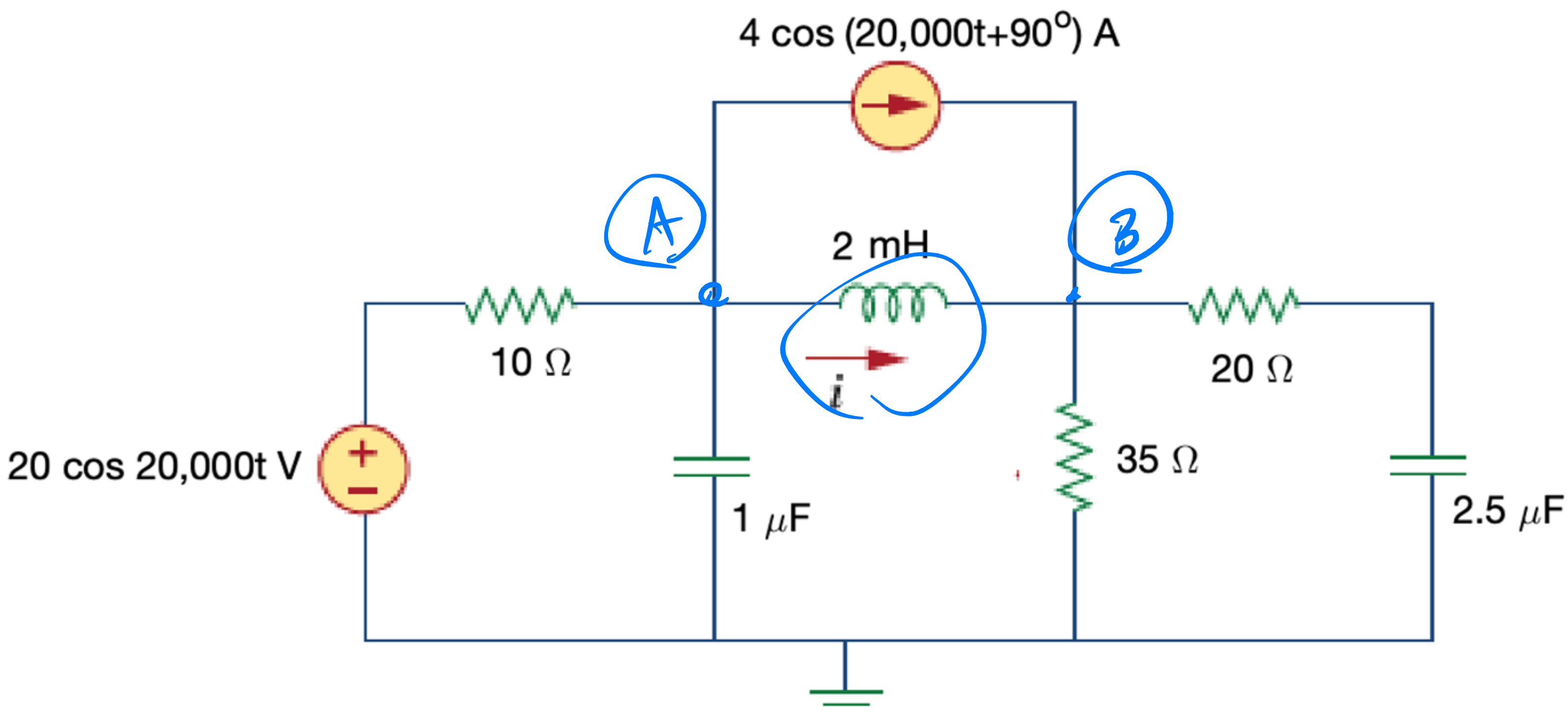
Phasors 6

more examples

Where Are We?

