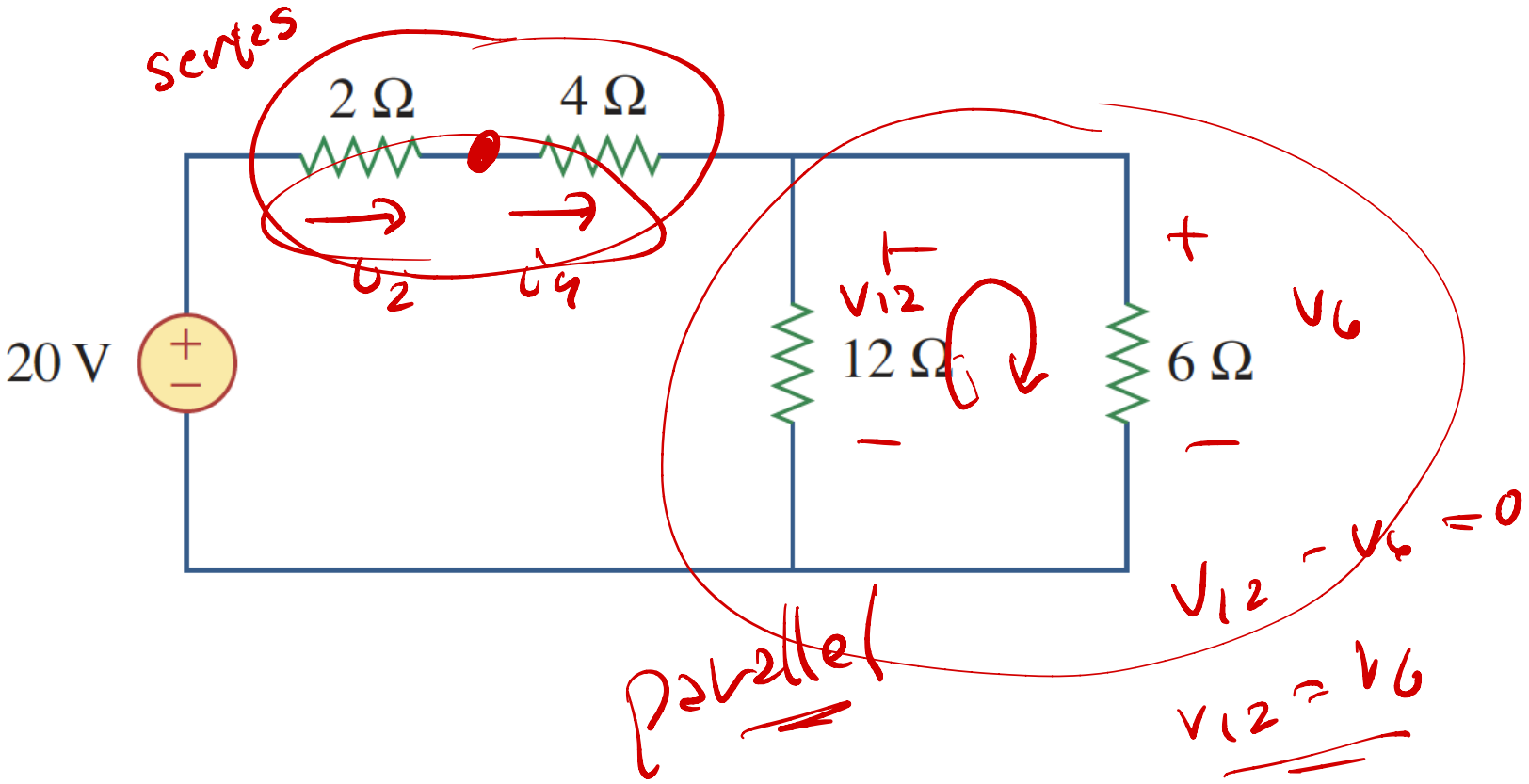
Basics – 4

series/parallel resistance;
voltage/current division

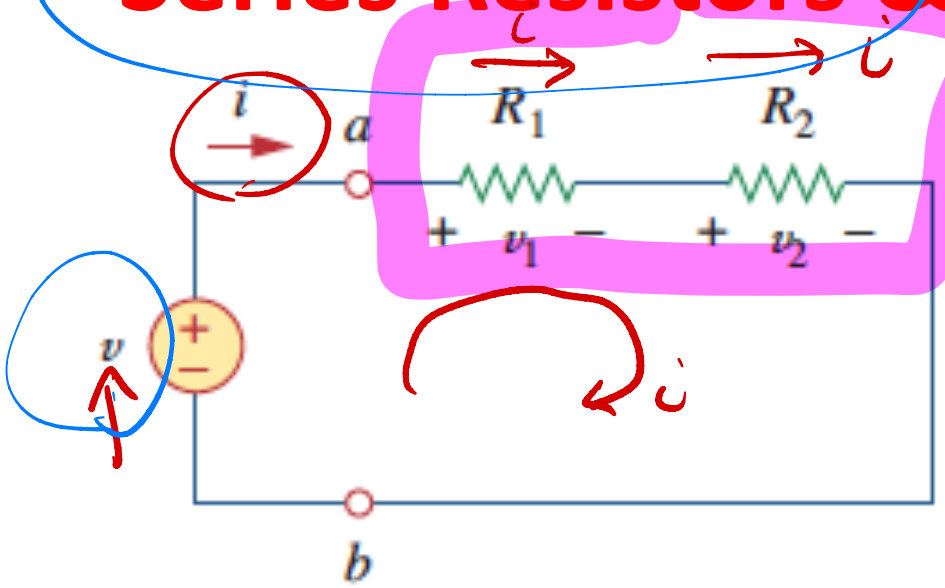
Applying KCL and KVL

Consider:

- Currents in the 2 and 4 Ω resistors (KCL)
- Voltages across the 6 and 12 Ω resistors (KVL)



