Phasors – 4

using phasors;

variation with frequency

Phasor Review

- Extend sinusoidal voltages/currents to phasors (complex)

$$V_s \cos(\omega t + \phi) \Rightarrow \mathbf{V} = V_s e^{j\phi}$$

$$I_s \cos(\omega t + \phi) \Rightarrow \mathbf{I} = I_s e^{j\phi}$$

- Convert components (R,L,C) to <u>impedances</u>

$$Z = \begin{cases} R & \text{resistor} \\ j\omega L & \text{inductor} \\ \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C} & \text{capacitor} \end{cases}$$

- Solve the problem using Ohm's Law, KVL/KCL, …

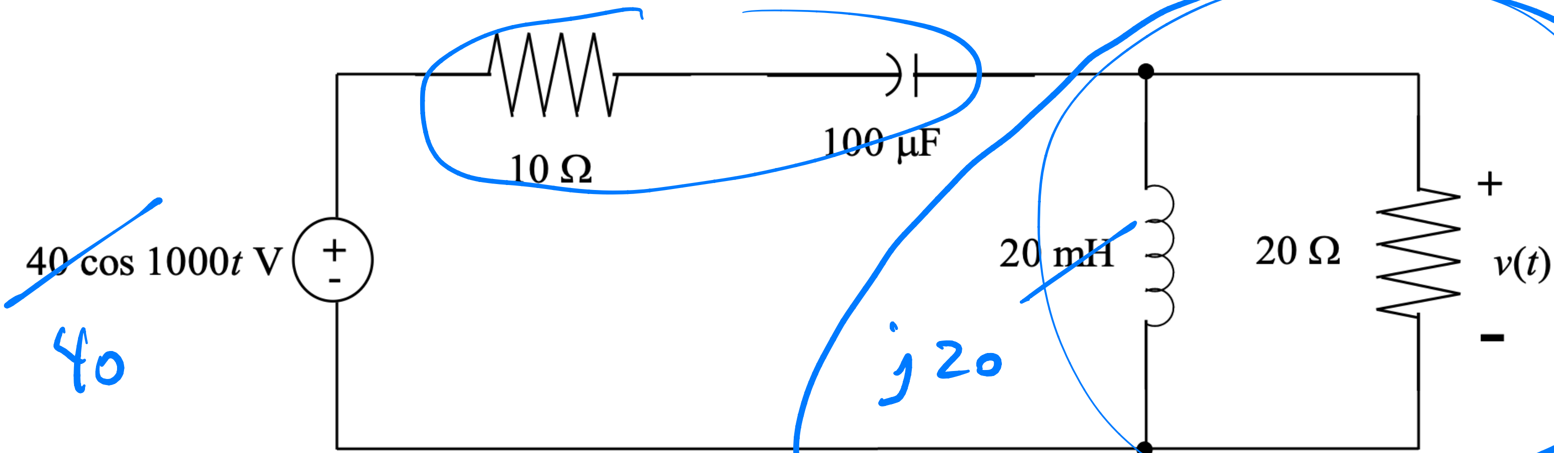
- Convert back

$$B\angle\theta \Rightarrow B\cos(\omega t + \theta)$$

Common Usage

- Find the voltage *v*(*t*)

$$-j\frac{10^6}{1000\cdot 100} = -j10$$



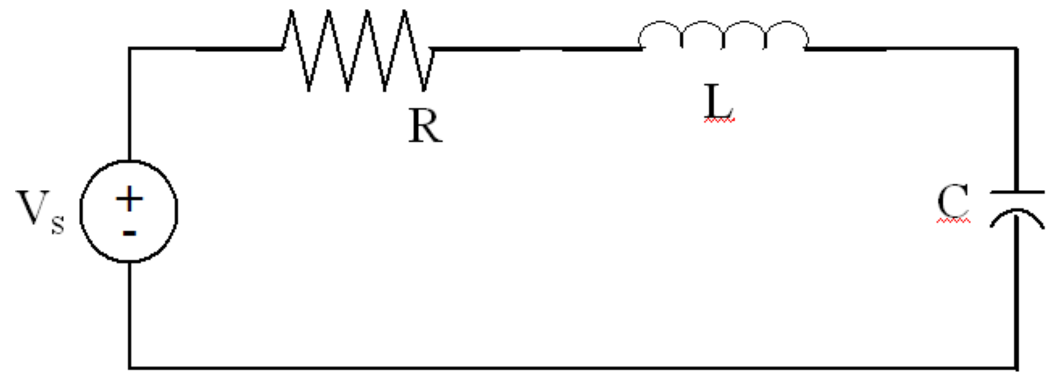
$$\frac{20\cdot j20}{20+j20} = Z$$

$$V = 40\cdot\frac{Z}{Z+10-j10}$$

$28.3\cos(1000t + 45°)\ V$

Consider Variation with Frequency

Consider voltage division:



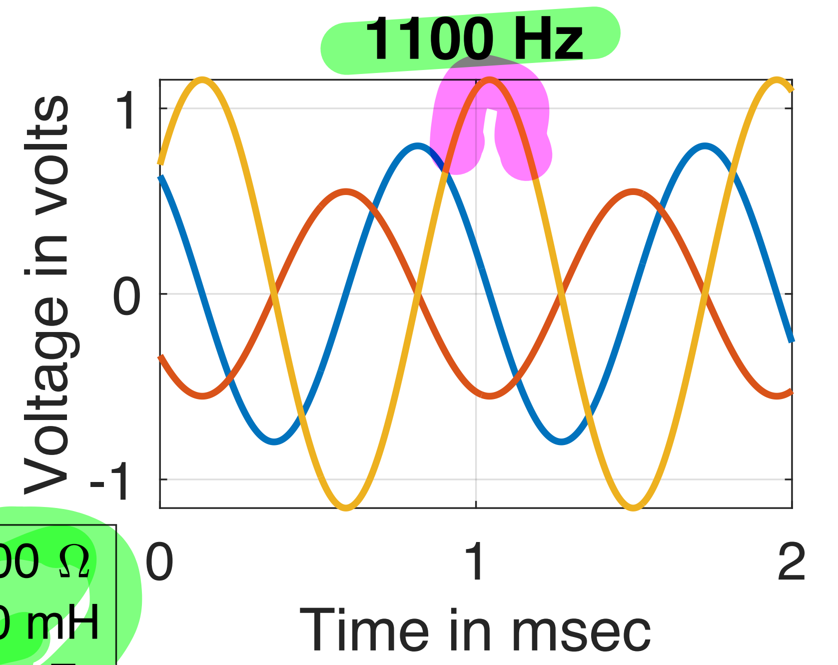
$$\boldsymbol{V}_R = \frac{R\boldsymbol{V}_s}{R + j\omega L + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