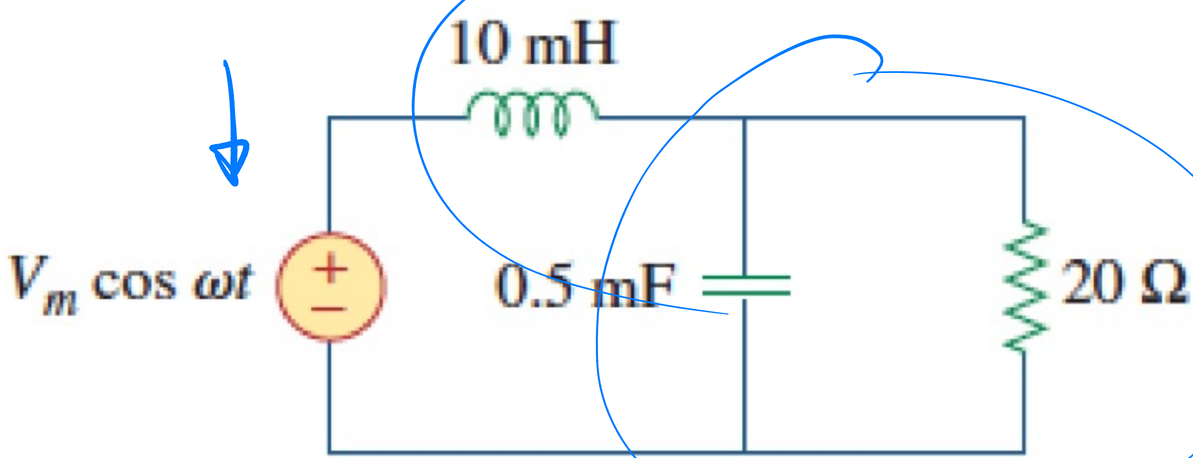
- What we know how to solve: find $i(t)$:



$1.91\cos(20{,}000t - 123°)\ A$

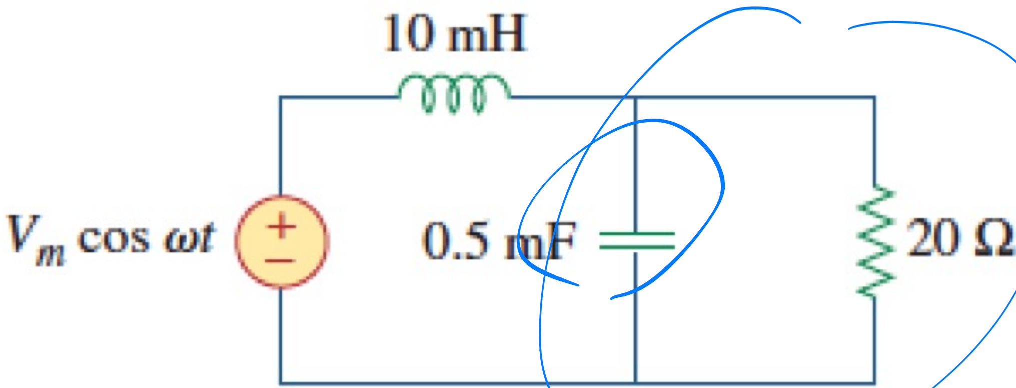
Other Question Types

Sample: for what frequency is the source current the largest?

