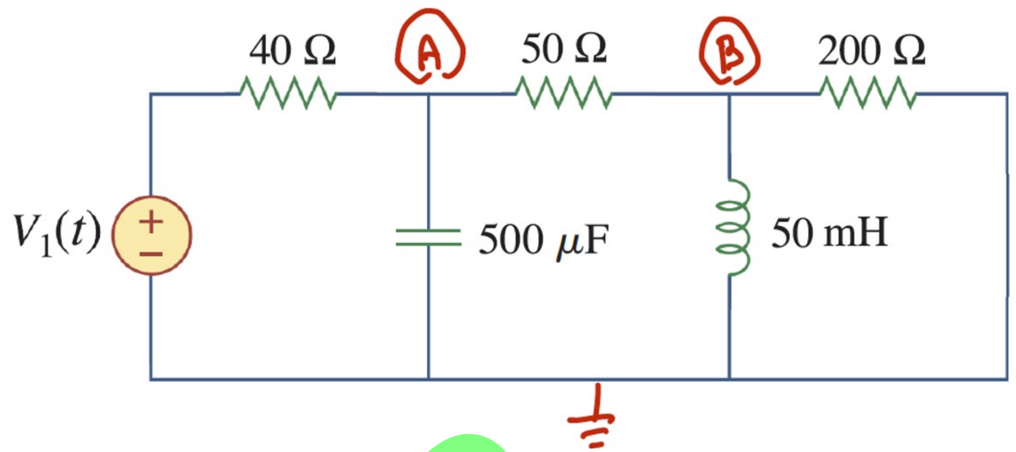
Phasors – 3

how phasors help

Reconsider our Example

- Use a phasor model for the source (and again, just **watch**)



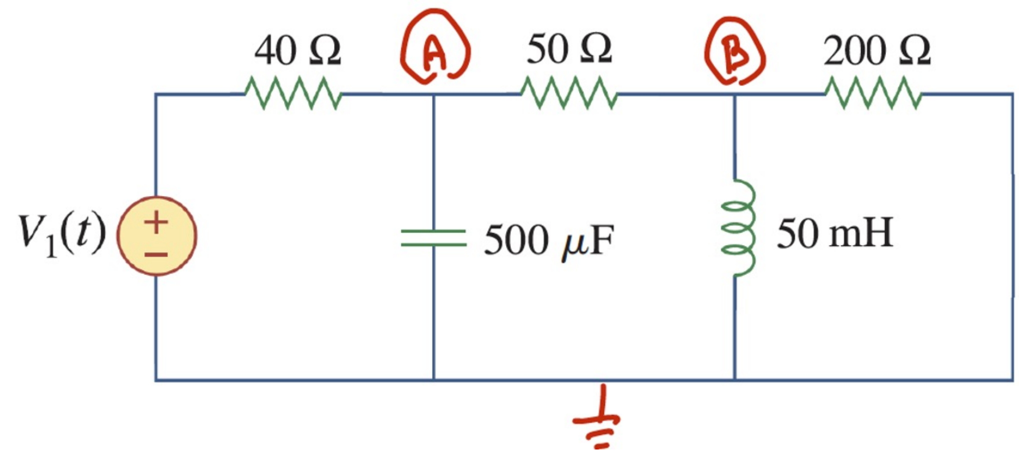
$$V_1(t) = \mathbf{V}e^{j\omega t}$$

$$\frac{A(t) - V_1(t)}{40} + i_C(t) + \frac{A(t) - B(t)}{50} = 0$$

$$\frac{B(t)}{200} + i_L(t) + \frac{B(t) - A(t)}{50} = 0$$

- If our forcing function is

$$V_1(t) = \mathbf{V}e^{j\omega t}$$



- Then the particular (steady-state) solution is also a complex exponential

Table 2.1 **Method of Undetermined Coefficients**

Term in $r(x)$	Choice for $y_p(x)$
$ke^{\gamma x}$	$Ce^{\gamma x}$
$kx^n \ (n = 0, 1, \cdots)$	$K_n x^n + K_{n-1}x^{n-1} + \cdots + K_1 x + K_0$
$k \cos \omega x$ $k \sin \omega x$	$K \cos \omega x + M \sin \omega x$ $= \boldsymbol{K\cos(\omega t + \theta)}$

