Potential Improvement in Detection via Censoring of Poor Data Sets

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Abstract

In order to assess the improvement in detection performance via the paradigm of poor data set censoring we study the problem of detection of a Rayleigh fading signal in white Gaussian noise. The intent is to develop an algorithm based on the hierarchical autonomous decision making approach that was used for Doppler estimation but now apply it to the problem of detection. Previous work has shown that large gains in performance are possible for a set of receiving sensors if confidence measures can be used to reject poor data sets. Additionally, the study illustrates a new facet of the important statistical principle of conditionality.

1. Problem Statement

We are given the outputs of M sensors modeled as

$$x_i[n] = \alpha A_i + w_i[n] \tag{1}$$

for $n = 0, 1, \ldots, N-1$ and $i = 1, 2, \ldots, M$. The signal amplitudes are modeled as $A_i \sim \mathcal{N}(0, \sigma_A^2)$ and are independent and identically distributed (IID) and the noises are independent from sensor to sensor which means that $w_i[m]$ is independent of $w_j[n]$ for $i \neq j$ and for all m and n, and also independent of the A_i's. Each sensor noise $w_i[n]$ is assumed to be white Gaussian noise (WGN) with the same variance σ^2. This modeling is typically used in communications (due to multipath reception), radar (due to multiple reflectors of a target), and also sonar (due to multipath and/or multiple target reflectors). The problem is one of detection for which the two hypotheses are $\mathcal{H}_0 : \alpha = 0$ and $\mathcal{H}_1 : \alpha = 1$. The parameters which are σ_A^2 and σ^2 are assumed known. There are two cases to consider. The first case we will call the unconditional case, which is the usual one assumed in practice, and the second is the conditional case, which we will study to ascertain potential performance improvements in detection. These two cases are described as follows:

1. (unconditional) The observer does not have access to the actual amplitude A_i for the ith sensor that was drawn from the $\mathcal{N}(0, \sigma_A^2)$ population. This results in a hypothesis testing problem of the *covariances* under the two hypotheses. It leads to what is sometimes called an *incoherent detector*. The data is described by the probability density function (PDF) given by $p(\mathbf{X}; \alpha)$, where $\mathbf{X}$ is the $MN \times 1$ vector containing the data from all the sensors, and with α the PDF parameter to be tested.

2. (conditional) For this case suppose that once A_i is drawn, *this information is made known to the observer*. What is unknown and what is chosen by "nature" is whether or not $\alpha = 0$ or $\alpha = 1$ in generating the data as given by (1). The PDF for this case is a conditional one and is given by $p(\mathbf{X}|\mathbf{A}; \alpha)$, where $\mathbf{A} = [A_1\, A_2 \ldots A_M]^T$. For any single realization of the amplitudes (if indeed $\mathcal{H}_1$ is in effect) the amplitudes are given by $\mathbf{A}$. Clearly, over multiple experiments the unconditional PDF is given by the "predictive PDF" of

$$p(\mathbf{X}; \alpha) = \int p(\mathbf{X}|\mathbf{A}; \alpha) p(\mathbf{A}) d\mathbf{A}. \tag{2}$$

We are interested in the difference in detection performance if we are given the signal amplitudes for each experiment and hence can base our detector on $p(\mathbf{X}|\mathbf{A};\alpha)$, which describes the *actual signal amplitude used for that particular experiment*. This is in contrast to Case 1 in which we do not observe the signal amplitudes directly under $\mathcal{H}_1$ so that we need to use (2) or *an average signal amplitude*. There are important testing performance distinctions between these cases which are usually referred to as the *conditionality principle* [Cox and Hinkley 1974]. What we will see is that with the extra information provided by the knowledge of the signal, Case 2 provides better detection performance since we can design a detector based on the actual signal generated and not the signal that *might have been generated as summarized by* $p(\mathbf{A})$. For example, knowing that a particular signal amplitude is small (if $\mathcal{H}_1$ is in effect) discourages our use of that sensor data for detection. Then we could with that knowledge label the data as "poor data" and therefore discard it. This strategy has already been incorporated for improved distributed estimation [Kay 2022].