Series Resistors & Voltage Division



$$V_1 = R_1 \cdot i$$

$$V_2 = R_2 \cdot i$$

$$V - V_1 - V_2 = 0$$

$$V - R_1 i - R_2 i = 0$$

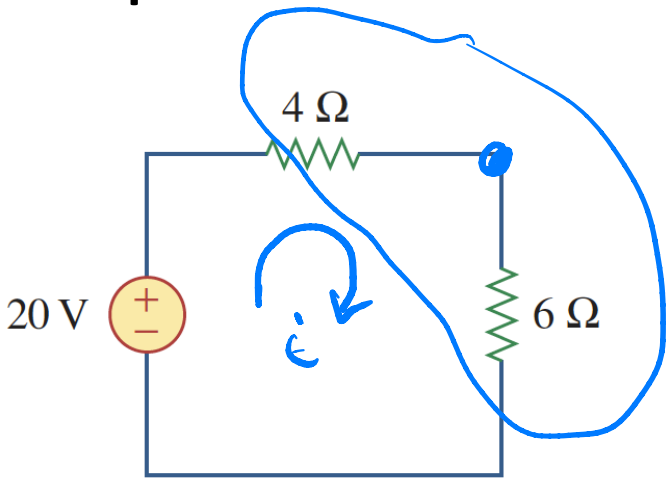
$$V = i\,(R_1 + R_2)$$