- Phasor notation:

$$A(t) \Longrightarrow \mathbf{A}e^{j500t} \quad B(t) \Longrightarrow \mathbf{B}e^{j500t} \quad V_1(t) \Longrightarrow 10e^{j500t}$$

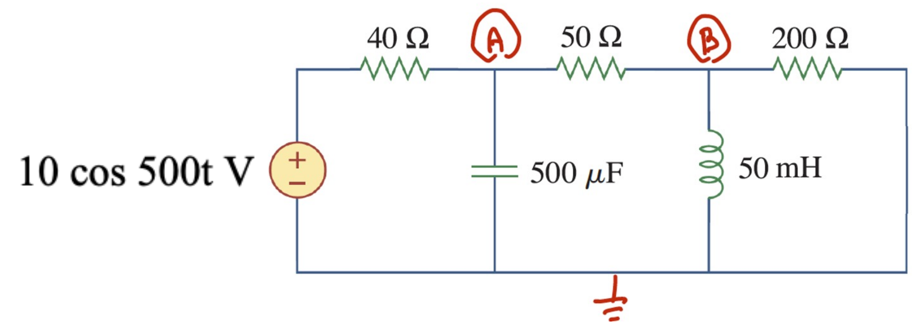
$$\mathbf{A} = Ae^{\phi} \quad \mathbf{B} = Be^{j\theta} \quad \mathbf{V_1} = 10$$

- Consider the capacitor and inductor currents with this general model; if the voltage is $v(t) = \mathbf{V}e^{j\omega t}$ then

$$i_C(t) = C\frac{dv}{dt} = C\frac{d}{dt}\left(\mathbf{V}e^{j\omega t}\right) = j\omega C\,\mathbf{V}e^{j\omega t} = j\frac{1}{4}\mathbf{V}e^{j\omega t}$$

$$i_L(t) = \frac{1}{L}\int^{t} v(u)\,du = \frac{1}{L}\int^{t} \mathbf{V}e^{j\omega u}du = \frac{\mathbf{V}e^{j\omega t}}{j\omega L} = \frac{\mathbf{V}e^{j\omega t}}{j25}$$

- Back to the original node equations

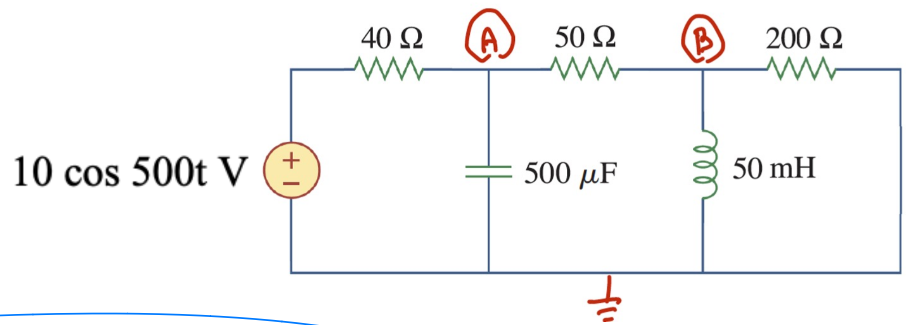


$$\frac{\mathbf{A}e^{j500t} - 10e^{j500t}}{40} + j\frac{1}{4}\mathbf{A}e^{j500t} + \frac{\mathbf{A}e^{j500t} - \mathbf{B}e^{j500t}}{50} = 0$$

$$\frac{\mathbf{B}e^{j500t}}{200} + \frac{\mathbf{A}e^{j500t}}{j25} + \frac{\mathbf{B}e^{j500t} - \mathbf{A}e^{j500t}}{50} = 0$$

- Can cancel the common e^{j500t} term

- Result is a set of simultaneous equations with complex coefficients

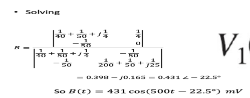


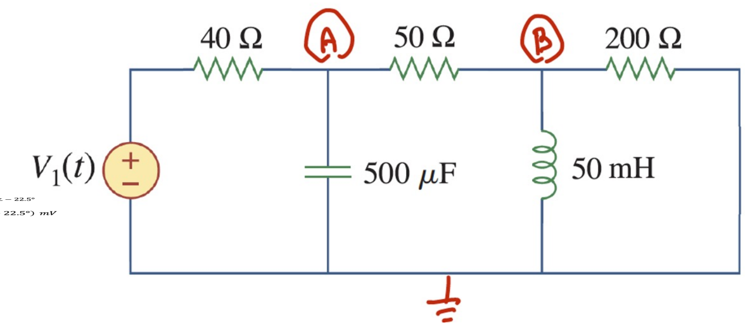
$$\frac{\mathbf{A}-10}{40}+j\frac{\mathbf{A}}{4}+\frac{\mathbf{A}-\mathbf{B}}{50}=0$$

$$\frac{\mathbf{B}}{200}+\frac{\mathbf{A}}{j25}+\frac{\mathbf{B}-\mathbf{A}}{50}=0$$