In Appendix A it is shown that for Case 1 the optimal Neyman-Pearson (NP) detector decides a signal is present if

$$T_1(\mathbf{X}) = \sum_{i=1}^{M} \hat{A}_i^2 > \gamma_1 \tag{3}$$

where $\hat{A}_i = (1/N)\sum_{n=0}^{N-1} x_i[n]$ is the sample mean for the ith sensor. This is termed an *incoherent detector* or energy detector [Kay 1998]. On the other hand, for Case 2 the NP optimal detector decides a signal is present if

$$T_2(\mathbf{X}) = \sum_{i=1}^{M} A_i\hat{A}_i > \gamma_2 \tag{4}$$

which is termed a *coherent detector* or matched filter [Kay 1998]. Note that the sufficient statistics which are the estimated amplitudes $\hat{A}_i$ should be weighted by the *known* amplitudes A_i. This is because the "SNR" of $\hat{A}_i$ can be argued to be $A_i^2/(\sigma^2/N)$ (in the conditional sense). Also, the ith sensor contributes $A_i\hat{A}_i$ to the test statistic so that one can think of the A_i as a "weight" that either emphasizes $\hat{A}_i$ if $|A_i|$ is large or deemphasizes $\hat{A}_i$ if $|A_i|$ is small. Clearly, then, it would be advantageous to know the value of A_i so that poor data sets, i.e., ones that have small $|A_i|$ could be eliminated from the test statistic. It is this advantage that we wish to quantify before embarking on *how to discern the poor data sets*.

Now the detectors given by (3) and (4) will give very different performances. The difference in performance is akin to that of a matched filter versus an energy detector. The former assumes knowledge of the signal (the conditional case) while the latter assumes the signal is unknown but has some statistical (or average) characterization (the unconditional case). Note that the two detectors are to be compared with respect to the original experiment which proceeds as follows:

Case 1. Nature chooses the amplitudes according to $\mathcal{N}(0, \sigma_A^2)$ *but does not reveal the values to the observer*. Nature then generates either $x_i[n] = w_i[n]$ or $x_i[n] = A_i + w_i[n]$ and presents this data to the observer.

Case 2. Nature chooses the amplitudes according to $\mathcal{N}(0, \sigma_A^2)$ and *reveals them to the observer*. Nature then generates either $x_i[n] = w_i[n]$ or $x_i[n] = A_i + w_i[n]$ and presents this data to the observer.

In either case a repeated experiment will yield a distribution of amplitudes so we evaluate the detection performance based on an this average basis.

2. Asymptotic Detection Performance Comparison and a Key Result

The detection performance of the two test statistics of (3) and (4) is derived in Appendix B. To gain some intuition we have derived the asymptotic performance. This asymptotic result assumes that the signal variance σ_A^2 is small and also that M is large so that $\sigma_A^2 = c/M$, for c a constant, goes to zero as M becomes large. This type of analysis is especially appropriate in detection theory in that the signal to be detected is usually small and so it is only the availability of multiple samples and/or multiple sensor outputs that make the detection possible. This provides a very nice result alluded to in [Kay 2020] and which quantifies the conditionality principle with a specific example as opposed to just a philosophy. Asymptotically both test statistics are Gaussian. Also it is shown that each one can be said to be a test statistic for the "Gauss-Gauss" problem. For this problem the deflection coefficient quantifies the probability of detection in that this probability is monotonically increasing with the deflection coefficient. Specifically, the detector for the Gauss-Gauss problem assumes that $T \sim \mathcal{N}(\mu_0, \sigma^2)$ under $\mathcal{H}_0$ and $T \sim \mathcal{N}(\mu_1, \sigma^2)$ under $\mathcal{H}_1$. It is shown in Appendix B that the deflection coefficients for the test statistics $T_1(\mathbf{X})$ and for $T_2(\mathbf{X})$ are given by (under the assumption that $\sigma_A^2/(\sigma^2/N) \ll 1$)

$$\begin{aligned} d_1^2 &= \frac{1}{2}M\left(\frac{\sigma_A^2}{\sigma^2/N}\right)^2 \\ d_2^2 &= M\left(\frac{\sigma_A^2}{\sigma^2/N}\right) \end{aligned}$$

where $\sigma_A^2/(\sigma^2/N)$ the "SNR" of a single sensor. The usual unconditional test will always have a lower deflection coefficient since