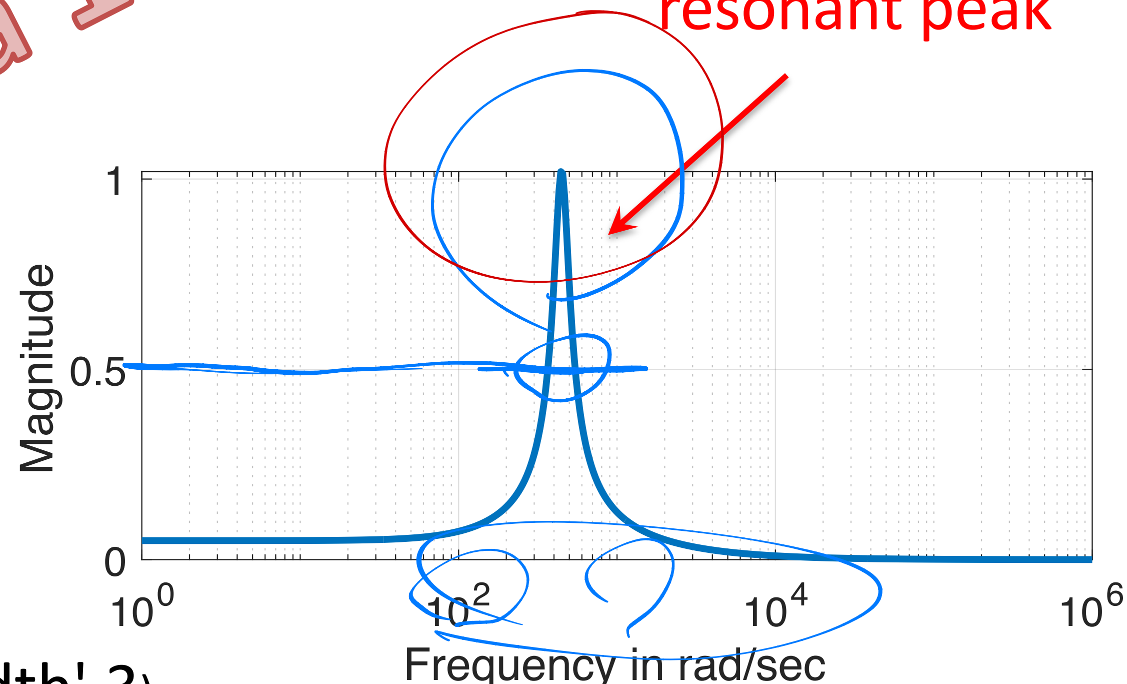


– Method?
  - Numerical calculation
  - Analysis

$$I = \frac{V_m}{Z}$$



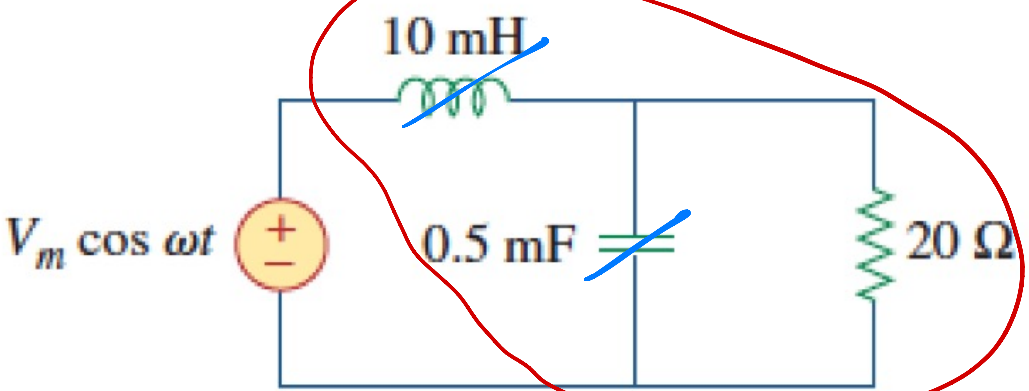
Idea 1 – compute it




```
om = logspace(0,6,500);
ZC = 1./(1j*om*0.5e-3);
ZP = 20*ZC./(20+ZC);
I0 = 1./(1j*om*10e-3+ZP);
semilogx(om,(abs(I0)),'linewidth',3)
```

$$j\omega\frac{10}{1000} = j\frac{\omega}{100}$$

$$|I| = \left|\frac{V_m}{Z}\right| = \frac{|V_m|}{|Z|}$$



$$-j\frac{1000}{\omega .5} = -j\frac{2000}{\omega}$$

Idea 2 – analyze it

$$Z_p = \frac{(20)\left(-j\frac{2000}{\omega}\right)}{20 - j\frac{2000}{\omega}} \cdot \frac{\omega}{\omega} = \frac{-j\,40{,}000}{20\omega - j\,2000} = \frac{-j\,2000}{\omega - j100}$$

$$|Z| = \left| j\frac{\omega}{100} + \frac{-j\,2000}{\underbrace{\omega - j100}_{\omega^2+10^4}} \cdot \frac{\omega + j100}{\omega + j100} \right| = \left| j\frac{\omega}{100} + \frac{2\cdot 10^5}{\omega^2+10^4} + \frac{-j\,2000\,\omega}{\omega^2+10^4} \right|$$

$$= \left| \frac{2\cdot 10^5}{\omega^2+10^4} + j\left(\frac{\omega}{100} - \frac{2000\,\omega}{\omega^2+10^4}\right) \right|$$