- Solve using various methods (Cramer, matrix inverse, etc)

- Solving





$$B = \frac{\begin{vmatrix} \frac{1}{40}+\frac{1}{50}+j\frac{1}{4} & \frac{1}{4} \\ -\frac{1}{50} & 0 \end{vmatrix}}{\begin{vmatrix} \frac{1}{40}+\frac{1}{50}+j\frac{1}{4} & -\frac{1}{50} \\ -\frac{1}{50} & \frac{1}{200}+\frac{1}{50}+\frac{1}{j25} \end{vmatrix}}$$

$$= 0.398 - j0.165 = 0.431\angle -22.5°$$

So $B(t) = 431\cos(500t - 22.5°)\ mV$

Same as last time !!

So How to Use Phasors?

- Extend sinusoidal voltages/currents to phasors (complex)

$$V_s \cos(\omega t + \phi) \Rightarrow \mathbf{V} = V_s e^{j\phi}$$

$$I_s \cos(\omega t + \phi) \Rightarrow \mathbf{I} = I_s e^{j\phi}$$

- Convert components (R,L,C) to <u>impedances</u>

$$Z = \begin{cases} R & \text{resistor} \\ j\omega L & \text{inductor} \\ \dfrac{1}{j\omega C} = -j\dfrac{1}{\omega C} & \text{capacitor} \end{cases}$$

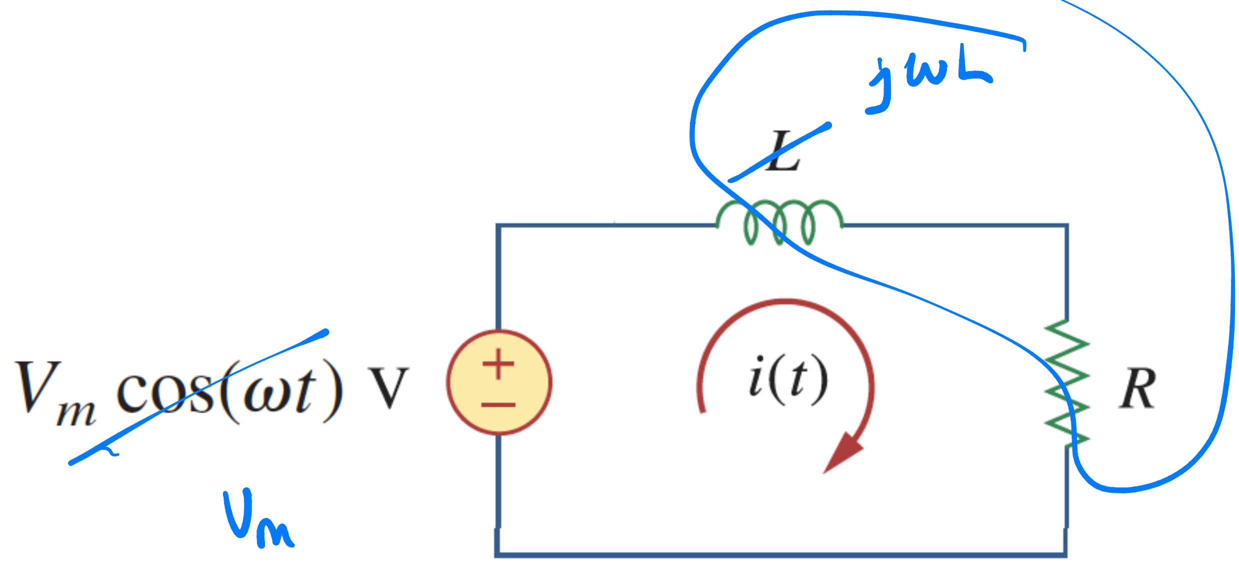
- Solve the problem using Ohm's Law, KVL/KCL, …

