A

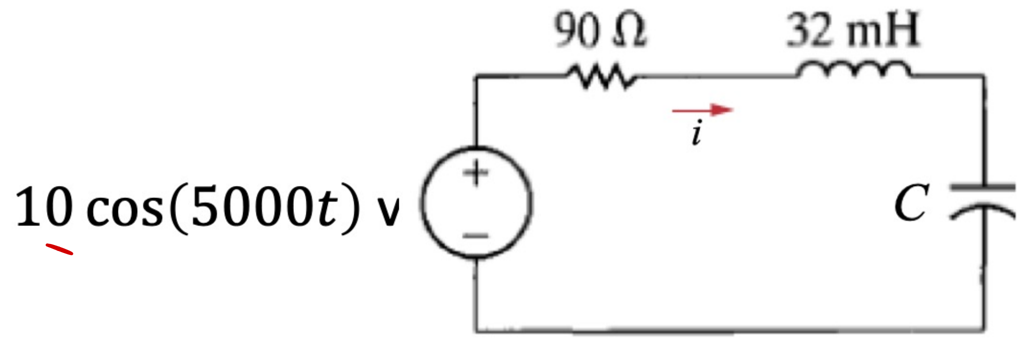
- Choose components to achieve a certain goal.
- Example:



– Can you choose a capacitor C so that the steady state current i has a phase angle of $-45°$ relative to the source ? If so, what is the current's amplitude?

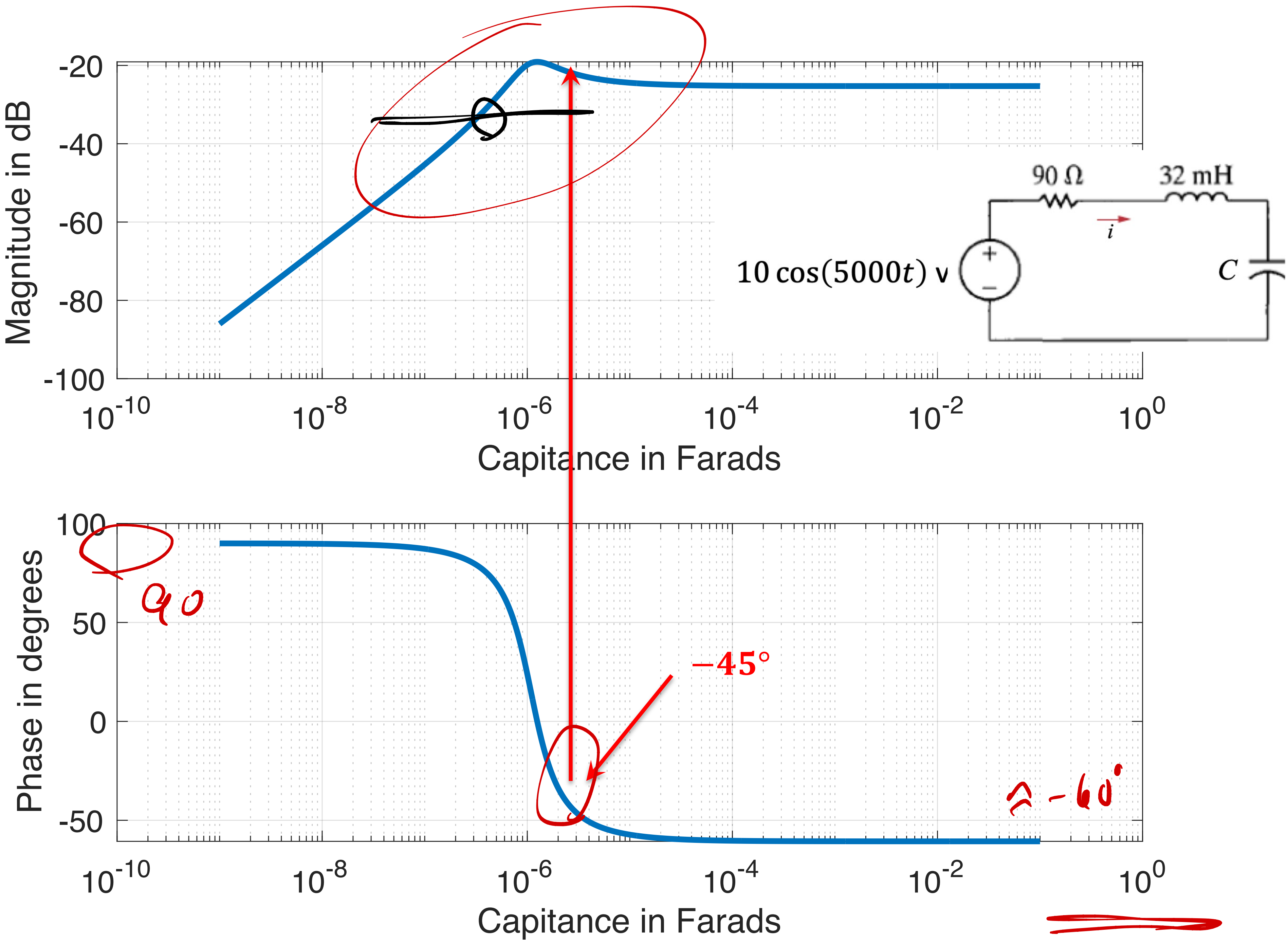
- Considerations:
  - Is the request even possible? How many degrees of freedom do you have versus the number of quantitative goals? Is more than one solution possible?
  - For our example, what range of angles is even possible?





