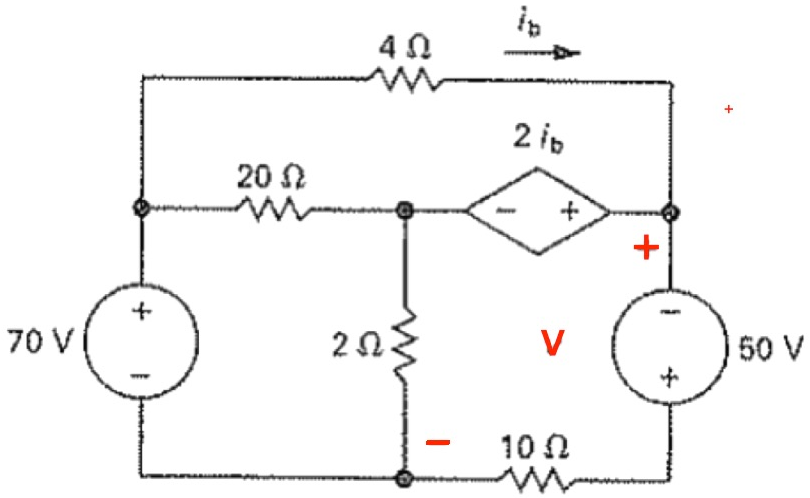
Node – 1

basic concepts

General Methods to Analyze Circuits

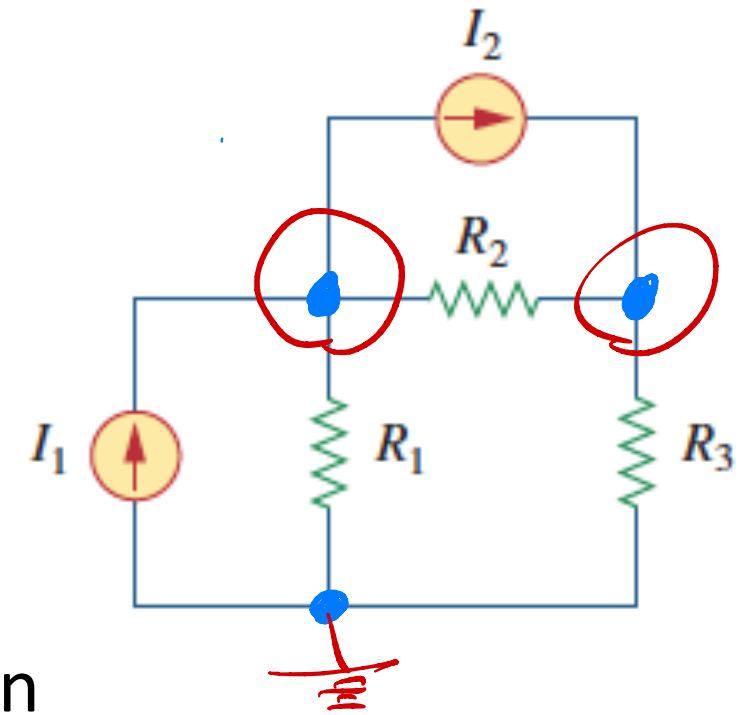
- What to do first?
  - KVL?
  - KCL?
  - Ohm's Law?



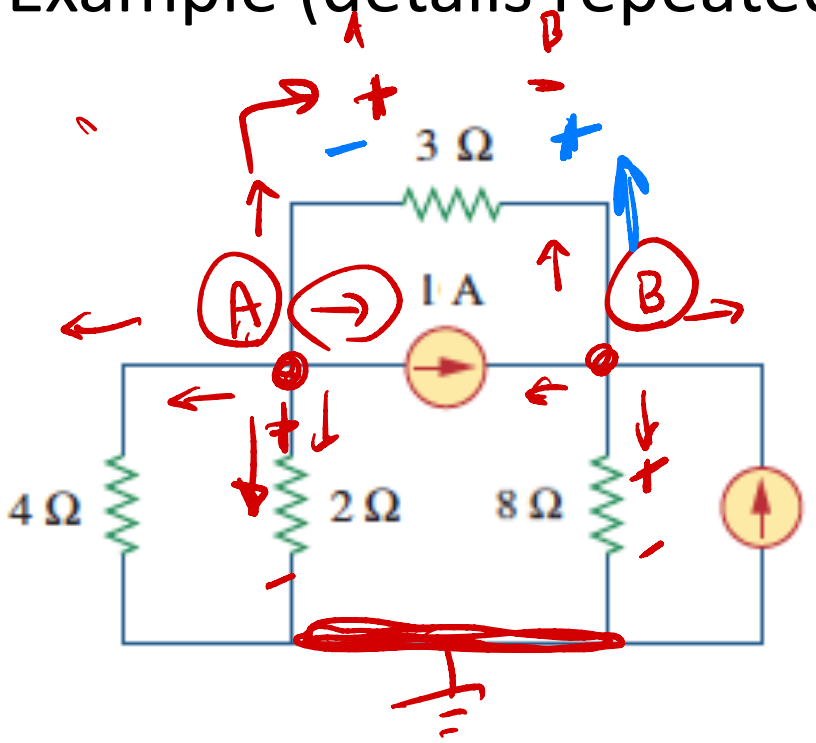
- We need a more direct approach:
  - Nodal analysis (KCL based)
  - Mesh analysis (KVL based, **end of semester**)
- Review appendix A (also online)