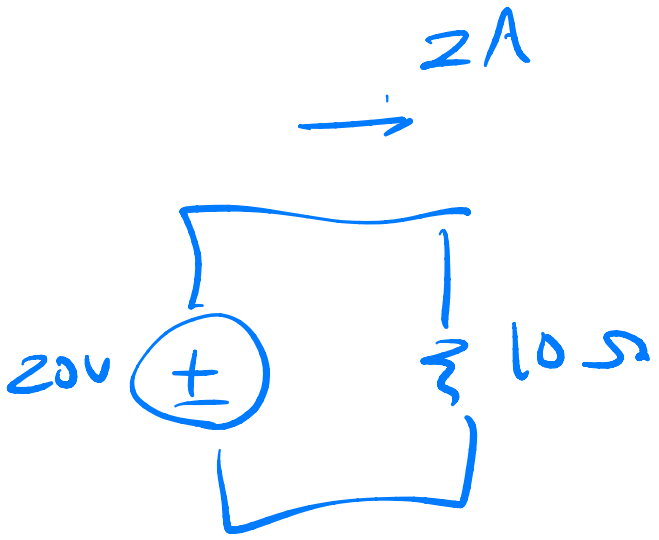
$$i = \frac{V}{R_1 + R_2}$$

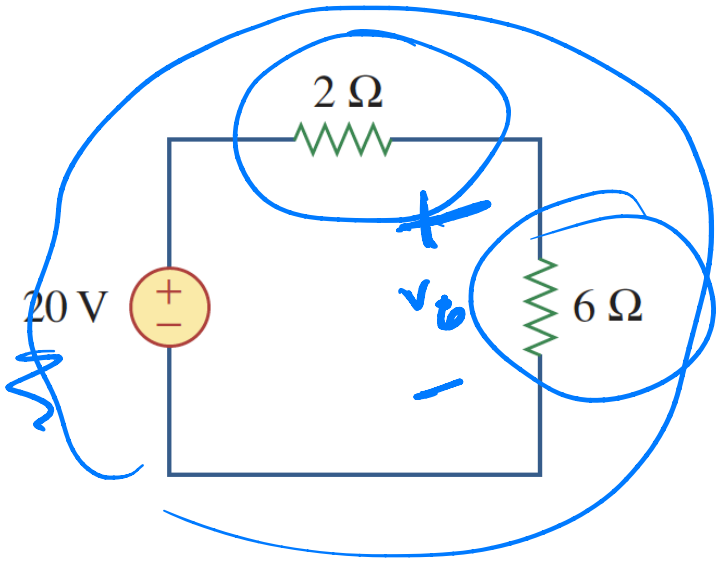
$$V_1 = \frac{R_1}{R_1 + R_2} \cdot V$$

$$V_2 = \frac{R_2}{R_1 + R_2} \cdot V$$

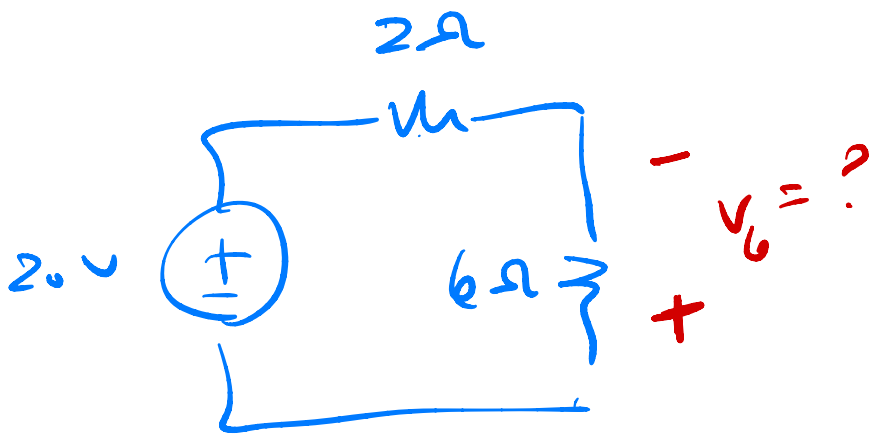
