$$\hat{\sigma}_1^2 = \frac{1}{N-p}\underline{x}^T\left(\underline{I} - \underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T\right)\underline{x}$$

FOR DC LEVEL $\underline{H} = \underline{1}$

$$\Rightarrow \hat{\sigma}_1^2 = \frac{1}{N-1}\underline{x}^T\left(\underline{I} - \frac{1}{N}\underline{1}\underline{1}^T\right)\underline{x}$$

$$= \frac{1}{N-1}\left(\sum_{n=0}^{N-1} x^2(n) - N\bar{x}^2\right)$$

(UNBIASED ESTIMATOR FOR σ^2 UNDER $\mathcal{H}_1$)

<u>ASIDE:</u> F DISTRIBUTIONS (SEE CHAPTER 2)

1) IF $x_1 \sim \chi^2_{\nu_1}$, $x_2 \sim \chi^2_{\nu_2}$ AND x_1, x_2 ARE INDEPENDENT,

$$y = \frac{x_1/\nu_1}{x_2/\nu_2} \sim F_{\nu_1,\nu_2} \quad \text{CENTRAL F}$$

$Q_{F_{\nu_1,\nu_2}}(x)$ = RIGHT-TAIL PROBABILITY MUST BE NUMERICALLY EVALUATED

2) IF $x_1 \sim \chi'^2_{\nu_1}(\lambda)$, $x_2 \sim \chi^2_{\nu_2}$

AND X_1, X_2 ARE INDEPENDENT,

$$y = \frac{X_1/\nu_1}{X_2/\nu_2} \sim F'_{\nu_1,\nu_2}(\lambda)$$

NONCENTRAL F WITH ν_1 NUMERATOR DEGREES OF FREEDOM, ν_2 DENOMINATOR DEGREES OF FREEDOM AND NON-CENTRALITY PARAMETER λ.

$Q_{F'_{\nu_1,\nu_2}(\lambda)}(x)$ = RIGHT-TAIL PROBABILITY,

MUST BE EVALUATED NUMERICALLY

$Q_{F'_{\nu_1,\nu_2}(\lambda)}(\alpha)$ MONOTONICALLY INCREASING WITH λ.

IF $\lambda = 0$, $\quad F'_{\nu_1,\nu_2}(\lambda) = F_{\nu_1,\nu_2}$.

THIS IS A VERY POWERFUL THEOREM. SEE APPENDIX 9A FOR PROOF.

EXAMPLE : UNKNOWN DC LEVEL IN
WGN WITH UNKNOWN σ^2

ALREADY SHOWED THAT GLRT
DECIDES $\mathcal{H}_1$ IF

$$2 \ln L_G(\underline{x}) = N \ln \frac{1}{1 - \frac{\bar{x}^2}{\hat{\sigma}_0^2}} > \gamma'$$

ALSO UNDER $\mathcal{H}_0$

$$2 \ln L_G(\underline{x}) \overset{a}{\sim} \chi_1^2$$

AND UNDER $\mathcal{H}_1$

$$2 \ln L_G(\underline{x}) \overset{a}{\sim} \chi_1'^2(\lambda)$$

$$\lambda = NA^2/\sigma^2$$

THESE WERE ASYMPTOTIC PDFS. CAN
NOW OBTAIN EXACT PDFS FROM
THEOREM.

TO USE THEOREM NOTE THAT
UNDER $\mathcal{H}_1$

$$\underline{x} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}}_{\underline{H}} \underbrace{A}_{\theta} + \underline{w}$$

WISH TO TEST

$\mathcal{H}_0: A = 0, \ \sigma^2 > 0 \qquad \sigma^2$ UNKNOWN

$\mathcal{H}_1: A \neq 0, \ \sigma^2 > 0 \qquad \sigma^2$ UNKNOWN

$\underline{A}\,\underline{\theta} = \underline{b} \iff A = 0$

$\Rightarrow \underline{A} = 1, \ \underline{\theta} = A, \ \underline{b} = 0$

$\uparrow r = 1 \qquad \uparrow p = 1$

ALSO, $\underline{H} = \underline{1} \quad (N \times 1)$

$$\hat{\underline{\theta}}_1 = (\underline{H}^T\underline{H})^{-1}\underline{H}^T\underline{x} = \frac{1}{N}\underline{1}^T\underline{x} = \bar{x}$$