Node Analysis

- Consider a circuit with current sources and resistors only
- Identify nodes
  - Select one as “ground”
  - Label others
  - Write KCL on these other nodes
  - Use Ohm’s Law for current in the resistive branches
  - Solve resulting equations



Example (details repeated on next slide)



$$12\left(1 + \frac{A}{2} + \frac{A}{4} + \frac{A-B}{3}\right) = 0 \quad \text{(A)}$$

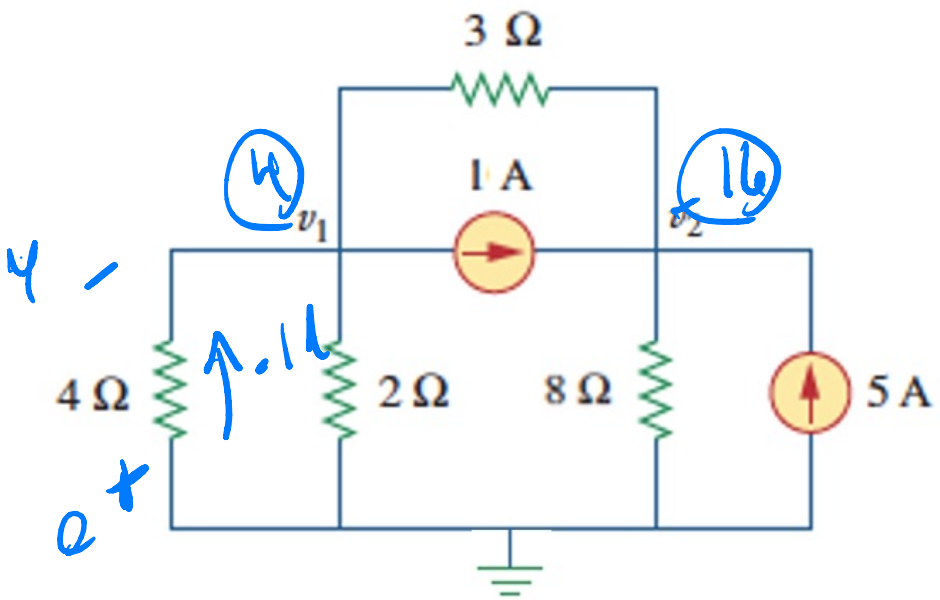
$$24\left(-5 + \frac{B}{8} - 1 + \frac{B-A}{3}\right) = 0 \quad \text{(B)}$$