$$\begin{aligned} \frac{d_1^2}{d_2^2} &= \frac{\frac{1}{2}M\left(\frac{\sigma_A^2}{\sigma^2/N}\right)^2}{M\left(\frac{\sigma_A^2}{\sigma^2/N}\right)} \\ &= \frac{\frac{1}{2}\sigma_A^2}{\sigma^2/N} \\ &= \frac{c/2}{M(\sigma^2/N)} < 1 \end{aligned}$$

for large enough M. It is also shown in Appendix B that

$$\frac{1}{2}d_2^2 = \frac{1}{2}d_1^2 + \frac{1}{2}M\ln\left(1 + \frac{\sigma_A^2}{\sigma^2/N}\right)$$

so that the improvement in the deflection coefficient (divided by 2) for the conditional case is

$$\frac{1}{2}M\ln\left(1 + \frac{\sigma_A^2}{\sigma^2/N}\right) \tag{5}$$

which is recognized as the *maximum mutual information (MI)* for the Gaussian channel, i.e., the channel capacity. This result is an example of the conditionality relationship cited in [Kay 2020] that succinctly says that

$$\text{KLD} = \text{SNR} - \text{MI}.$$

Explicity it says that

$$\underbrace{D(p_1(\mathbf{X})||p_0(\mathbf{X}))}_{\text{KLD} = d_1^2/2} = \underbrace{E_A\left[D(p_1(\mathbf{X}|\mathbf{A})||p_0(\mathbf{X}))\right]}_{\text{KLD (or SNR)} = d_2^2/2} - \underbrace{D(p_1(\mathbf{X},\mathbf{A})||p_1(\mathbf{X})p_1(\mathbf{A}))}_{\text{mutual information under } \mathcal{H}_1} \tag{6}$$

where the mutual information is given by (5), $p_0(\mathbf{X})$ is the PDF under $\mathcal{H}_0$, $p_1(\mathbf{X}|\mathbf{A})$ is the conditional PDF of $\mathbf{X}$ under $\mathcal{H}_1$, $p_1(\mathbf{A})$ is the PDF of $\mathbf{A}$ under $\mathcal{H}_1$ and finally $p_1(\mathbf{X})$ is the marginal PDF of $\mathbf{X}$ under $\mathcal{H}_1$ (also called the predictive PDF). *The loss in performance by not being able to observe the outcomes of the signal amplitudes is thus given by the mutual information between the data* $\mathbf{X}$ *and the signal amplitudes* $\mathbf{A}$. This loss occurs because knowledge of the amplitudes indicates which PDF $p_1(\mathbf{x})$ or $p_1(\mathbf{x})$ is most likely to be true. Note that the actual probability of detection can be quantified, at least asymptotically, using (6) in Stein's lemma [Kay 2020].

In summary, Case 2, the conditional one, leads us to design the detector in a conditional sense (assuming of course that we observe the amplitudes). Any other detector will have poorer performance in the long run. Said another way we should design the detector based on the data that was actually observed and not on the data that might have been observed. This is the essence of the *conditionality principle*.

4. Computer Simulation

We next simulate the performance and compare against the asymptotic predictions given in the previous section. To do so we assume that we have $N = 50$ data points at each of $M = 100$ sensors. The deflection coefficient of the unconditional detector is $d_1^2 = 0.5$ while that for the conditional detector is $d_2^2 = 10$. Hence, the performance improvement by using the conditional detector, with of course knowleddge of the signal amplitudes, is a factor of 20 or 13 dB. The results are shown in Figure 1 and are portrayed as a receiver operating characteristic of probability of detection P_d versus probability of false alarm P_{FA}. As can be seen the true performance as obtained via Monte Carlo computer simulation matches very well the predicted performance.