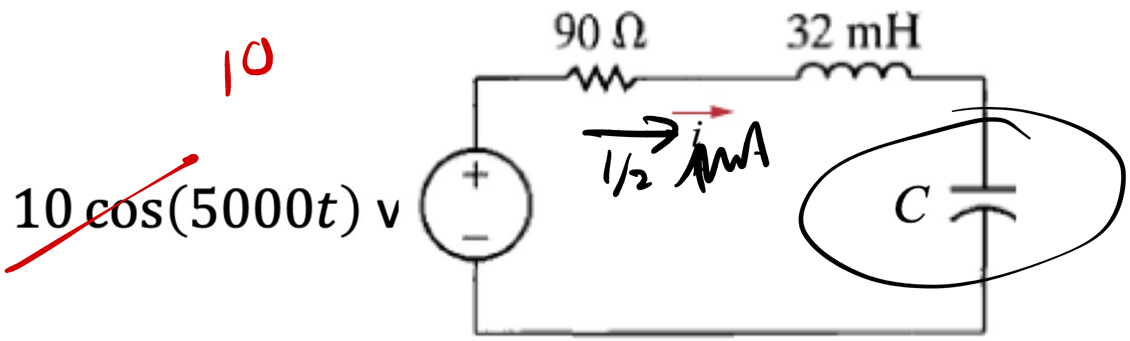

```
om = 5000;
R = 90;
L = 32e-3;
ZL = 1j*om*L;
C = logspace(-9,-1,1000);
ZC = 1./(1j*om*C);
I = 10./(R+ZL+ZC);
```

```
subplot(211)
semilogx(C,20*log10(abs(I)),'linewidth',3)
xlabel('Capitance in Farads')
ylabel('Magnitude in dB')
set(gca,'fontsize',16)
grid on
subplot(212)
semilogx(C,180/pi*angle(I),'linewidth',3)
xlabel('Capitance in Farads')
ylabel('Phase in degrees')
set(gca,'fontsize',16)
grid on
```



Magnitude in dB
-20
-40
-60
-80
-100
10^-10
10^-8
10^-6
10^-4
10^-2
10^0
Capitance in Farads
90 Ω
32 mH
i
10 cos(5000t) v
C
Phase in degrees
100
50
0
-50
90
−45°
≈ -60°
Capitance in Farads

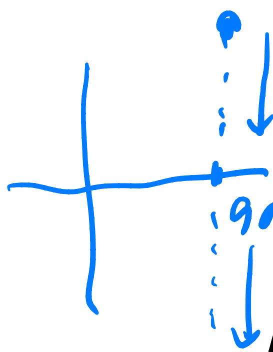
Let's actually solve for C and the current's amplitude



$$j\omega L = j\,\frac{5000 \cdot 32}{1000} = j160$$

$$\frac{1}{j\omega C} = -j\,\frac{1}{5000C}$$

$$I = \frac{10}{90 + j160 - j\frac{1}{5000C}} = \frac{10\angle 0^\circ}{\sqrt{90^2 + \left(160 - \frac{1}{5000C}\right)^2}\ \angle \tan^{-1}\left(\frac{160 - \frac{1}{5000C}}{90}\right)}$$



$$\angle I = 0 - \tan^{-1}\left(\frac{160 - \frac{1}{5000C}}{90}\right) = -45^\circ$$