$$6A + 3A + 4A - 4B = -12$$

$$3B + 8B - 8A = 144$$

$$13A - 4B = -12$$

$$-8A + 11B = 144$$



$$\frac{v_1}{2}+\frac{v_1}{4}+\frac{v_1-v_2}{3}+1 = 0$$

$$\frac{v_2}{8}+\frac{v_2-v_1}{3} = 1+5$$

$$6v_1+3v_1+4v_1-4v_2 = -12$$

$$3v_2+8v_2-8v_1 = 144$$

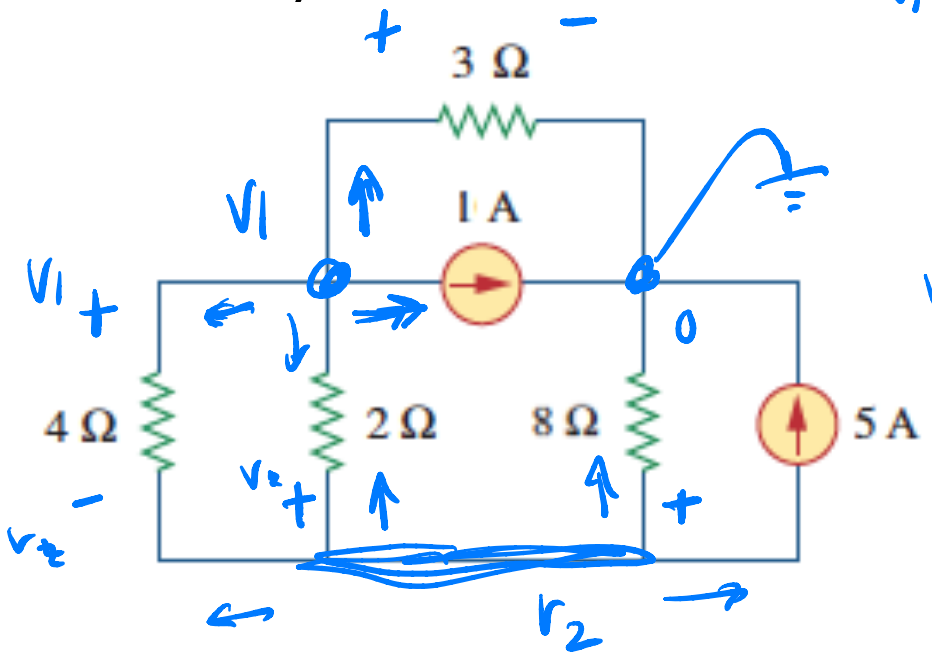
$$13v_1-4v_2 = -12$$

$$-8v_1+11v_2 = 144$$

$$v_1=\frac{\begin{vmatrix}-12 & -4\\ 144 & 11\end{vmatrix}}{\begin{vmatrix}13 & -4\\ -8 & 11\end{vmatrix}}=\frac{-132+576}{143-32}=\frac{444}{111}=4 \text{ volts}$$

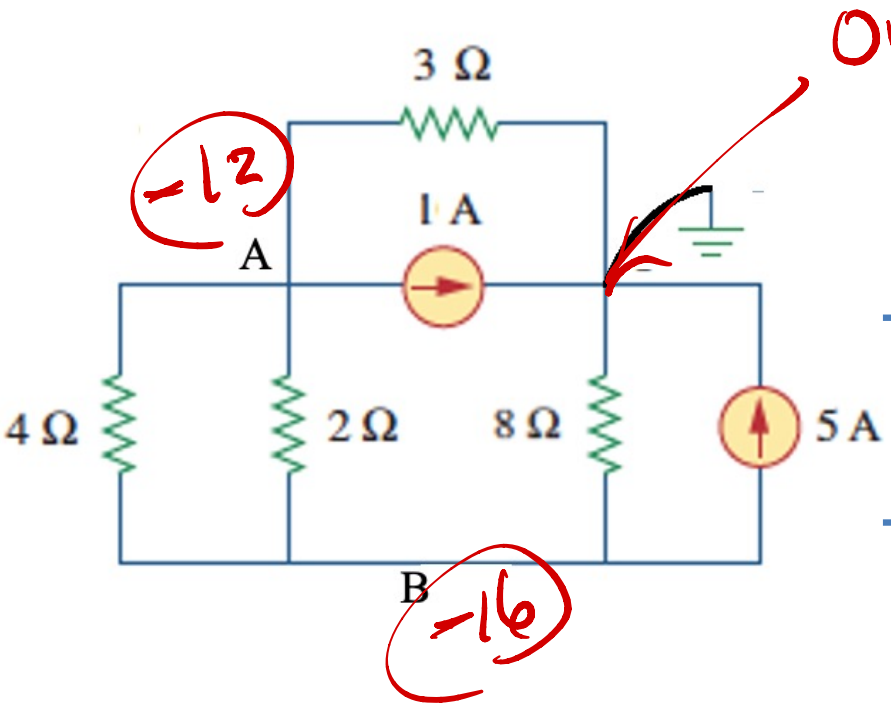
$$v_2=\frac{\begin{vmatrix}13 & -12\\ -8 & 144\end{vmatrix}}{\begin{vmatrix}13 & -4\\ -8 & 11\end{vmatrix}}=\frac{1872-96}{111}=\frac{1776}{111}=16 \text{ volts}$$

Example (same circuit, change ground to top right, details on next slide)



v_1: $1 + \frac{v_1}{3} + \frac{v_1 - v_2}{4} + \frac{v_1 - v_2}{2} = 0$

v_2: $5 + \frac{v_2}{8} + \frac{v_2 - v_1}{2} + \frac{v_2 - v_1}{4} = 0$



$$\frac{A-B}{2}+\frac{A-B}{4}+\frac{A}{3}+1 = 0$$
$$\frac{B-A}{4}+\frac{B-A}{2}+\frac{B}{8}+5 = 0$$