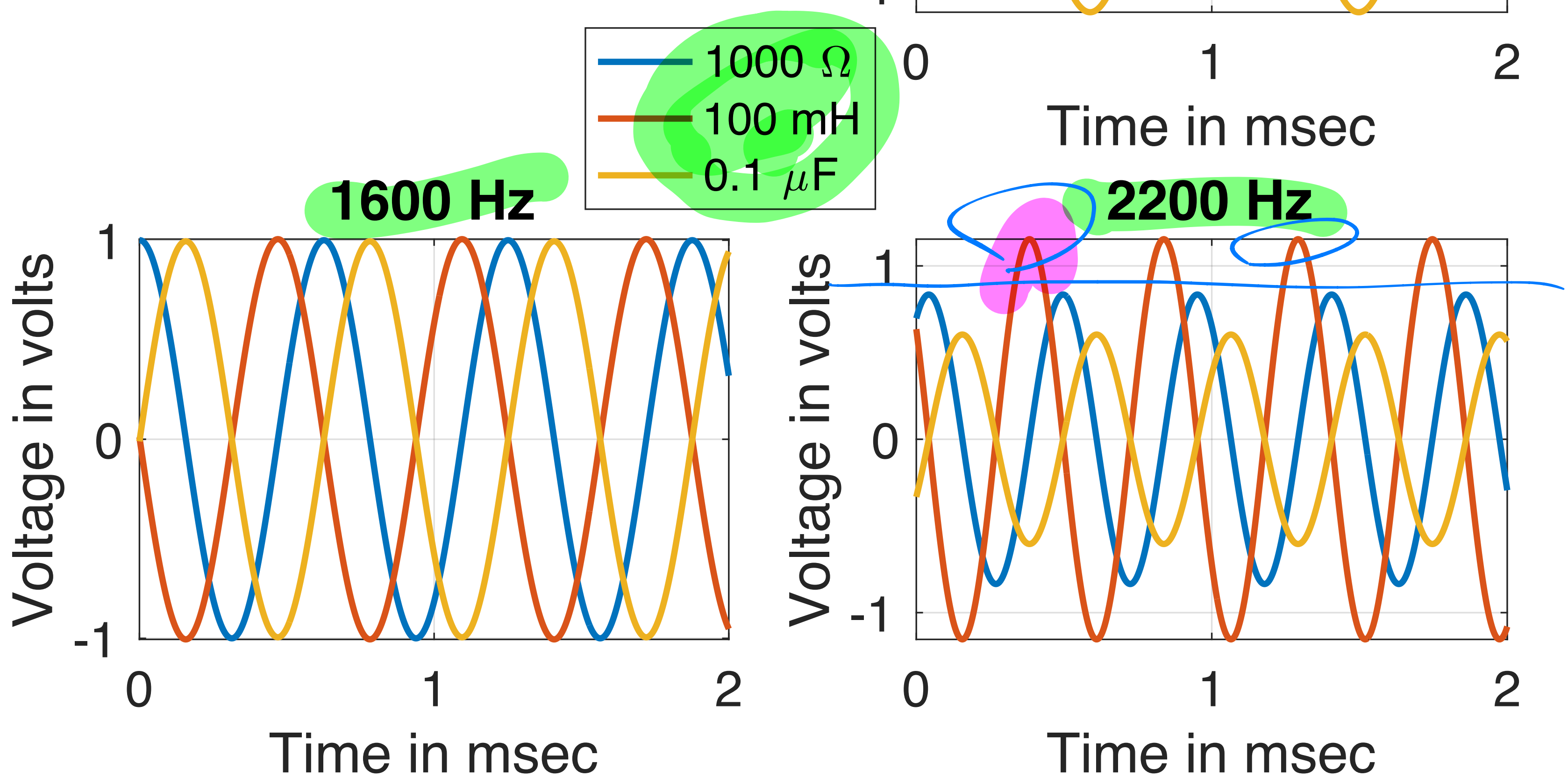
Similarly

$$\boldsymbol{V}_L = \frac{-\omega^2 LC}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

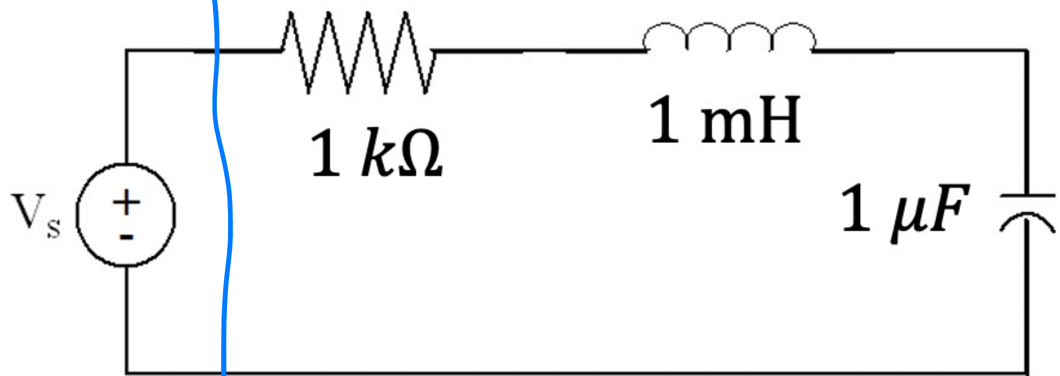
$$\boldsymbol{V}_C = \frac{1}{1 - \omega^2 LC + j\omega RC}\boldsymbol{V}_s$$

- Comparison of the component voltages for different frequencies $(V_s = 1)$





Consider combined impedance variation



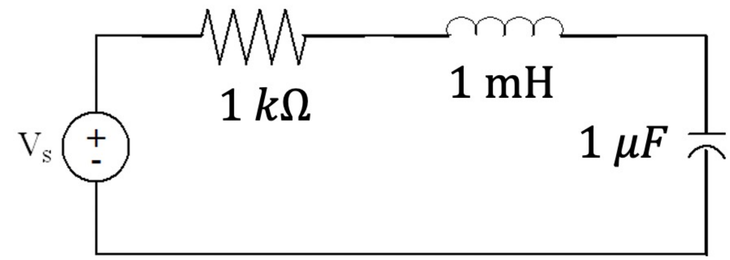
$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$= 1000 + j\boxed{\left(\frac{\omega}{1000} - \frac{10^6}{\omega}\right)} = 0$$

Question – at what frequency does this “appear” purely resistive?