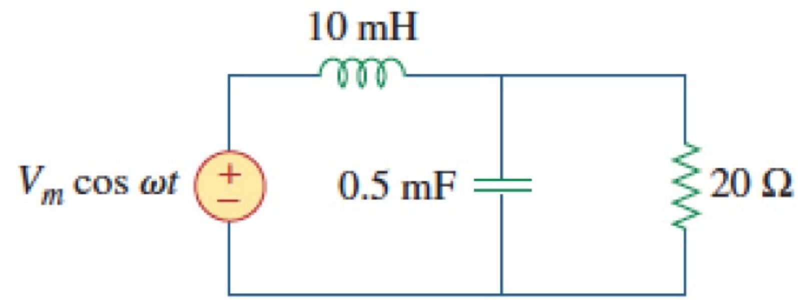
$$\min_{\omega}$$

$$\min_{w} \left| \frac{2\cdot 10^5}{w^2+10^4} + j\left(\frac{w}{100} - \frac{2000w}{w^2+10^4} \right) \right|$$

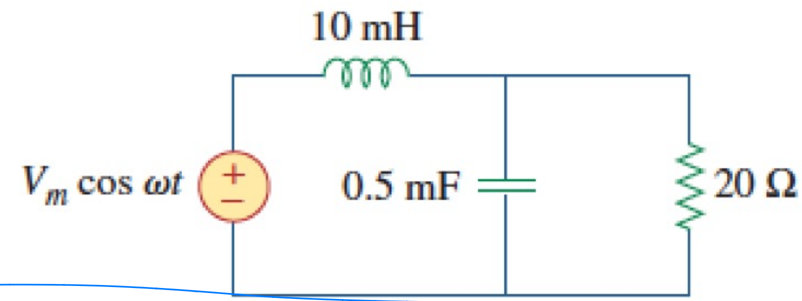
$$\Rightarrow \min_{w} \sqrt{\left(\frac{2\cdot 10^5}{w^2+10^4}\right)^2 + \left(\frac{w}{100} - \frac{2000w}{w^2+10^4}\right)^2}$$

$$\Rightarrow \min_{w} \left(\qquad \right)^2 + \left(\qquad \right)^2$$

```
>> syms w real
>> zc = 1/(1j*w*5e-4)

zc =

-2000i/w

>> zp = 20*zc/(20+zc)

zp =

40000i/(w*(2000i/w - 20))

>> z = 1j*w*1e-2 + zp

z =

(w*1i)/100 + 40000i/(w*(2000i/w - 20))
```