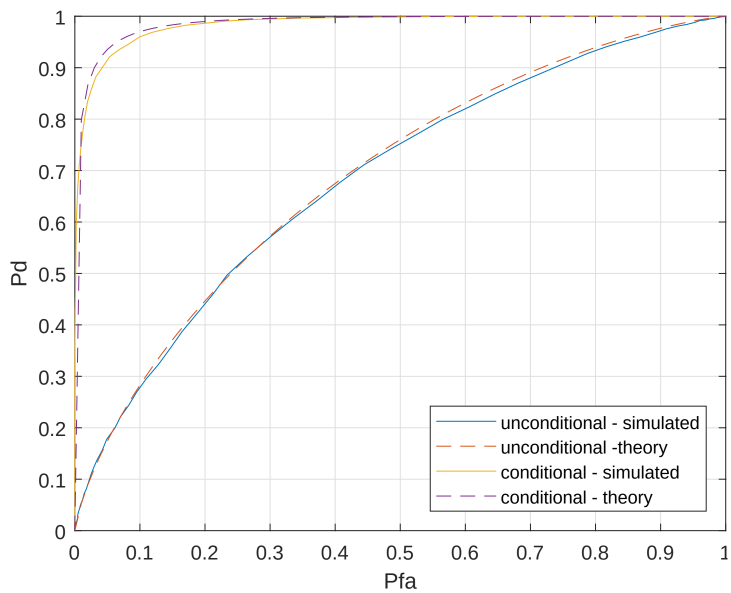

Figure 1: Receiver operating characteristic comparison between the unconditional and conditional detectors. Solid lines depict simulation results while the dashed line indicate the performance predicted using the asymptotic analysis.

References

Cox, D.R., D.V. Hinkley, *Theoretical Statistics*, Chapman and Hall, NY, 1974.

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Appendix A - Detector Derivation

Consider Case 1 (unconditional) and let $\mathbf{x}_i = [x_i[0]\, x_i[1] \ldots x_i[N-1]]^T$ and $\mathbf{X} = [\mathbf{x}_1^T\, \mathbf{x}_2^T \ldots \mathbf{x}_M^T]^T$. Then we have that

$$\mathbf{x}_i = \alpha A_i \mathbf{1} + \mathbf{w}_i$$

where $\mathbf{1} = [1\,1 \ldots 1]^T$ and is $N \times 1$. Note that all $\mathbf{x}_i$'s are IID and each has the distribution

$$\mathbf{x}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{C}_\alpha)$$

where

$$\mathbf{C}_\alpha = \alpha^2 \sigma_A^2 \mathbf{1}\mathbf{1}^T + \sigma^2 \mathbf{I}.$$

Now under $\mathcal{H}_0$ we have $\mathbf{C}_0^{-1} = (1/\sigma^2)\mathbf{I}$ and using Woodbury's identity

$$\mathbf{C}_1^{-1} = \frac{1}{\sigma^2}\mathbf{I} - \frac{1}{\sigma^4}\frac{\sigma_A^2 \mathbf{1}\mathbf{1}^T}{1 + N(\sigma_A^2/\sigma^2)}.$$

The likelihood ratio is

$$\begin{aligned} L_1(\mathbf{X}) &= \frac{\prod_{i=1}^M \frac{1}{(2\pi)^{N/2}|\mathbf{C}_1|^{1/2}} \exp\left[-\frac{1}{2}\mathbf{x}_i^T \mathbf{C}_1^{-1}\mathbf{x}_i\right]}{\prod_{i=1}^M \frac{1}{(2\pi)^{N/2}|\mathbf{C}_0|^{1/2}} \exp\left[-\frac{1}{2}\mathbf{x}_i^T \mathbf{C}_0^{-1}\mathbf{x}_i\right]} \\ &= \prod_{i=1}^M \frac{|\mathbf{C}_0|^{1/2}}{|\mathbf{C}_1|^{1/2}} \exp\left[\frac{1}{2}\mathbf{x}_i^T(\mathbf{C}_0^{-1} - \mathbf{C}_1^{-1})\mathbf{x}_i\right]. \end{aligned}$$

But

$$\mathbf{C}_0^{-1} - \mathbf{C}_1^{-1} = \frac{1}{\sigma^4}\frac{\sigma_A^2 \mathbf{1}\mathbf{1}^T}{1 + N(\sigma_A^2/\sigma^2)}$$

and therefore

$$\begin{aligned} \mathbf{x}_i^T(\mathbf{C}_0^{-1} - \mathbf{C}_1^{-1})\mathbf{x}_i &= \frac{1}{\sigma^2}\frac{\sigma_A^2}{\sigma^2 + N\sigma_A^2}\left(\mathbf{1}^T\mathbf{x}_i\right)^2 \\ &= \frac{N}{\sigma^2}\frac{\sigma_A^2}{\sigma_A^2 + \sigma^2/N}\hat{A}_i^2 \end{aligned}$$