How might we graph impedance vs frequency?

$$Z = 1000 + j\left(\frac{\omega}{1000} - \frac{10^6}{\omega}\right)$$



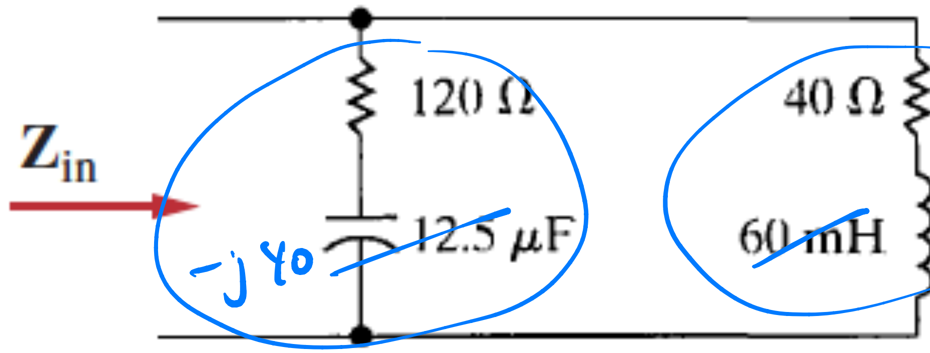
- Real/imag vs frequency?
- Real vs imag?

$$\frac{\omega^2}{\cancel{1000}} = \frac{\cancel{10^6}\ 10^3}{\omega}$$

$$\boxed{\omega = \sqrt{1000}}$$

$80 + j40\ \Omega$

Example: find Z_{in} if $\omega = 2000$ rad/sec



rad $\quad j\omega L$

$= j\, 2000 \cdot \frac{60}{1000}$

$-j\frac{1}{\omega C} = -j\frac{10^6}{2000 \cdot 12.5}$

25000

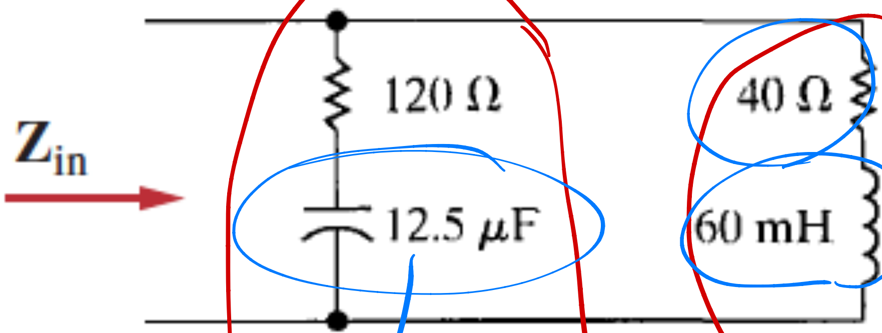
$= -j40$

$120 - j40 \qquad 40 + j120$

$$Z_{in} = \frac{(120-j40)(40+j120)}{\cancel{120} - j\cancel{40} + \cancel{40} + j\cancel{120}}$$

$$= \frac{40(3-j)\,40(1+3j)}{160 + j80} = \frac{40^{\cancel{2}}(6+8j)}{\cancel{40}(4+j2)} \frac{(4-j2)}{4-j2}$$

How does Z_{in} vary with frequency? Is it ever purely real?