- Convert back

$$B\angle\theta \Rightarrow B\cos(\omega t + \theta)$$

Example:

- Convert problem
- Combine in series
- Apply Ohm's Law
- Convert back



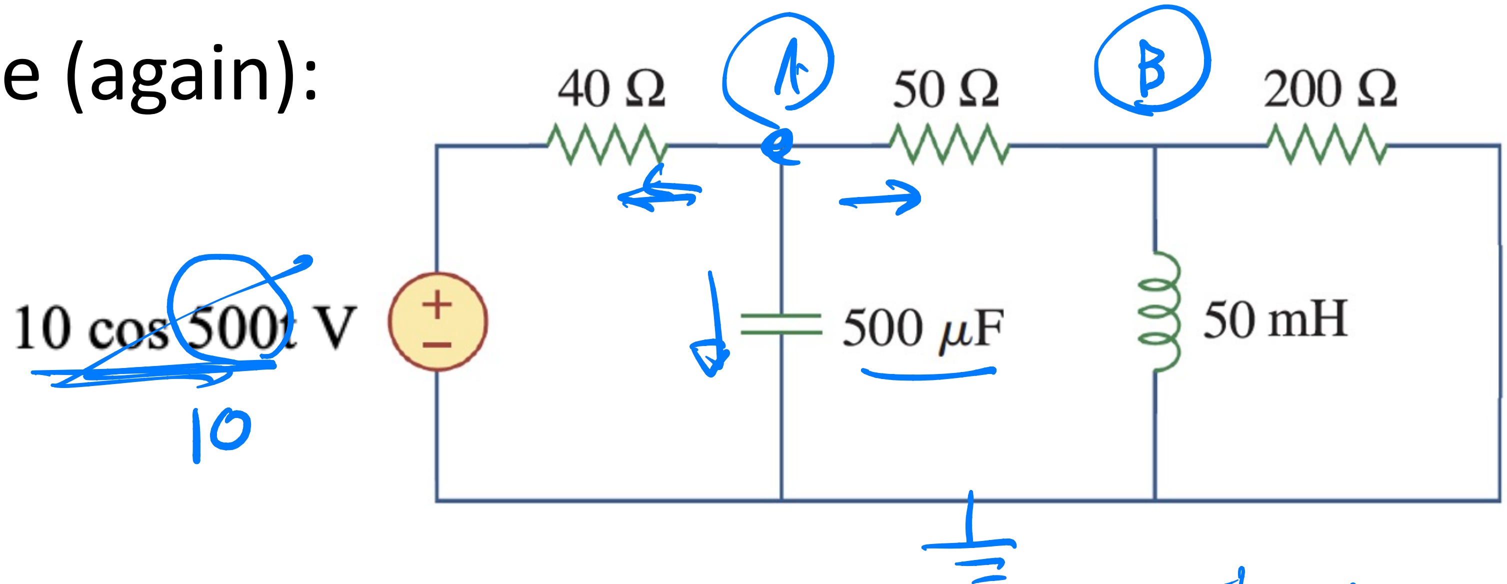
R+jωL

$$I = \frac{V_m}{R + j\omega L}$$

$$= \frac{V_m \angle 0^\circ}{\sqrt{R^2 + \omega^2 L^2}} \angle \tan^{-1}\frac{\omega L}{R}$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

Example (again):