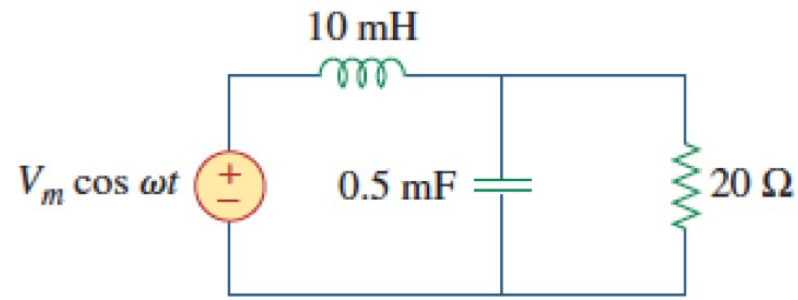


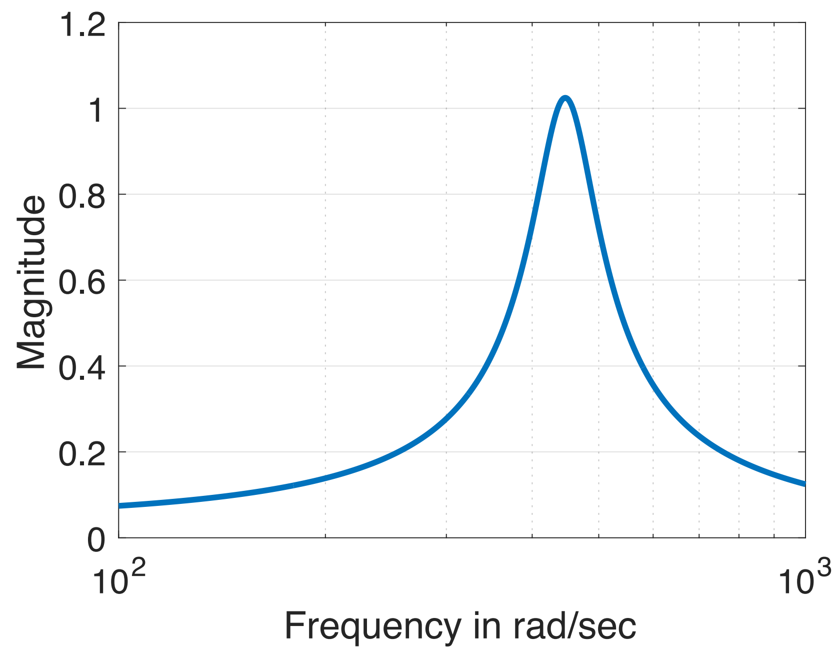
Or use a tool




```
>> H = 1/z;
>> aH = sqrt( real(H)^2 + imag(H)^2 );
>> pretty(aH)
        / /   w            800000          \2                                   \
        | |  --- - --------------------    |                                   |
        | | 100      / 4000000        \    |                                   |
        | |        w | ------- + 400  |    |                                   |
        | |          |     2          |    |                                   |
        | \          \    w           /    /          6400000000000000          |
sqrt|  ------------------------------ + ------------------------ |
        |                #1                       4    / 4000000       \2 |
        |                                    w   #1 | ------- + 400  |  |
        |                                             |     2          |  |
        \                                             \    w           /  /

where

            /     6400000000000000       /   w            800000          \2 \2
   #1 ==  | ------------------------ + |  --- - --------------------    |   |
            |  4 / 4000000       \2   | 100       / 4000000        \   |   |
            | w  | ------- + 400 |       |          w | ------- + 400 |  |   |
            |    |     2         |       |            |     2         |  |   |
            \    \    w          /       \            \    w          /  /   /
```

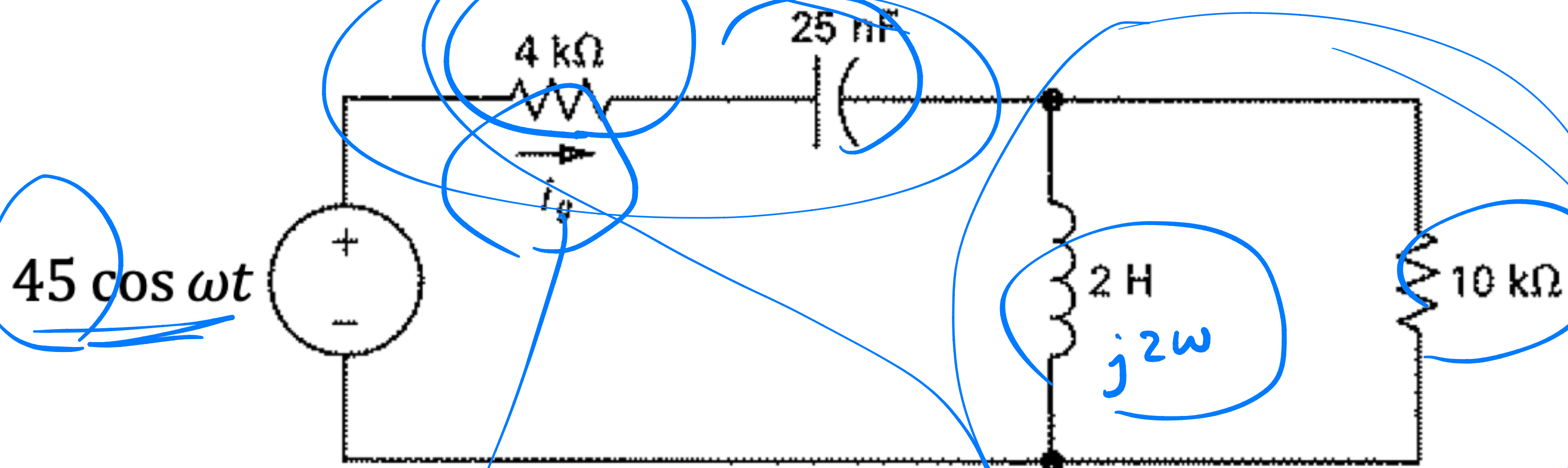
```
>> daH = diff(aH,w);
>> solve(daH,w)

ans =

  (20000*110^(1/2) - 10000)^(1/2)
 -(20000*110^(1/2) - 10000)^(1/2)

>> eval(ans(1))

ans =

  446.9472
```