where

$$\hat{A}_i = \frac{1}{N}\sum_{n=0}^{N-1} x_i[n]$$

and is recoginized as the sample mean for the ith sensor. As a result, we have

$$L_1(\mathbf{X}) = \frac{|\mathbf{C}_0|^{M/2}}{|\mathbf{C}_1|^{M/2}} \exp\left[\frac{N}{\sigma^2}\frac{\sigma_A^2}{\sigma_A^2+\sigma^2/N}\sum_{i=1}^M \hat{A}_i^2\right]$$

or

$$l_1(\mathbf{X}) = \ln L_1(\mathbf{X}) = c_1 + c_2 \sum_{i=1}^M \hat{A}_i^2$$

where c_1 and $c_2 > 0$ are constants, not depending on $\mathbf{X}$. Therefore, for Case 1 the NP detector decides $\mathcal{H}_1$ if

$$T_1(\mathbf{X}) = \sum_{i=1}^M \hat{A}_i^2 > \gamma_1.$$

Next consider Case 2, the conditional case. Then we need $p(\mathbf{X}|\mathbf{A};\alpha)$ so that the likelihood ratio (LR) (actually conditional LR) is

$$L_2(\mathbf{X}|\mathbf{A}) = \frac{p(\mathbf{X}|\mathbf{A};\alpha=1)}{p(\mathbf{X}|\mathbf{A};\alpha=0)}$$

and we note that this is just the case of a known signal under $\mathcal{H}_1$ of $A_i\mathbf{1}$. Hence, we have $\mathbf{x}_i|A_i \sim \mathcal{N}(\alpha A_i \mathbf{1}, \sigma^2\mathbf{I})$ and the $\mathbf{x}_i$'s are independent. Therefore

$$\begin{aligned} L_2(\mathbf{X}) &= \frac{\prod_{i=1}^M \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left[-\frac{1}{2\sigma^2}(\mathbf{x}_i - A_i\mathbf{1})^T(\mathbf{x}_i - A_i\mathbf{1})\right]}{\prod_{i=1}^M \frac{1}{(2\pi\sigma^2)^{\frac{N}{2}}} \exp\left[-\frac{1}{2\sigma^2}\mathbf{x}_i^T\mathbf{x}_i\right]} \\ &= \prod_{i=1}^M \exp\left[-\frac{1}{2\sigma^2}(-2A_i\mathbf{1}^T\mathbf{x}_i + NA_i^2\right] \\ &= \prod_{i=1}^M \exp\left[\frac{A_i}{\sigma^2}N\hat{A}_i - \frac{N}{2\sigma^2}A_i^2\right] \end{aligned}$$

and thus

$$l_2(\mathbf{X}) = \frac{N}{\sigma^2}\sum_{i=1}^M A_i\hat{A}_i - \frac{N}{2\sigma^2}\sum_{i=1}^M A_i^2.$$

The NP detector decides $\mathcal{H}_1$ if

$$T_2(\mathbf{X}) = \sum_{i=1}^M A_i\hat{A}_i > \gamma_2.$$

Appendix B - Detector Performance

We obtain the asymptotic performance of the two detectors by letting $M \to \infty$ and $\sigma_A^2 \to 0$ such that $M\sigma_A^2 = c$ for c a constant. This provides some nice intuition which the exact analysis, although possible, does not. For Case 1 we have that $T_1(\mathbf{X}) = \sum_{i=1}^M \hat{A}_i^2$, where $\hat{A}_i = \alpha A_i + \bar{w}_i \sim \mathcal{N}(0, \alpha^2\sigma_A^2 + \sigma^2/N)$, and where the $\hat{A}_i$'s are IID. As a result

$$\frac{T_1(\mathbf{X})}{\alpha^2\sigma_A^2 + \sigma^2/N} \sim \chi_M^2. \tag{7}$$

Now for large M we use the central limit theorem to assert that the test statistic is approximately Gaussian. Then, the mean and variance are easily found from (7). We have that

$$\begin{aligned} E[T_1(\mathbf{X})] &= M(\alpha^2\sigma_A^2 + \sigma^2/N) \\ \mathrm{var}(T_1(\mathbf{X})) &= 2M(\alpha^2\sigma_A^2 + \sigma^2/N)^2 \end{aligned}$$

and letting $\sigma_A^2 = c/M$

$$\begin{aligned} E[T_1(\mathbf{X})] &= M(\alpha^2 c/M + \sigma^2/N) \\ \mathrm{var}(T_1(\mathbf{X})) &= 2M(\alpha^2 c/M + \sigma^2/N)^2 \to \frac{2M\sigma^4}{N^2}. \end{aligned}$$