GLRT DECIDES $\mathcal{H}_1$ IF

$$T(\underline{x}) = \frac{N-p}{r}\,\frac{(\underline{A}\hat{\underline{\theta}}_1 - \underline{b})^T\left[\underline{A}(\underline{H}^T\underline{H})^{-1}\underline{A}^T\right]^{-1}(\underline{A}\hat{\underline{\theta}}_1 - \underline{b})}{\underline{x}^T(\underline{I} - \underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T)\underline{x}}$$

$$= (N-1)\,\frac{\bar{x}\left[N^{-1}\right]^{-1}\bar{x}}{\underline{x}^T\underline{x} - \frac{\underline{x}^T\underline{1}\,\underline{1}^T\underline{x}}{N}}$$

$$= (N-1)\,\frac{N\bar{x}^2}{\sum_n x^2(n) - N\bar{x}^2} = \frac{(N-1)\,\bar{x}^2}{\frac{1}{N}\sum x^2(n) - \bar{x}^2}$$

$$= (N-1)\,\frac{\bar{x}^2}{\hat{\sigma}_1^2} > \gamma'$$

NOTE NORMALIZATION IS BY $\hat{\sigma}_1^2$.
($T(\underline{x})$ IS SCALE INVARIANT $\Rightarrow$
EXACTLY CFAR IN AGREEMENT
WITH F DISTRIBUTION).

THIS IS SAME AS BEFORE SINCE

$$T(\underline{x}) = (N-1)\left(L_G(\underline{x})^{2/N} - 1\right)$$

(T IS MONOTONIC WITH L_G)

$$\Rightarrow L_G(\underline{x}) = \left(\frac{T(\underline{x})}{N-1} + 1\right)^{N/2}$$

$$= \left(\frac{\bar{x}^2}{\hat{\sigma}_1^2} + 1\right)^{N/2}$$

$$2 \ln L_G(\underline{x}) = N \ln \frac{\bar{x}^2 + \hat{\sigma}_1^2}{\hat{\sigma}_1^2}$$

BUT $\hat{\sigma}_1^2 = \hat{\sigma}_0^2 - \bar{x}^2$

$$2 \ln L_G(\underline{x}) = N \ln \frac{\hat{\sigma}_0^2}{\hat{\sigma}_0^2 - \bar{x}^2} = N \ln \frac{1}{1 - \frac{\bar{x}^2}{\hat{\sigma}_0^2}}$$

DETECTION PERFORMANCE IS

$$P_{FA} = Q_{F_{1,N-1}}(\gamma')$$

$$P_D = Q_{F'_{1,N-1}(\lambda)}(\gamma')$$

WHERE

$$\lambda = \frac{(\underline{A}\underline{\theta}_1 - \underline{b})^T [\underline{A}(\underline{H}^T\underline{H})^{-1}\underline{A}^T]^{-1}(\underline{A}\underline{\theta}_1 - \underline{b})}{\sigma^2}$$

$$= \frac{A[N^{-1}]^{-1}A}{\sigma^2} = NA^2/\sigma^2$$

ASYMPTOTICALLY (AS $N \to \infty$), DETECTION PERFORMANCE WILL REDUCE TO USUAL GLRT χ^2 PDFS

EXAMPLE: SIGNAL WITH UNKNOWN LINEAR PARAMETERS IN WGN WITH UNKNOWN σ^2

$$\underline{s} = \underline{H}\underline{\theta} \leftarrow \text{UNKNOWN}$$

WISH TO TEST $\mathcal{H}_0: \underline{\theta} = \underline{0}$

$\mathcal{H}_1: \underline{\theta} \neq \underline{0}$

APPLY THEOREM WITH $\underline{A} = \underline{I}$, $\underline{b} = \underline{0}$

$r = p$

DECIDE $\mathcal{H}_1$ IF

$$T(\underline{x}) = \frac{N-p}{p} \frac{\hat{\underline{\theta}}_1^T \underline{H}^T \underline{H} \hat{\underline{\theta}}_1}{\underline{x}^T(\underline{I} - \underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T)\underline{x}} > \gamma'$$

WHERE $\hat{\underline{\theta}}_1 = (\underline{H}^T\underline{H})^{-1}\underline{H}^T\underline{x}$