node A: $\frac{A-10}{40} + \frac{A-B}{50} + \frac{A}{-j4} = 0$

node B: $\frac{B-A}{50} + \frac{B}{200} + \frac{B}{j25} = 0$

$j\omega L = j \cdot 500 \cdot \frac{50}{1000}$

$= j25$

$\frac{1}{j\omega C}$

$= \frac{1}{j\, 500 \cdot 500 \cdot 10^{-6}}$

$= -j \frac{10^6}{25 \cdot 10^4}$

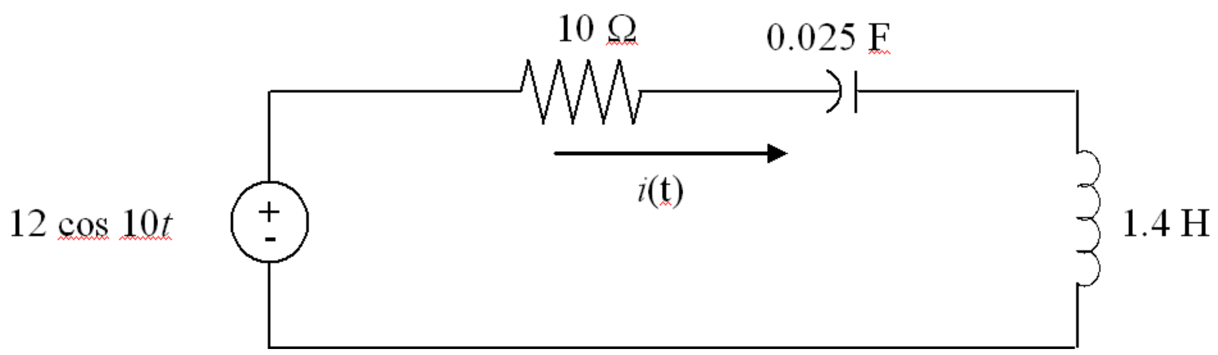
$= -j4$

- Previous equations:

$$\frac{\mathbf{A} - 10}{40} + j\frac{\mathbf{A}}{4} + \frac{\mathbf{A} - \mathbf{B}}{50} = 0$$

$$\frac{\mathbf{B}}{200} + \frac{\mathbf{A}}{j25} + \frac{\mathbf{B} - \mathbf{A}}{50} = 0$$