$2.86\ \mu F; 78.6\ mA$

$$-\tan^{-1}\left(\frac{160-\frac{1}{5000C}}{90}\right)=-45^\circ$$

need

$$160-\frac{1}{5000C}=90$$

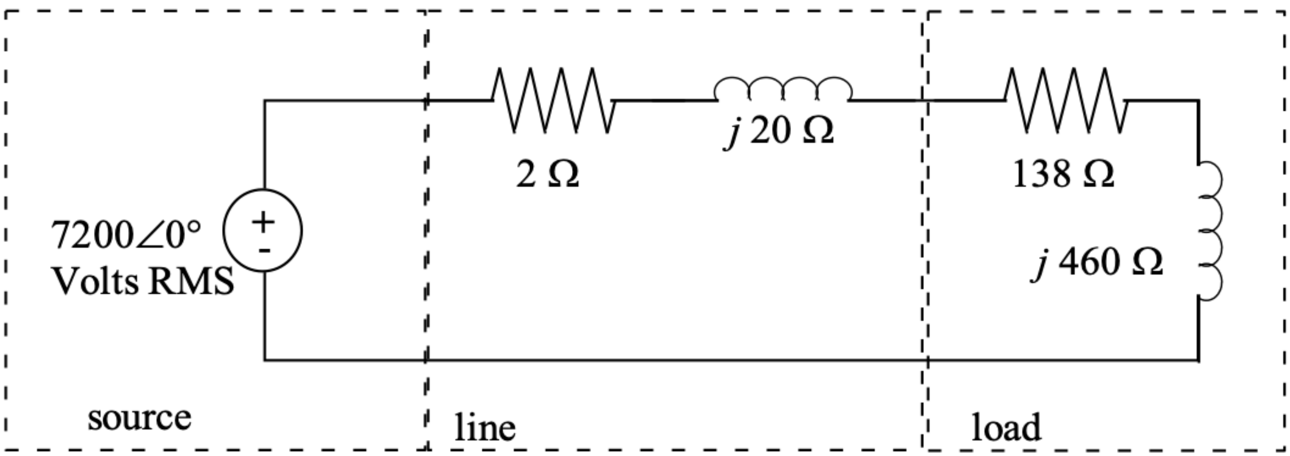
$$+\frac{1}{5000C}=+70$$

$$5000C=\frac{1}{70}$$

$$C=\frac{1}{70\cdot 5000}$$

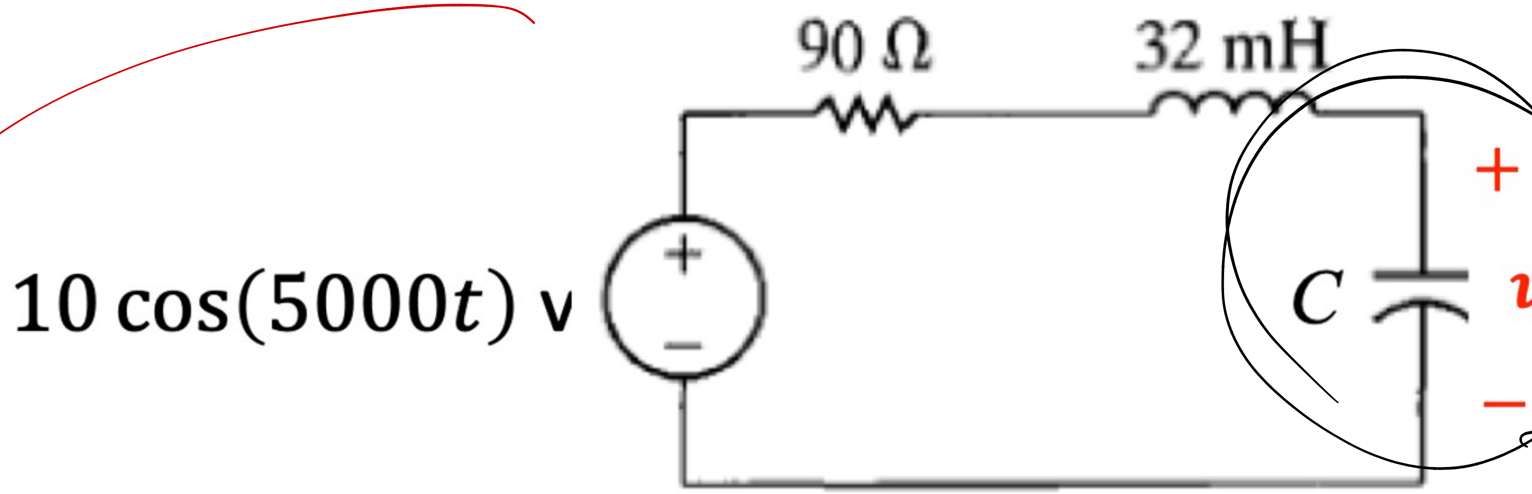
Practice problem:

- Find the average power dissipated by the line
- Find the shunt capacitance to make the load appear purely resistive
- Find the load resistance resulting from this shunt capacitance
- Find the average power dissipated by the line with the shunt capacitor installed



$207\ W; -501\ \Omega;\ 1.67\ k\Omega; 18.6\ W$

Practice problem: for the same circuit, Can you choose a capacitor C so that its steady state voltage v has a phase angle of $-45°$ relative to the source ?



$$\angle V_c = \angle 10^\circ \; \angle \frac{\frac{1}{j5000C}}{90 + j160 - j\frac{1}{5000C}}$$

$$-90 - \tan^{-1}(\;)$$

0.118 nF