TO INTERPRET GLRT:

$$T(\underline{x}) = \frac{N-p}{p} \frac{[(\underline{H}^T\underline{H})^{-1}\underline{H}^T\underline{x})^T \underline{H}^T\underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T\underline{x}}{\underline{x}^T(\underline{I} - \underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T)\underline{x}}$$

$$= \frac{N-p}{p} \frac{\underline{x}^T\underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T\underline{x}}{\underline{x}^T(\underline{I} - \underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T)\underline{x}}$$

LET $\underline{P}_H = \underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T$

THIS IS A PROJECTION MATRIX

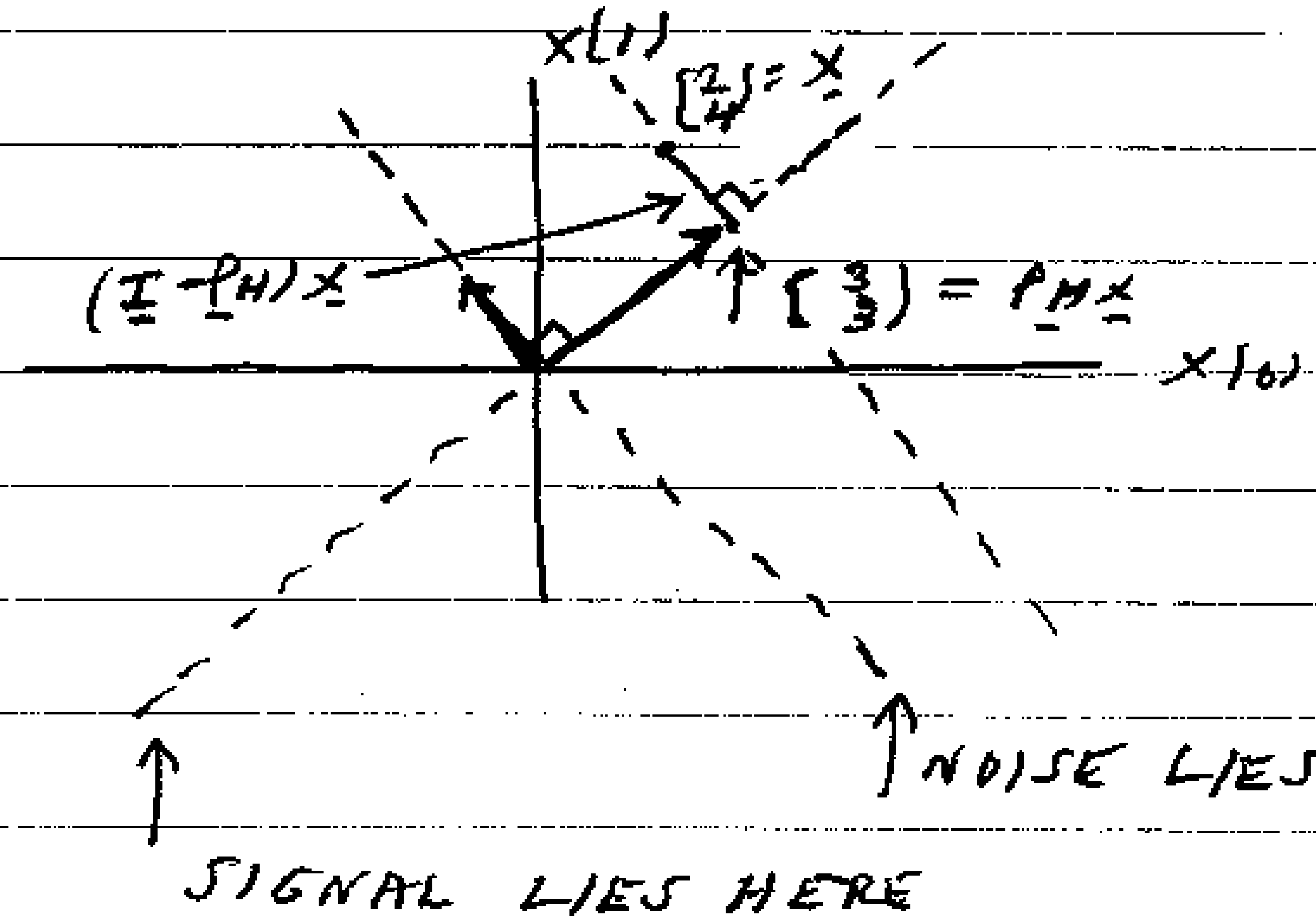
<u>EXAMPLE</u>: DC LEVEL IN WGN

$$N = 2 \qquad \underline{H} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\underline{P}_H = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \frac{1}{2} [1 \; 1] = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