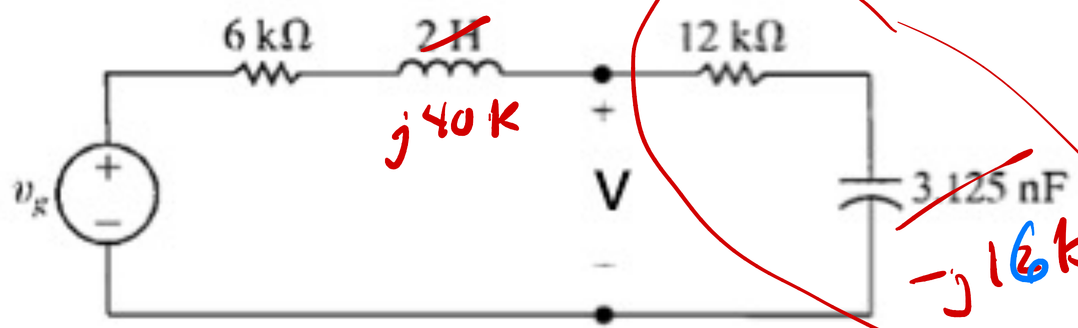
Example: find the current $i(t)$



$0.848\ \cos(10t - 45°)$ A

Example: find V(t) by voltage division

$v_g(t) = 75\cos 20{,}000t\ \text{V}$



1- current + phasor

2. voltage division

$jωL = j\,20000 \cdot 2$

$\frac{1}{jωC} = -j\frac{10^9}{20000 \cdot 3.125}$

$$V = 75 \cdot \frac{12000(1-j)}{18000 + j28000}$$

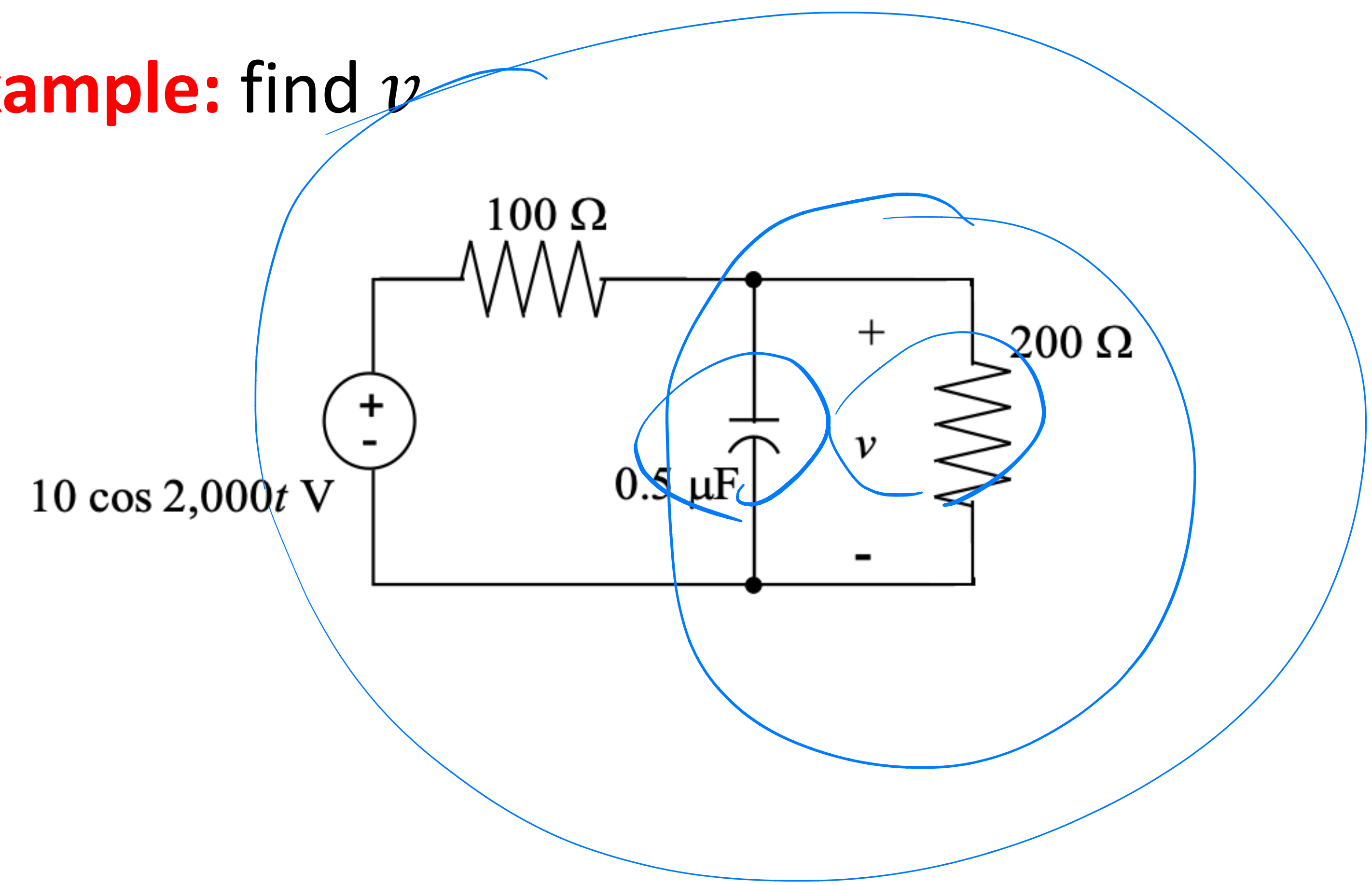
$$= 75 \cdot \frac{12(1-j)}{18 + j28} = \frac{75 \cdot 12(1-j)}{2(9+j14)} = \frac{75 \cdot 12 \cdot \sqrt{2}\ \angle -45^\circ}{2\sqrt{9^2+14^2}\ \angle \tan^{-1}\frac{14}{9}}$$