$j\omega \cdot \frac{60}{1000}$

$40 + j\,\frac{6\omega}{100}$

$-j\,\frac{10^6}{12.5\,\omega}$

$120 - j\,\frac{80000}{\omega}$

$$Z_{eq} = \frac{\left(120 - j\,\frac{80{,}000}{\omega}\right)\left(40 + j\,\frac{6\omega}{100}\right)}{160 + j\left(\frac{6\omega}{100} - \frac{80000}{\omega}\right)} \qquad \frac{100\,\omega}{100\,\omega}$$

$$\frac{\left(120^{w} - j\,\frac{80{,}000}{\cancel{w}}\right)\left(40 + j\,\frac{6w}{\cancel{100}}^{100}\right)}{160_{100w} + j\left(\frac{6ww}{\cancel{100}} - \frac{80000^{100}}{\cancel{w}}\right)}$$

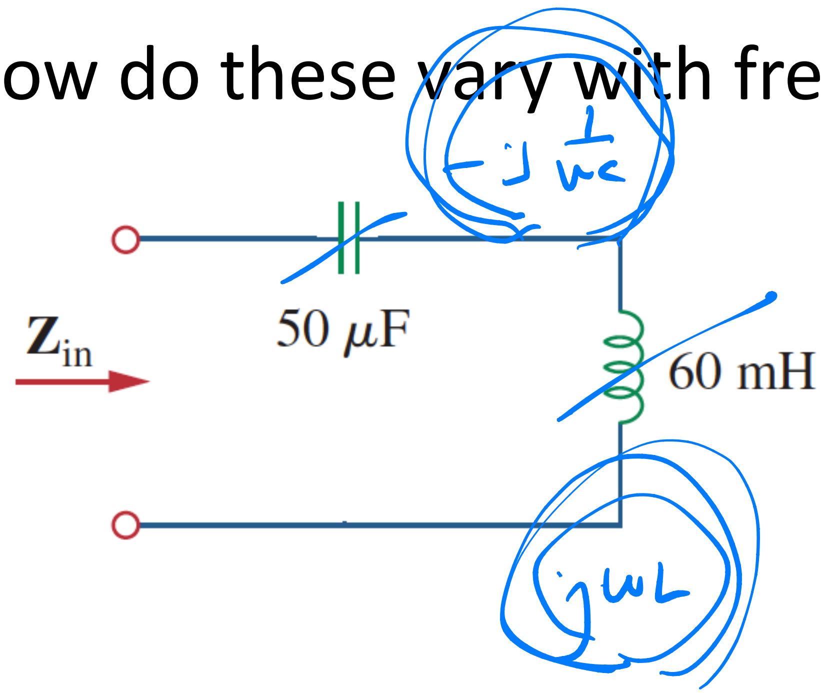
$$\frac{(120w - j\,80{,}000)(4000 + j\,6w)}{16000\,w - j\,(6w^2 - 8\cdot 10^6)}$$

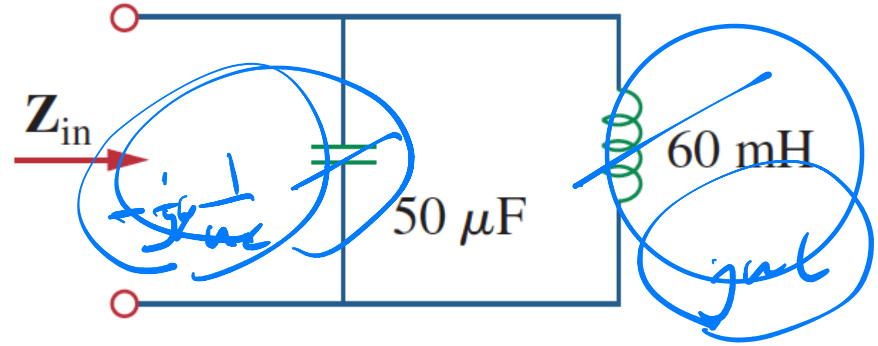
100w

100

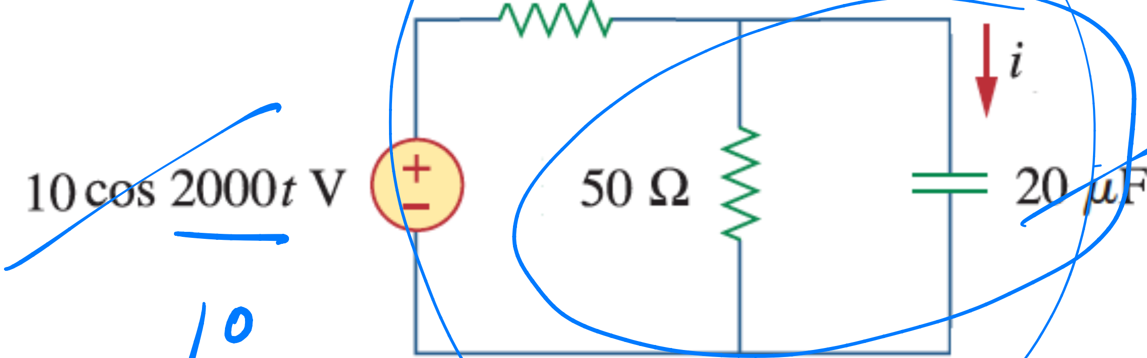
w

How do these vary with frequency?