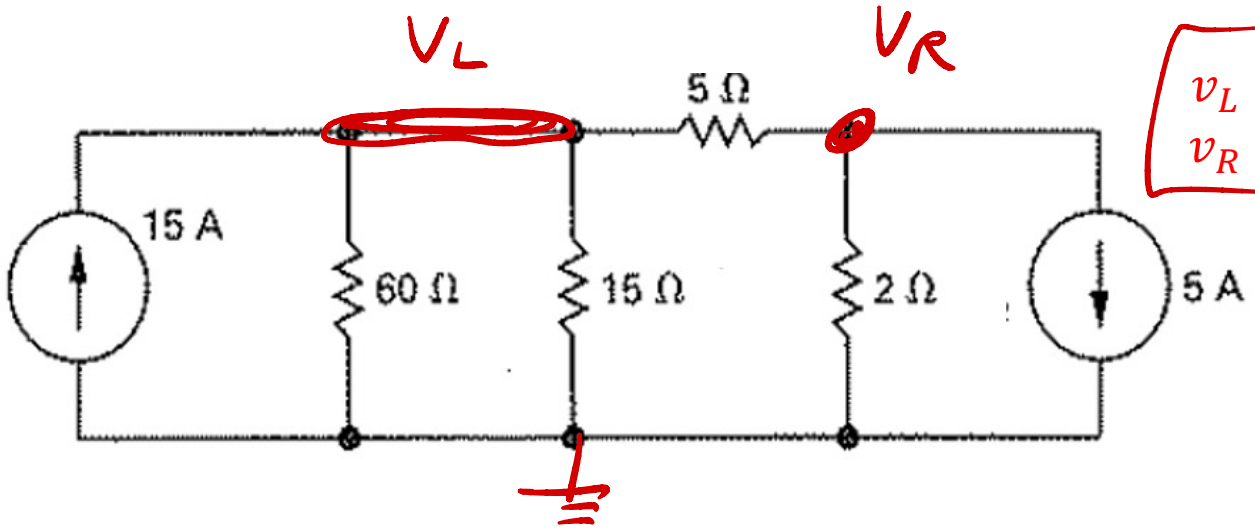
$$6A-6B+3A-3B+4A = -12$$
$$2B-2A+4B-4A+B = -40$$

$$13A-9B = -12$$
$$-6A+7B = -40$$

$$A=\frac{\begin{vmatrix}-12 & -9\\ -40 & 7\end{vmatrix}}{\begin{vmatrix}13 & -9\\ -6 & 7\end{vmatrix}}=\frac{-84-360}{91-54}=\frac{-444}{37}=-12 \text{ volts}$$

$$B=\frac{\begin{vmatrix}13 & -12\\ -6 & -40\end{vmatrix}}{\begin{vmatrix}13 & -9\\ -6 & 7\end{vmatrix}}=\frac{-520-72}{35}=\frac{-592}{37}=-16 \text{ volts}$$

Example:



$v_L = 60\ V$
$v_R = 10\ V$

write KCL

V_L:

$$60\left(-15 + \frac{V_L}{60} + \frac{V_L}{15} + \frac{V_L - V_R}{5} = 0\right)$$

V_R:

$$10\left(5 + \frac{V_R}{2} + \frac{V_R - V_L}{5}\right) = 0$$

$$16\,V_L - 12\,V_R = 900$$

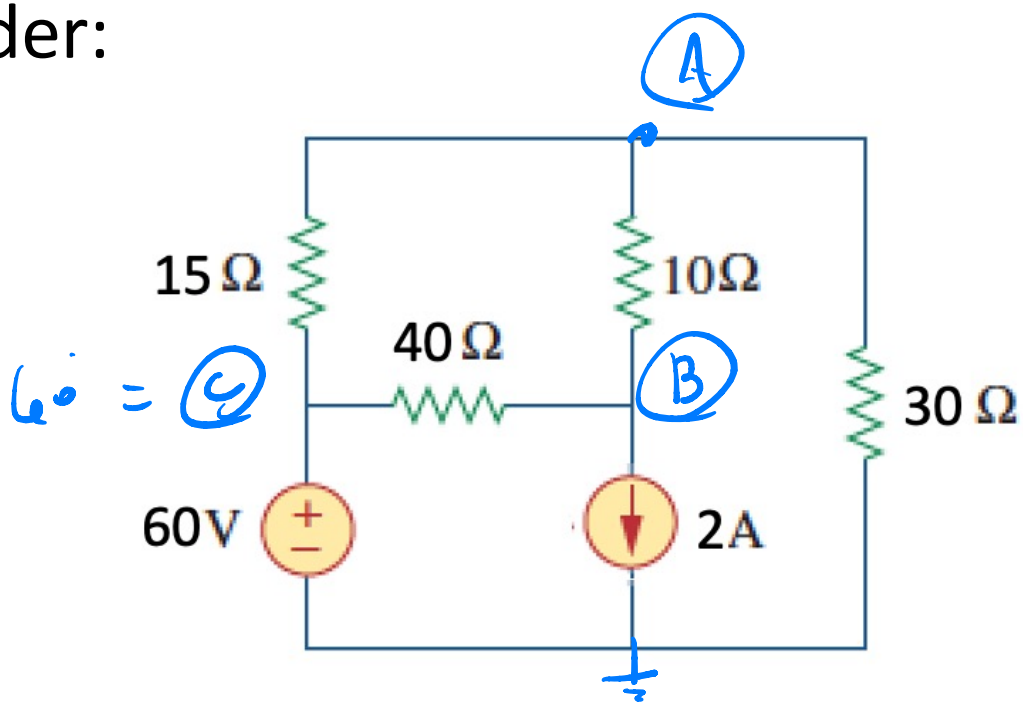
$$-2\,V_L + 7\,V_R = -50$$

$$\begin{bmatrix} 16 & -12 \\ -2 & 7 \end{bmatrix}\begin{bmatrix} V_L \\ V_R \end{bmatrix} = \begin{bmatrix} 900 \\ -50 \end{bmatrix}$$

$$\begin{bmatrix} V_L \\ V_R \end{bmatrix} = \frac{1}{88}\begin{bmatrix} 7 & 12 \\ 2 & 16 \end{bmatrix}\begin{bmatrix} 900 \\ -50 \end{bmatrix}$$

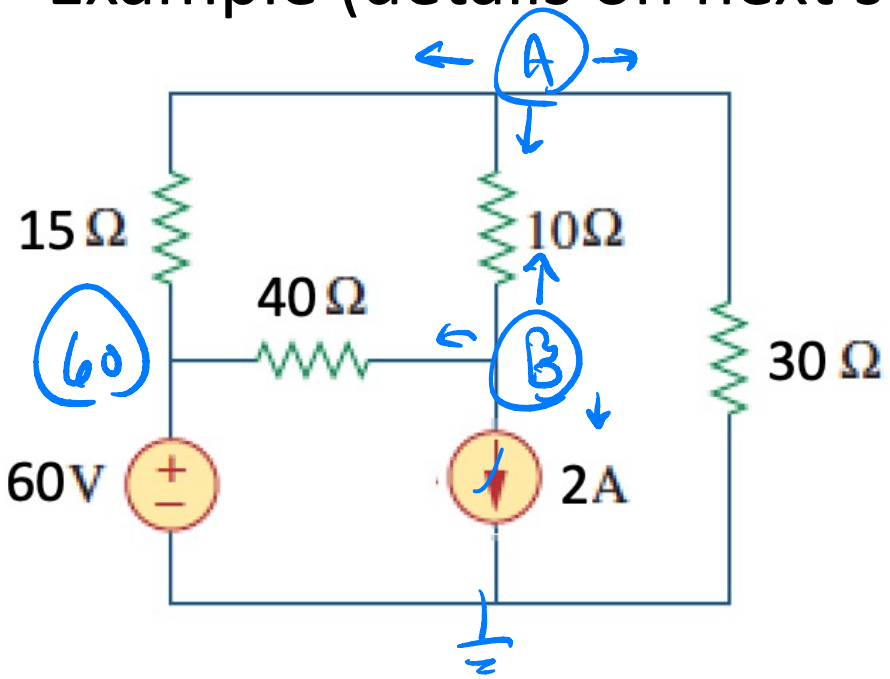
Extension #1 – a V-only branch

- Consider:



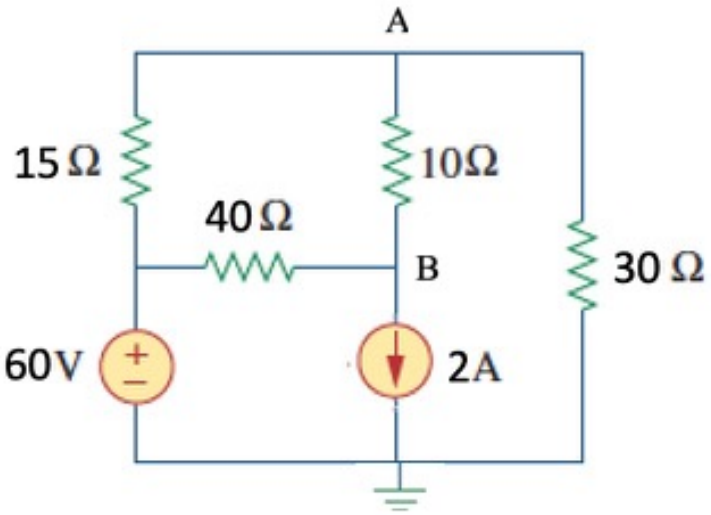
  - **IF** connected to ground, it's just one less node voltage to worry about