Examples



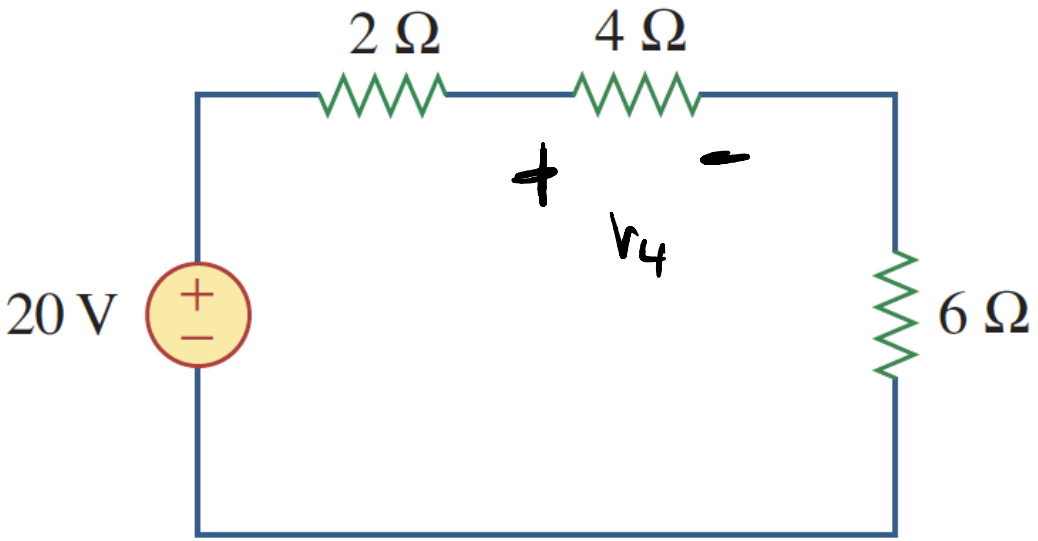


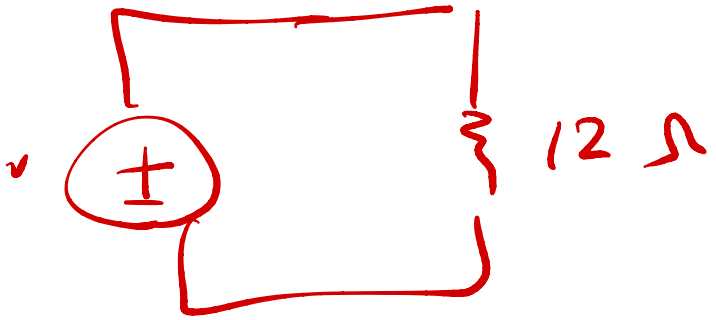


$$V_2 = \frac{6}{6+2} \cdot 20 = 15\ V$$



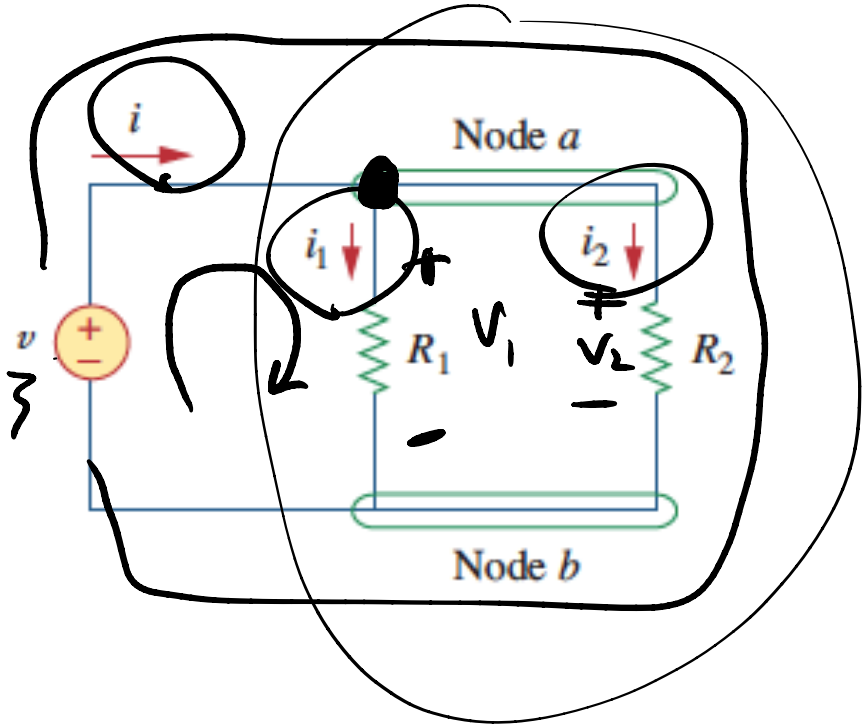
Example:





$$v_4 = \frac{4}{2+4+6} \cdot 20 = \frac{80}{12} \text{ V}$$

Parallel Resistors & Current Division



KVL $= V_1 = V$

$- V_2 = V$

Ohm $i_1 = \frac{V_1}{R_1} = \frac{V}{R_1}$

$i_2 = \frac{V_2}{R_2} = \frac{V}{R_2}$

$i = V \cdot \frac{R_1 R_2}{R_1 R_2} \cdot \left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

$= V \left(\frac{R_2 + R_1}{R_1 R_2}\right)$

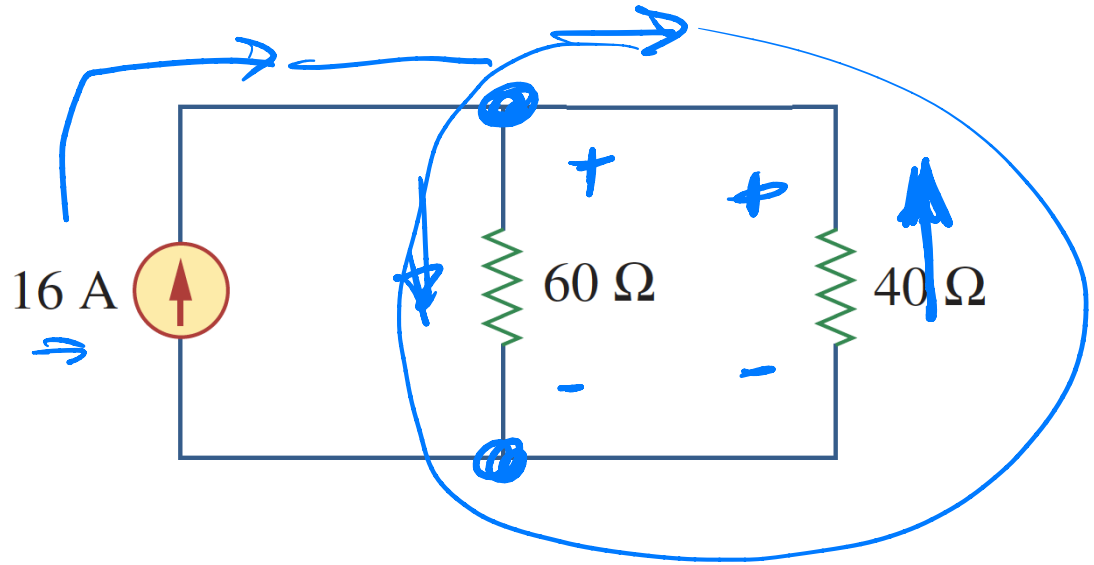
KCL $i = i_1 + i_2$

$= \frac{V}{R_1} + \frac{V}{R_2}$

$i = \frac{V}{\frac{R_1 R_2}{R_1 + R_2}}$

$i = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$

Examples:

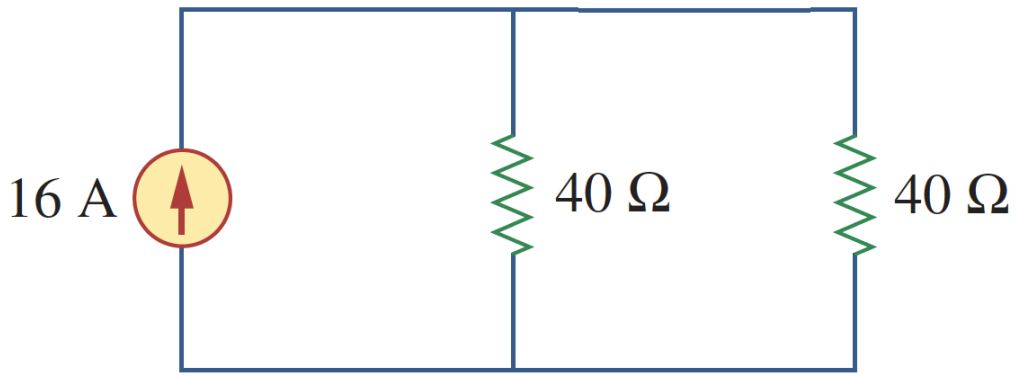


$$R_{eq} = \frac{60 \cdot 40}{60 + 40}$$

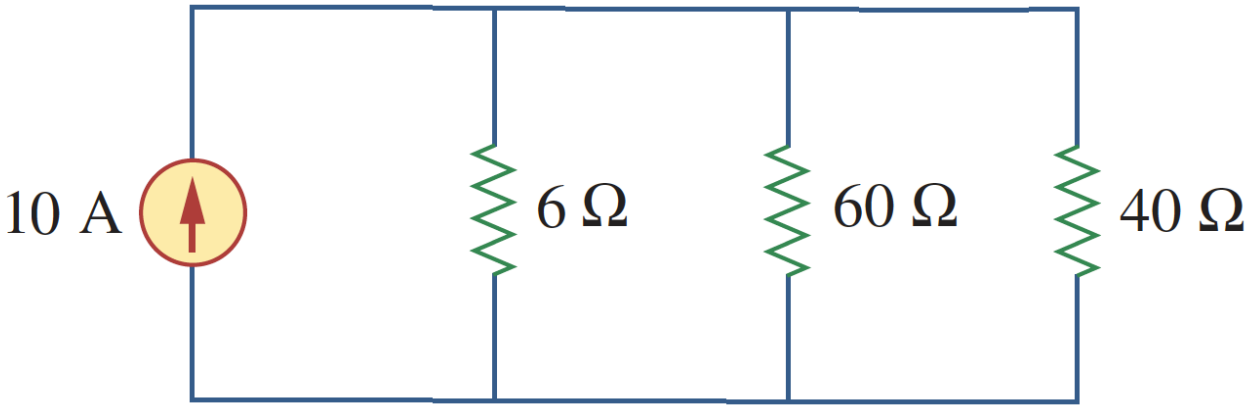
$$= \frac{2400}{100} = 24\,\Omega$$

$$i_{60} = 16 \cdot \frac{40}{40+60}$$

$$i_{40} = 16 \cdot \frac{60}{40+60}$$

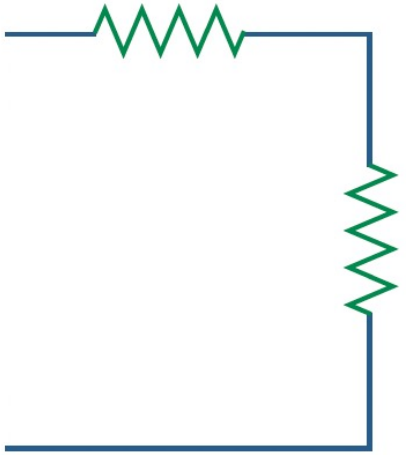


Example:

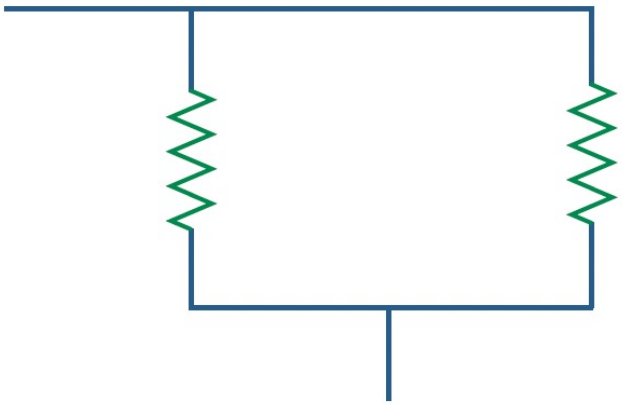


Series/Parallel Summary

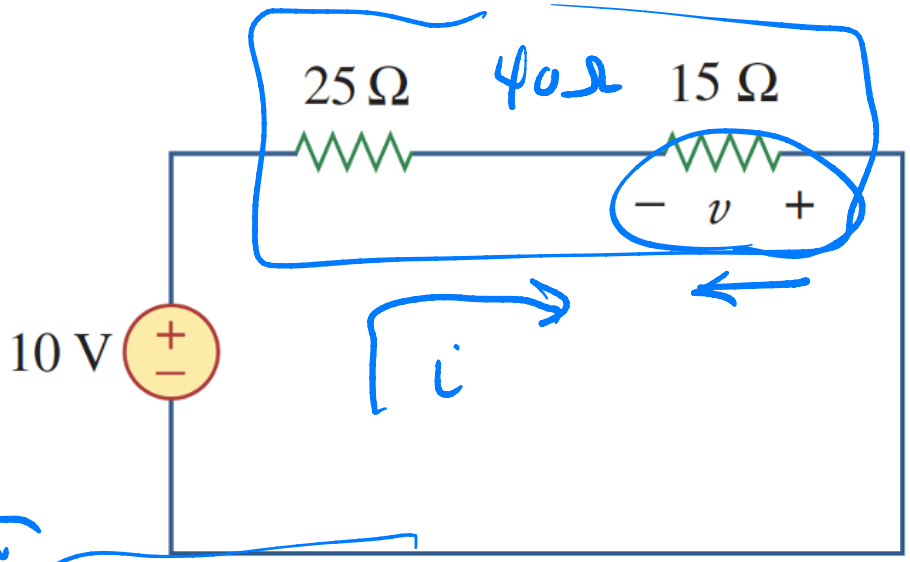
- Series: resistances add
  - Nothing connected in the middle
  - **Same current (KCL)**
  - Voltage divides proportionally



- Parallel: resistances add inversely
  - Connected at both ends
  - **Same voltage (KVL)**
  - Current divides proportionally