- **Example:** find $i(t)$



10

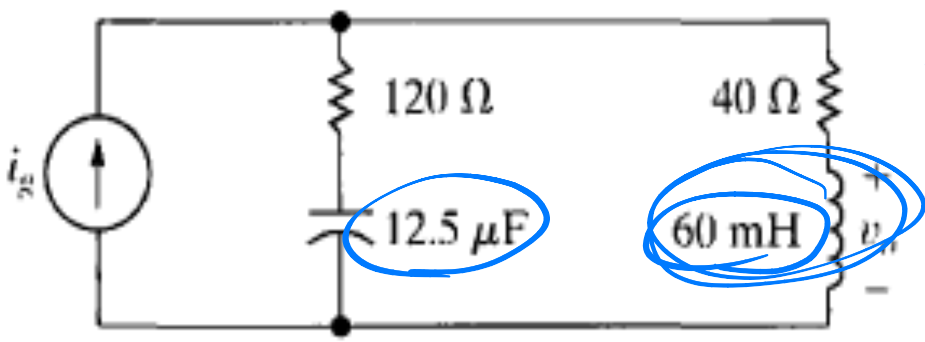
$-j\,\dfrac{10^6}{2000 \cdot 20}$

$= -j\,50$

$141\cos(2000t + 45°)\ mA$

Example: find $v_o(t)$ if $i_g(t)$ = 500 cos 2000 t mA

$\rightarrow$ 1/2



$$\downarrow = \frac{1}{2} \cdot \frac{120 - j40}{160 + j80}$$

$120 - j40$

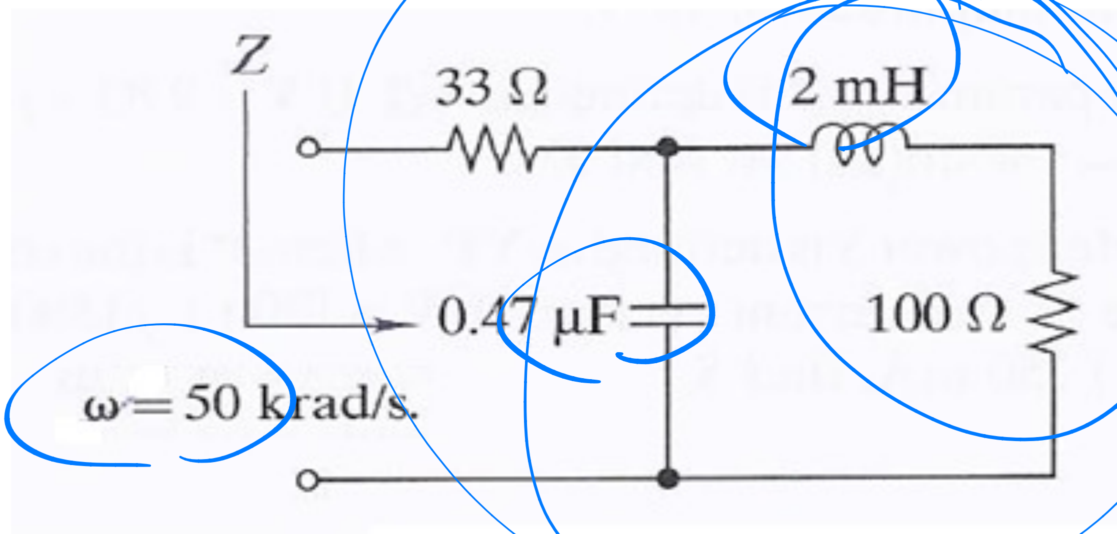
$40 + j120$

$1.59\ \cos(2000t - 17.0°)$ V

Example: find Z

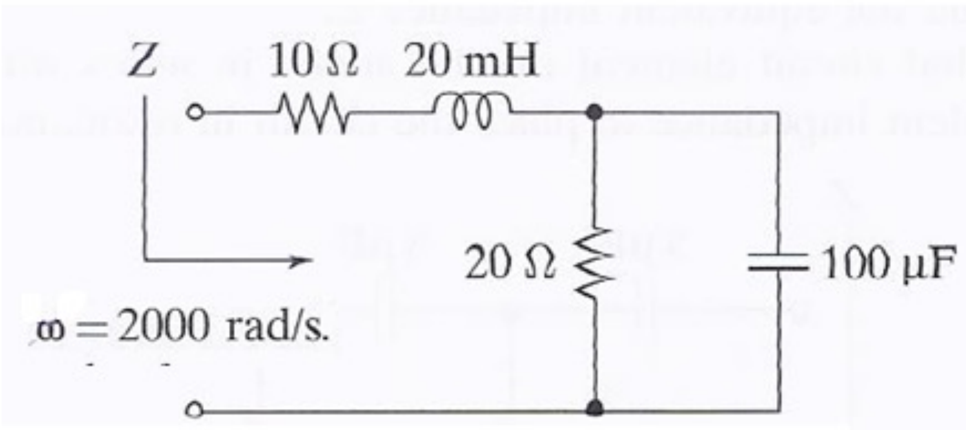
$\boxed{46.4 - j50.4;}$

$46.4\ \Omega\ \ 0.397\ \mu F$



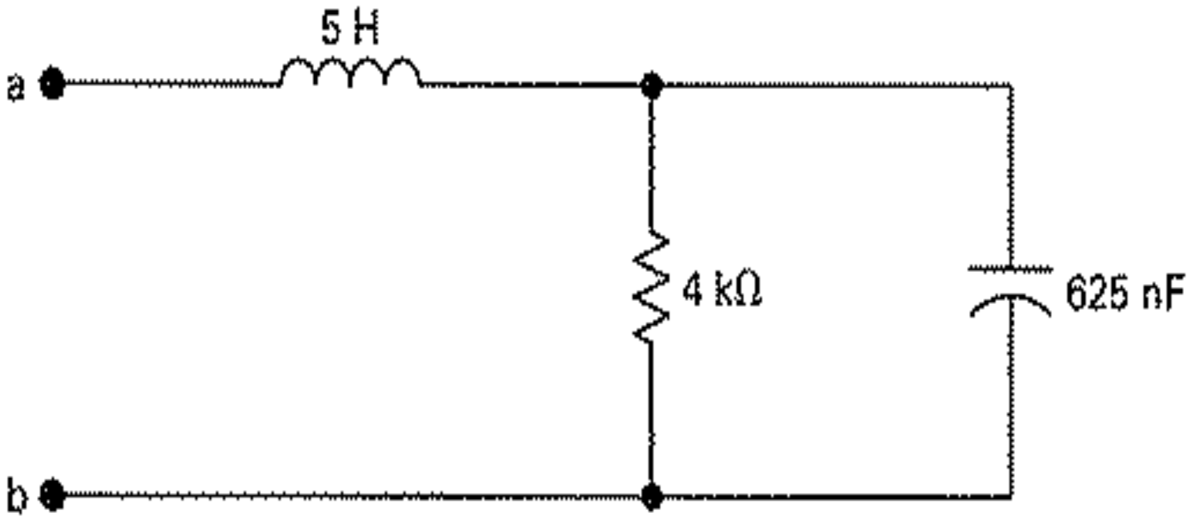
Practice problem: find Z

$11.2 + j35.3;$
$11.2\ \Omega \quad 17.6\ mH$



Practice problem: at what frequency does this circuit seem purely resistive? What is the resistance?

$$400\frac{rad}{sec}; 2\ k\Omega$$



- **Practice problem:** consider the parallel connection of a 220 Ω resistor, a 0.5 μF capacitor, and a 5 mH inductor.
  - What is the equivalent impedance of this circuit at 1000 Hz?
  - At 5000 Hz?
  - At what frequency is the impedance purely real?

$11.6 + j49.2\ \Omega;\ 1.42 - j17.6\ \Omega;\ 1.59\ kHz$

Practice problem: Find the time expression for $v_o(t)$.
Note that $\sin\omega t = \cos(\omega t - 90°)$

$17.1 \cos 200t \ V$