3- write answer in polar form

$$\approx 37\angle -80^\circ$$

$50\cos(20{,}000t - 106°)\ V$

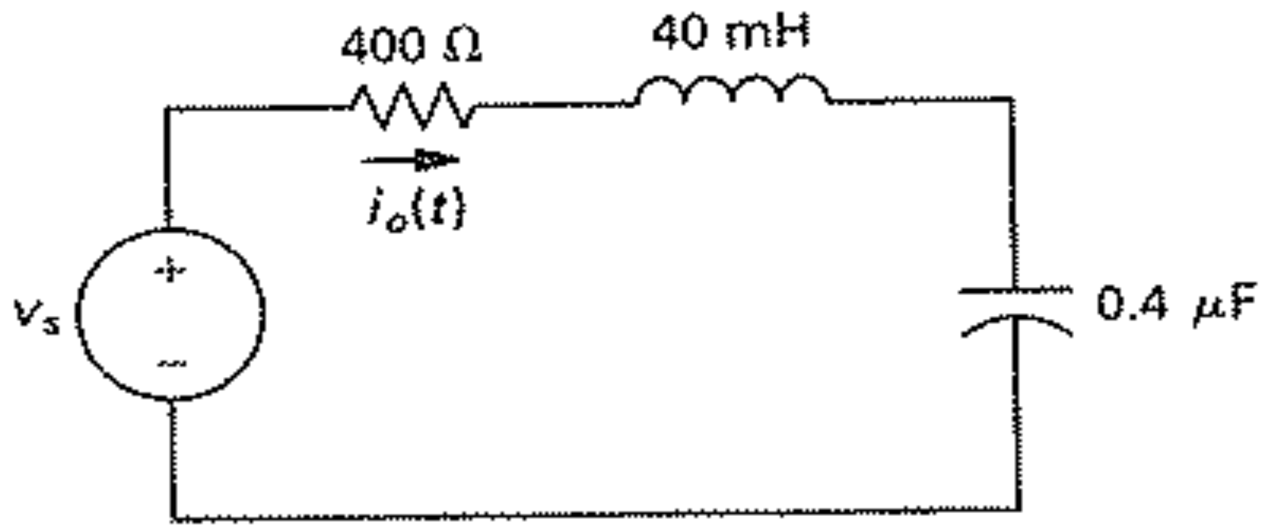
Example: find v



$$6.65\ \cos(2000t - 3.81°)\ V$$

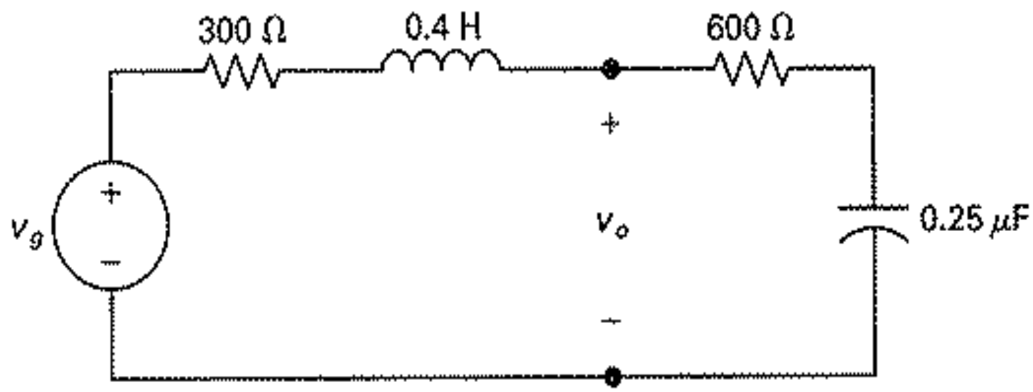
Practice problem: find the current if

$v_s(t) = 750 \cos 5000t$



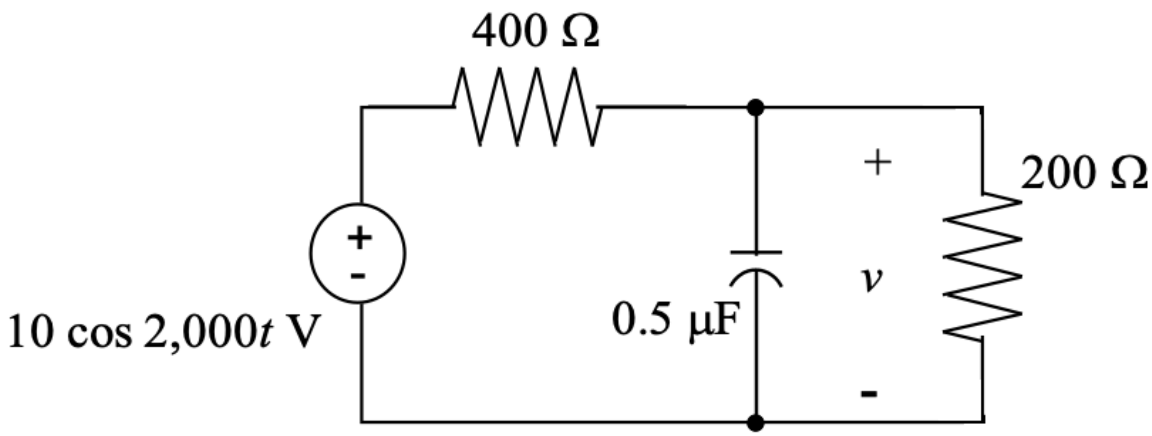
$1.5 \cos(5000t + 36.9°)\ A$

Practice problem: if $v_g(t)$ = 75 cos 5000t V, find the time expression for v_o



$130\cos(5000t - 93.4°)$ V

Practice problem: find v



$3.30\ \cos(2000t - 7.60°)\ V$