Since asymptotically the variances are the same under $\mathcal{H}_0$ and $\mathcal{H}_1$ the detection performance is characterized by the deflection coefficient, which is

$$\begin{aligned} d_1^2 &= \frac{(E_1[T_1] - E_0[T_1])^2}{\mathrm{var}(T_1)} \\ &= \frac{[(c + M\sigma^2/N) - (M\sigma^2/N)]^2}{\frac{2M\sigma^4}{N^2}} \\ &= \frac{c^2}{\frac{2M\sigma^4}{N^2}} = \frac{M^2\sigma_A^4}{\frac{2M\sigma^4}{N^2}} \\ &= \frac{1}{2}M\left(\frac{\sigma_A^2}{\sigma^2/N}\right)^2. \end{aligned}$$

Next for Case 2 we have that $T_2(\mathbf{X}) = \sum_{i=1}^M A_i\hat{A}_i$ and note that $\xi_i = A_i\hat{A}_i$ are IID for $i = 1, 2, \ldots, M$. Thus, asymptotically the ξ_i's are Gaussian and we need only find the first two moments. Clearly, then

$$\begin{aligned} E[T_2] &= ME[\xi_i] && (8) \\ \mathrm{var}(T_2) &= M\mathrm{var}(\xi_i). && (9) \end{aligned}$$

Now

$$\begin{aligned} \xi_i &= A_i\hat{A}_i \\ &= A_i\left(\frac{1}{N}\sum_{n=0}^{N-1} x_i[n]\right) \\ &= A_i\frac{1}{N}\sum_{n=0}^{N-1}(\alpha A_i + w_i[n]) \\ &= \alpha A_i^2 + A_i\bar{w}_i \end{aligned}$$

where $\bar{w}_i = (1/N)\sum_{n=0}^{N-1} w_i[n]$. It follows that

$$\begin{aligned} E[\xi_i] &= \alpha E[A_i^2] + E[A_i\bar{w}_i] \\ &= \alpha\sigma_A^2 + E[A_i]E[\bar{w}_i] \\ &= \alpha\sigma_A^2 \end{aligned}$$

due to independence of A_i and $w_i[n]$. Also,

$$\begin{aligned} E[\xi_i^2] &= E[\alpha^2 A_i^4 + 2\alpha A_i^3\bar{w}_i + A_i^2\bar{w}_i^2] \\ &= \alpha^2 E[A_i^4] + E[A_i^2]E[\bar{w}_i^2] \end{aligned}$$

since $A_i \sim \mathcal{N}(0, \sigma_A^2)$ and has a zero third-order moment. Therefore,

$$E[\xi_i^2] = \alpha^2 3\sigma_A^4 + \sigma_A^2\sigma^2/N$$

and

$$\begin{aligned} \mathrm{var}(\xi_i) &= E[\xi_i^2] - E^2[\xi_i] \\ &= \alpha^2 2\sigma_A^4 + \sigma_A^2\sigma^2/N. \end{aligned}$$

Finally from (8) and (9) we have

$$\begin{aligned} E[T_2] &= M\alpha\sigma_A^2 \\ \mathrm{var}(T_2) &= M\alpha^2 2\sigma_A^4 + M\sigma_A^2\sigma^2/N \\ &= \alpha^2 2c^2/M + M\sigma_A^2\sigma^2/N \to M\sigma_A^2\sigma^2/N. \end{aligned}$$

As before, asymptotically the variance is the same under both $\mathcal{H}_0$ and $\mathcal{H}_1$. The deflection coefficient becomes

$$\begin{aligned} d_2^2 &= \frac{(M\sigma_A^2)^2}{M\sigma_A^2\sigma^2/N} \\ &= M\left(\frac{\sigma_A^2}{\sigma^2/N}\right). \end{aligned}$$