Example: find v



SR

$$i = \frac{10}{40} = \frac{1}{4}\,A$$

$$v_{15} = 15 \cdot (-i)$$

$$= 15 \cdot -\frac{1}{4} = -3.75\ V$$

Brute Force

$$v_{15} = 15 \cdot i_{15}$$

$$v_{25} = 25 \cdot i_{25}$$

$$i_{10} = -i_{25}$$

$$i_{15} = -i_{25}$$

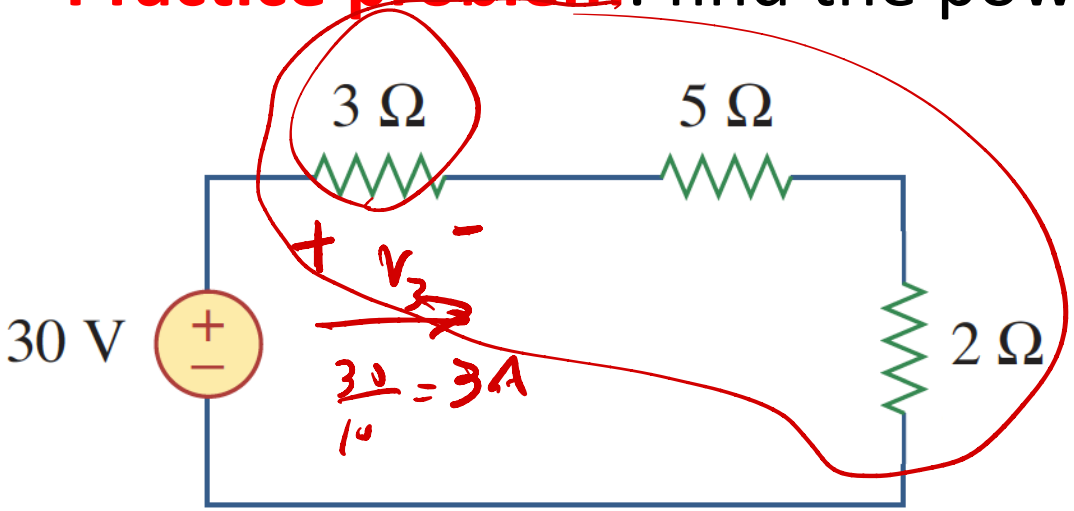
$$10 - v_{25} + v_{15} = 0$$

$$v_2 = -\frac{15}{15+25} \cdot 10$$

$-3.75\ V$

27 W

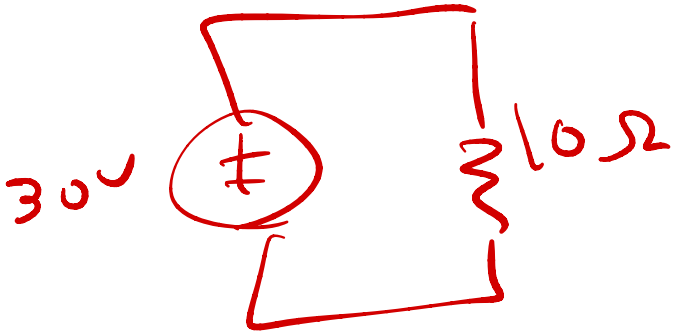
Practice problem: find the power in the 3 Ω resistor



$V_3 = 30 \cdot \frac{3}{10} = 9\ V$

$\frac{30}{10} = 3A$

$P = \frac{81}{3} = 27\ W$

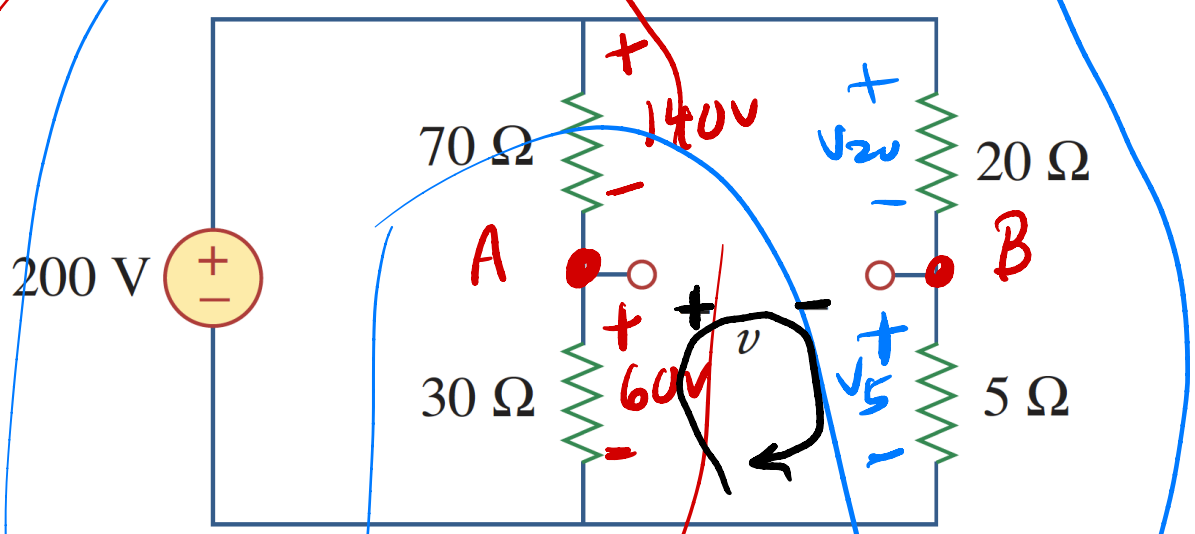


$P = i^2 R = 3^2 \cdot 3$

$= 27\ W$

20 V

Practice problem: find v



use voltage div.

$V_{30} = 200 \cdot \frac{30}{100} = 60\ V$

$V_{70} = 200 \cdot \frac{70}{100} = 140\ V$

use voltage drop or right

$V_5 = 40\ V$

$V_{20} = 160\ V$

KVL $\quad V_{30} - V - V_5 = 0$

$V = V_{30} - V_5 = 60 - 40 = 20\ V$

$3.2\ kW$

Practice problem: find the power in the 50 Ω resistor