Example: At what frequency ω is i_g in phase with the voltage source?

$$-j\frac{10^9}{25\omega} = -j\frac{40\cdot 10^6}{\omega}$$



$A\cos\omega t$

$$I = \frac{45}{Z}$$

$$Z = (j\omega L \parallel 10k) + 4k + \frac{1}{j\omega C}$$

$A + jB$

10^4 rad/sec

$$Z_p = \frac{10^4 \cdot j2\omega}{10^4 + j2\omega}$$

10^4 rad/sec

$$Z = \frac{10^4 \cdot j2\omega}{10^4 + j2\omega} + 4000 - j\frac{40 \cdot 10^6}{\omega}$$

$$I = \frac{45}{Z}$$

Z real

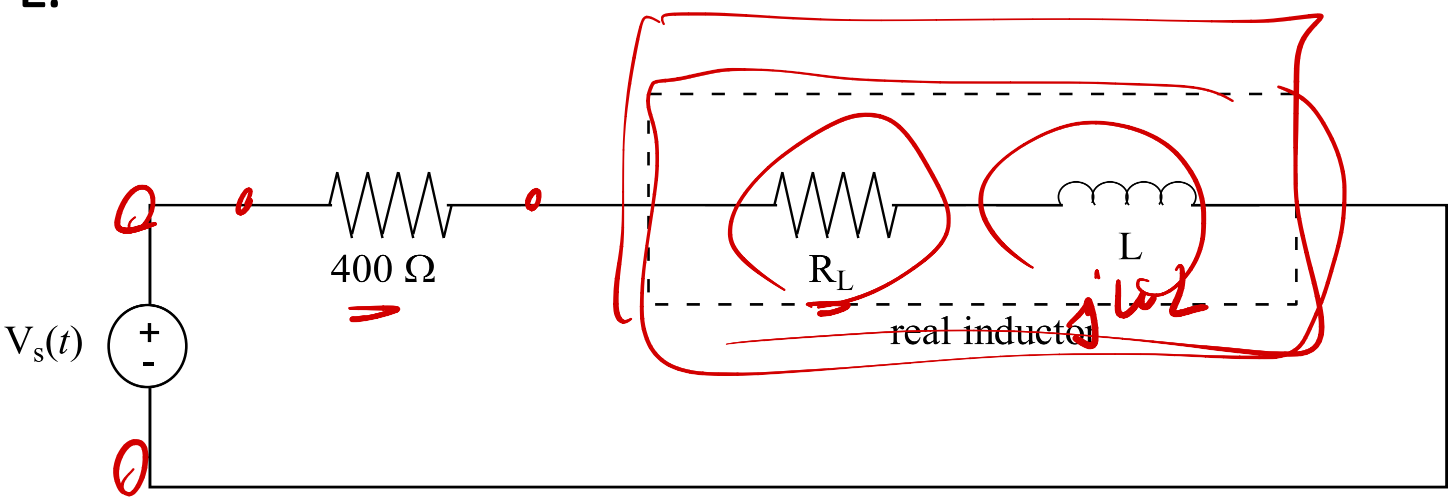
$$\frac{10^4\, j2\omega\,(10^4 - j2\omega)}{(10^4 + j2\omega)(10^4 - j2\omega)}$$

$\downarrow$ imag

$$\frac{2\, 10^8 \omega}{10^8 + 4\omega^2} = \frac{40 \cdot 10^6}{\omega}$$

$$2 \cdot 10^2 \omega^2 = 40 \cdot 10^1 (10^8 + 4\omega^2)$$

Example: We model a real inductor as shown with a series parasitic resistance R_L. To measure its parameters, R_L and L, we build the circuit shown (with a 60 Hz source) and use an AC voltmeter to measure the amplitudes of the component voltages. Given $|V_S| = 120\ V$, $|V_R| = 100\ V$, $|V_L| = 30$, find R_L and L.