Example (details on next slide)



$$A: \quad \frac{A-60}{15} + \frac{A-B}{10} + \frac{A}{30} = 0$$

$$B: \quad 2 + \frac{B-60}{40} + \frac{B-A}{10} = 0$$



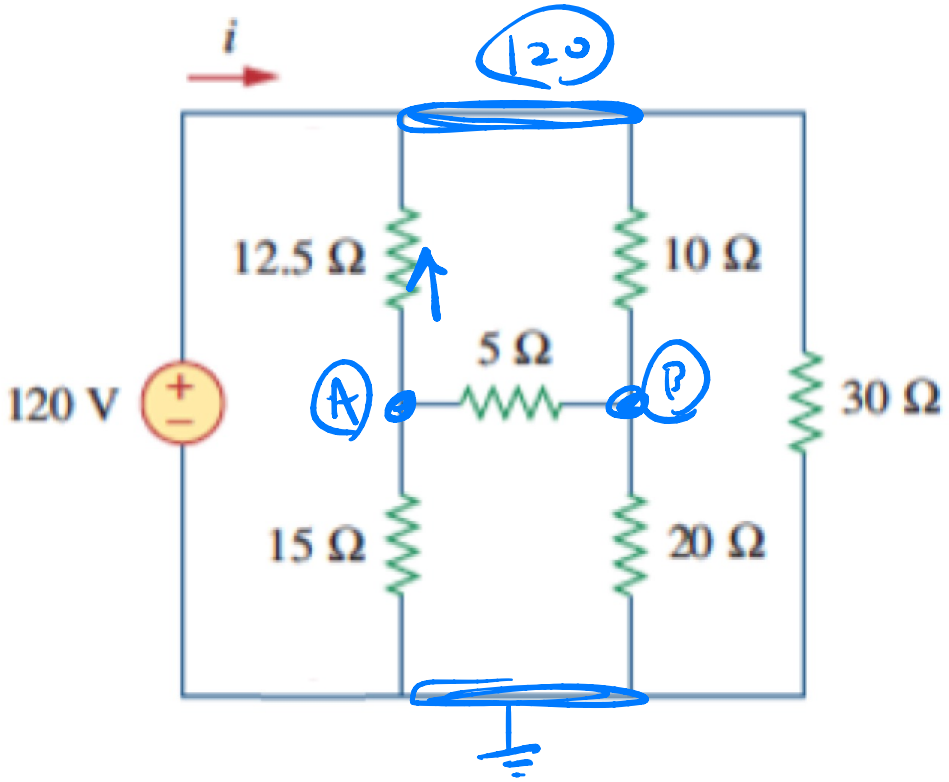
$$\frac{A-60}{15}+\frac{A-B}{10}+\frac{A}{30} = 0$$
$$\frac{B-60}{40}+\frac{B-A}{10}+2 = 0$$

$$6A-3B = 120$$
$$-4A+5B = -20$$

$$A = 30 \text{ volts}$$
$$B = 20 \text{ volts}$$

Example from Delta-Wye transformations

$v_L = 70.8\ V$
$v_R = 74.8\ V$
$i = 12.5\ A$



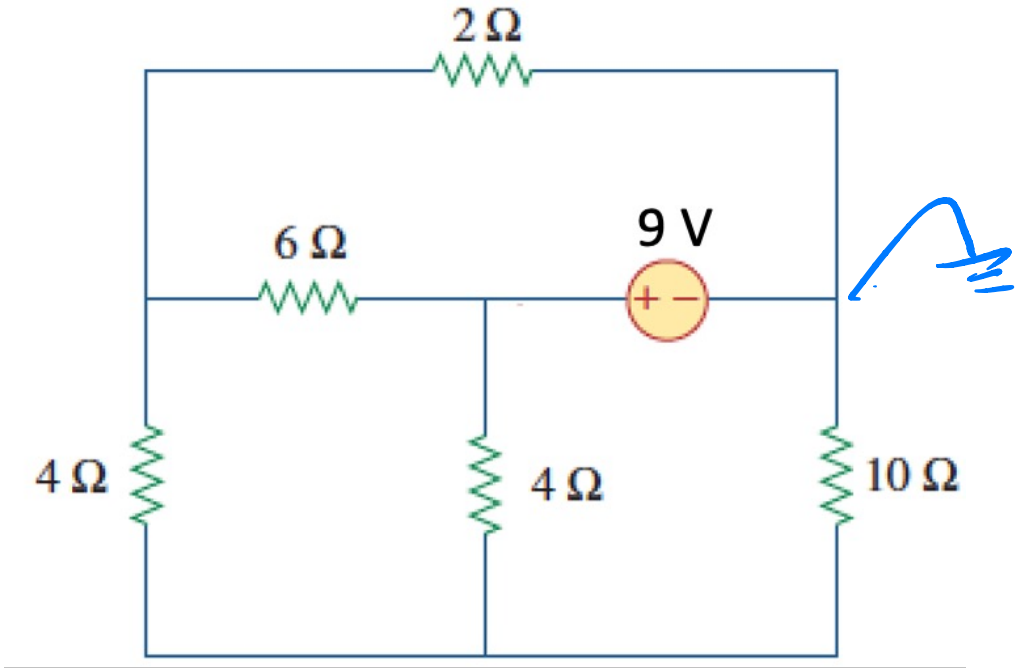
A:

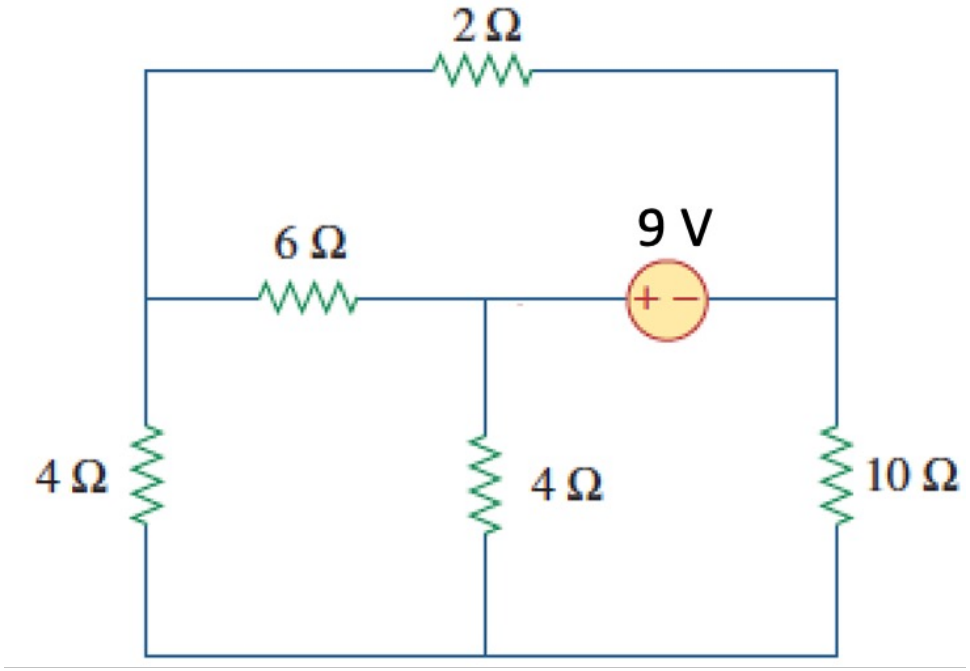
$$\frac{A-120}{12.5} + \frac{A}{15} + \frac{A-B}{5} = 0$$

B:

$$\frac{B}{20} + \frac{B-120}{10} + \frac{B-A}{5} = 0$$

Example: recall that we can place ground as needed; put it on the right and solve for left and bottom