Next examine the difference of the deflection coefficients. To do so we let

$$\rho = \frac{\sigma_A^2}{\sigma^2/N}$$

which is an SNR. Then, since $d_2^2 = M\rho$ and $d_1^2 = (M/2)\rho^2$, we have that

$$d_2^2 - d_1^2 = M(\rho - \frac{1}{2}\rho^2).$$

Since we have assumed that $\sigma_A^2 = c/M$, $\rho = (Nc/\sigma^2)/M$ and for large enough M, $\rho < 1$. Furthermore, since

$$\ln(1+x) = x - \frac{1}{2}x^2 + O(x^3)$$

as $x \to 0$, we have that

$$d_2^2 - d_1^2 = M\ln(1+\rho)$$

or finally that asymptotically

$$d_2^2 = d_1^2 + M\ln\left(1 + \frac{\sigma_A^2}{\sigma^2/N}\right).$$

Appendix C - MATLAB Listings

`detection_via_conditioning.m`

```
% detection_via_conditioning.m
%
% This program simulates the detection performance of a Rayleigh fading
% DC level in white Gaussian noise for a set of sensors.  The usual
```

```
% unconditional detector is compared against a detector that has knowledge
% of the random amplitudes at the sensors.  The unconditional detector
% does not have this knowledge.
%
% External subprograms required: roccurve.m
clear all
close all
randn('state',0)
N=50; % data record length
sig2=10; % WGN variance
d2_c=10  % deflection coefficient for conditional detector
c=(sig2/N)*d2_c;  % value of c to yield conditional deflection coeff.
M=100;  % number of sensors
sig2A=c/M;  % vaiance of random amplitude signal
d2_uc=(M/2)*((N*sig2A)/sig2)^2  % deflection coeff. for uncontional
                                % detector
nreal=5000;  % Monte Carlo realizations
for n=1:nreal
for i=1:M
    xi0=sqrt(sig2)*randn(N,1); % data received at each sensor under H0
    Ai=sqrt(sig2A)*randn(1,1);  % random amplitude
    xi1=Ai+xi0;    % data received at each sensor under H1
    Aihat0=mean(xi0);  %  means of data at each sensor under H0 and H1
    Aihat1=mean(xi1);
    q0uc(i,1)=Aihat0.*Aihat0;  % unconditional test statistic under H0
    q1uc(i,1)=Aihat1.*Aihat1;  % unconditional test statistic under H1
    q0c(i,1)=Ai.*Aihat0; % conditional test statistic under H0
    q1c(i,1)=Ai.*Aihat1; % conditional test statistic under H1
end
T0uc(n,1)=sum(q0uc);  % sum all test statistics for each sensor
T1uc(n,1)=sum(q1uc);
T0c(n,1)=sum(q0c);
T1c(n,1)=sum(q1c);
end

varuc=var(T0uc);  % estimated variance of unconditional test statistic
                  % under H0
d2uc_est=(mean(T1uc)-mean(T0uc))^2/varuc  % estimated deflection
```

```
                                        % coeff. for unconditional detector

varc=var(T0c); % estimated variance of conditional test statistic
               % under H0
d2c_est=(mean(T1c)-mean(T0c))^2/varc  % estimated deflection
                                    % coeff. for conditional detector

figure  % plot PDFs of unconditional detector
subplot(2,1,1)
pdf(T0uc,nreal,50,0,50,0.5)
subplot(2,1,2)
pdf(T1uc,nreal,50,0,50,0.5)
title('unconditional')

figure % plot PDFs of conditional detector
subplot(2,1,1)
pdf(T0c,nreal,50,-10,10,0.5)
subplot(2,1,2)
pdf(T1c,nreal,50,-10,10,0.5)
title('conditional')

[Pfauc,Pduc,gam]=roccurve(T0uc,T1uc,100);  % ROC calculations
[Pfac,Pdc,gam]=roccurve(T0c,T1c,100);
Pfa=[0:0.01:1]';
Pdtheoryuc=Q(Qinv(Pfa)-sqrt(d2_uc));
Pdtheoryc=Q(Qinv(Pfa)-sqrt(d2_c));

figure
plot(Pfauc,Pduc,'-',Pfa,Pdtheoryuc,'--',Pfac,Pdc,'-',Pfa,Pdtheoryc,'--')
grid
xlabel('Pfa')
ylabel('Pd')
legend('unconditional- simulated','unconditional - theory',...
    'conditional- simulated','conditional - theory','Location','SouthEast')

print c:/MATLAB_R2014b/Metron/Metron_report_2021_14_Fig1.eps -depsc
```