$$|V_R| = \left| \frac{120\angle\phi \cdot 400}{400 + R_L + j\omega L} \right| = 100$$

$$\omega = 2\pi \cdot 60$$

$$|V_L| = \left| \frac{120\angle\phi \cdot (R_L + j\omega L)}{400 + R_L + j\omega L} \right| = 30$$

$$\left| \frac{120\angle\phi \cdot 400}{400 + R_L + j\omega L} \right| = 100$$

$70\,\Omega,\ 259\ mH$

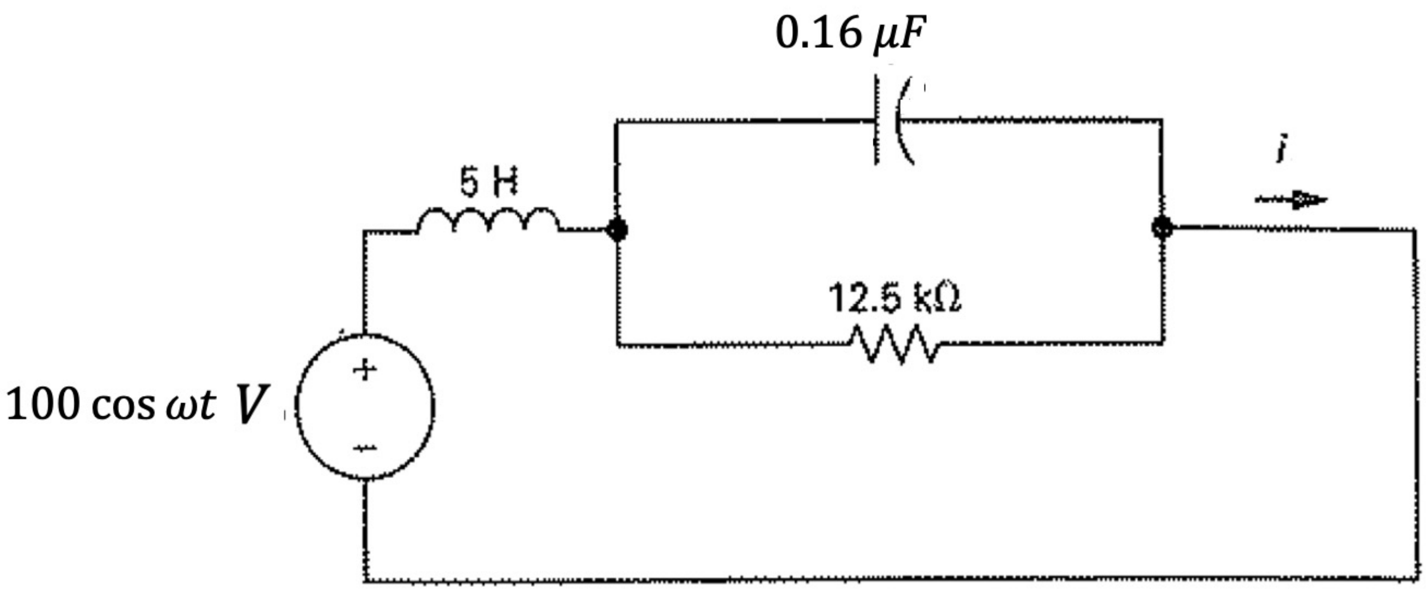
$\omega = 2\pi \cdot 60$

$$\left| \frac{120\angle\phi \cdot (R_L + j\omega L)}{400 + R_L + j\omega L} \right| = 30$$

$$\frac{120 \cdot 400}{\sqrt{(400+R_L)^2 + \omega^2 L^2}} = 100$$

$$\frac{120 \cdot \sqrt{R_L^2 + \omega^2 L^2}}{\sqrt{(400+R_L)^2 + \omega^2 L^2}} = 30$$

Practice problem: At what frequency does the current i have the largest magnitude? What is that magnitude?



$$1120\,\frac{rad}{sec};\,43.8\;mA$$