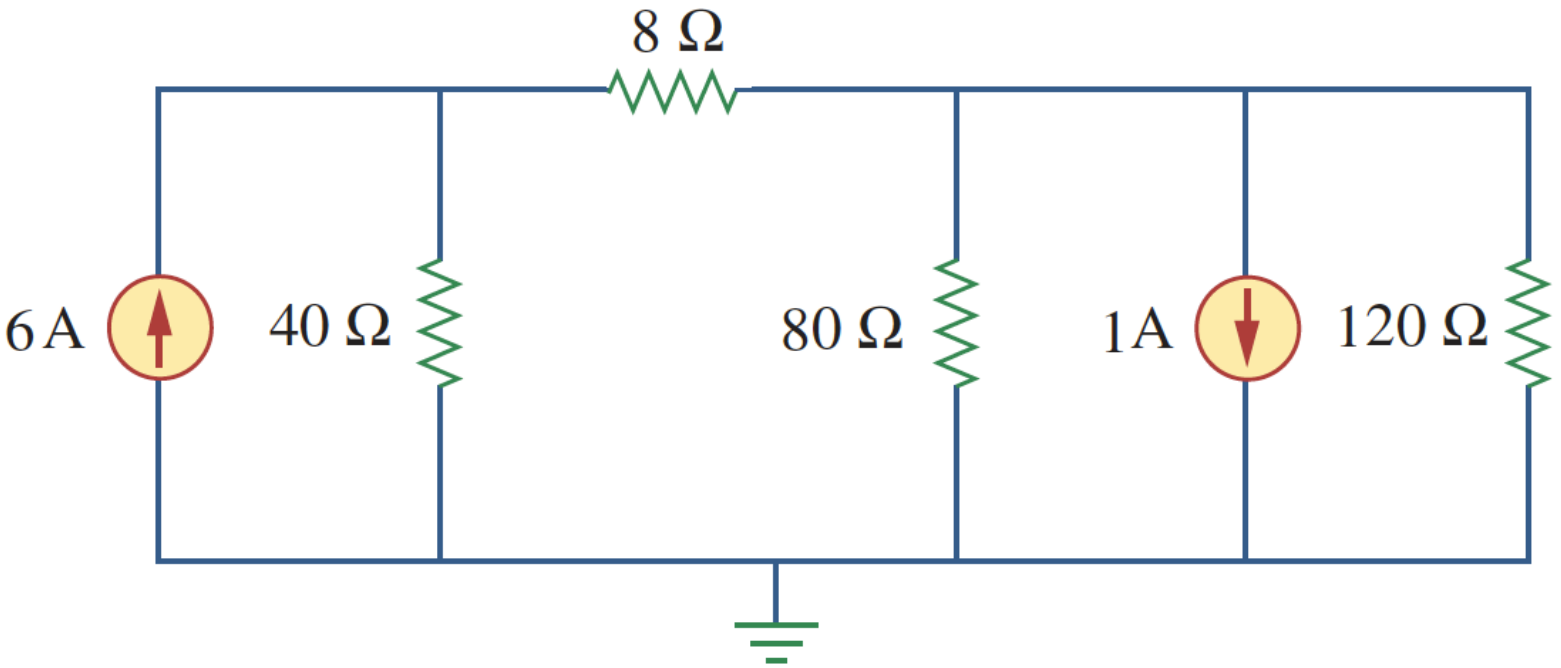


$v_L = 3\ V$

$v_B = 5\ V$

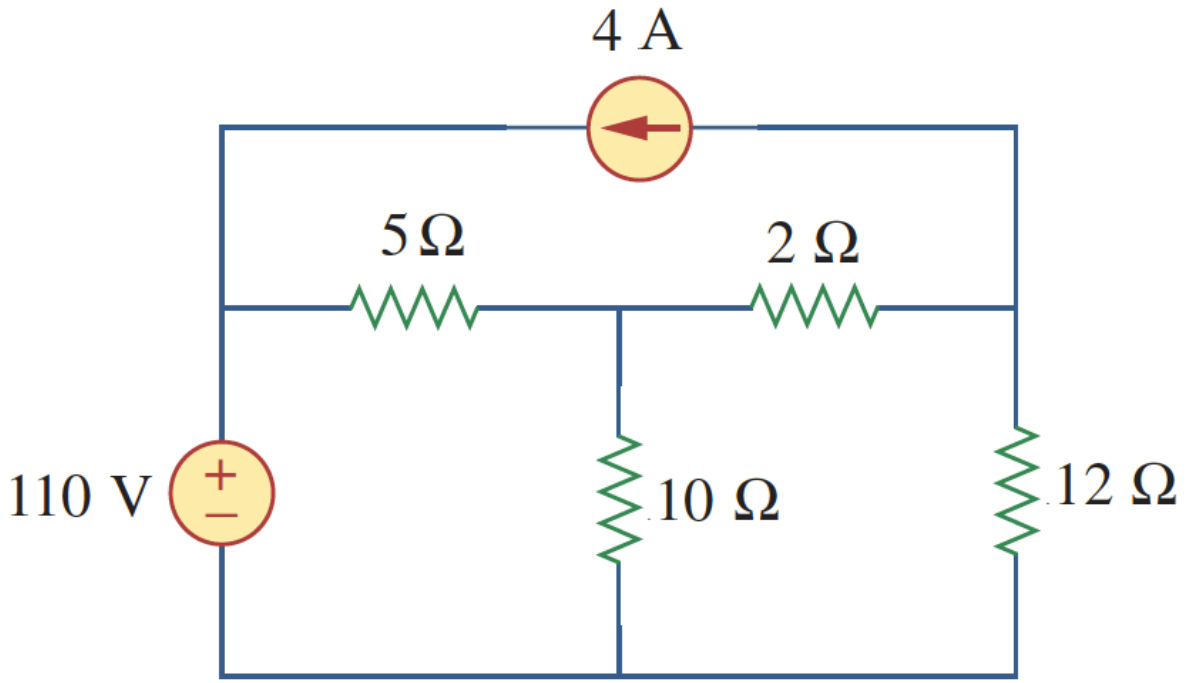
$120\ V, 96\ V$

Practice problem: find the nodes voltages to the left and right of the 8 Ω resistor



$50\ V, 36\ V$

Practice problem: assuming ground on the bottom, find the nodes voltages to the left and right of the 2 Ω resistor



$70\ V, 60\ V$

Practice problem: find the nodes voltages to the left and right of the $10\ k\Omega$ resistor, ground at the bottom