LET $\underline{x} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

$\underline{P_H} \underline{x} = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$



$\underline{P_H}\underline{x}$ PROJECTS $\underline{x}$ ONTO SIGNAL SUBSPACE (SPAN OF COLUMNS OF $\underline{H}$)

ALSO, $\underline{x} - \underline{P_H}\underline{x} = (\underline{I} - \underline{P_H})\underline{x}$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$= \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

= ERROR VECTOR

OR PROJECTION ONTO NOISE SUBSPACE (ORTHOGONAL TO SIGNAL SUBSPACE)

$\underline{I} - \underline{P}_H = \underline{P}_H^{\perp}$ = PROJECTION ONTO ORTHOGONAL COMPLEMENT SUBSPACE

NOTE : $\underline{P}_H^{\perp} \underline{P}_H = \underline{0}$ WHY ?

NOW $T(\underline{x}) = \frac{N-p}{p} \frac{\underline{x}^T \underline{P}_H \underline{x}}{\underline{x}^T \underline{P}_H^{\perp} \underline{x}}$

AND $\underline{P}_H \underline{P}_H = \underline{P}_H \qquad \underline{P}_H^T = \underline{P}_H$

$\underline{P}_H^{\perp} \underline{P}_H^{\perp} = \underline{P}_H^{\perp} \qquad \underline{P}_H^{\perp T} = \underline{P}_H^{\perp}$

$$\Rightarrow T(\underline{x}) = \frac{N-p}{p} \frac{\underline{x}^T \underline{P}_H^T \underline{P}_H \underline{x}}{\underline{x}^T \underline{P}_H^{\perp T} \underline{P}_H^{\perp} \underline{x}}$$

$$= \frac{1}{p} \frac{\|\underline{P}_H \underline{x}\|^2}{\|\underline{P}_H^{\perp} \underline{x}\|^2 / N-p}$$

$$\sim \frac{1}{p} \frac{\text{SIGNAL ENERGY}}{\text{NOISE POWER}}$$

DECOMPOSE $\underline{x}$ INTO SIGNAL SUBSPACE AND NOISE SUBSPACE COMPONENTS AND COMPARE THEM.

RAO TEST - UNKNOWN SIGNAL PLUS NOISE PARAMETERS

RECALL THAT WE NEED MLE ONLY UNDER $\mathcal{H}_0$. WHEN SIGNAL PARAMETERS ARE UNKNOWN, RAO TEST IS USEFUL. HOWEVER, ...

EXAMPLE: DETECT $A\cos 2\pi f_0 n$ IN WGN WITH UNKNOWN VARIANCE σ^2. ASSUME A AND f_0 ARE UNKNOWN.

$$\Rightarrow \quad \mathcal{H}_0: A = 0, f_0, \sigma^2 > 0$$
$$\mathcal{H}_1: A \neq 0, f_0, \sigma^2 > 0$$

$$\underline{\theta} = [A \; f_0 \; \sigma^2]^T$$
$$\underline{\theta}_r = A \qquad \underline{\theta}_s = [f_0 \; \sigma^2]^T$$

$$T_R(\underline{x}) = \left.\frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \underline{\theta}_r}\right|^T_{\underline{\theta}=\tilde{\underline{\theta}}} \left(\underline{I}^{-1}(\tilde{\underline{\theta}})\right)_{\theta_r\theta_r} \cdot \left.\frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \underline{\theta}_r}\right|_{\underline{\theta}=\tilde{\underline{\theta}}}$$

$$\underline{\tilde{\theta}} = [\,\underline{\theta}_{r_0}^T \ \hat{\underline{\theta}}_{s_0}^T\,]^T$$

$\leftarrow$ MLE UNDER $\mathcal{H}_0$

$$= [\,0 \ \underbrace{\hat{f}_0 \ \hat{\sigma}_0^2}_{\text{MLE UNDER } \mathcal{H}_0}\,]^T$$

BUT UNDER $\mathcal{H}_0$ THERE IS NO SIGNAL $\Rightarrow$ CANNOT ESTIMATE f_0

$\Rightarrow$ $[I^{-1}(\tilde{\underline{\theta}})]_{\theta_r\theta_r}$ DOES NOT EXIST

CAN'T USE RAO TEST FOR SIGNALS WITH NUISANCE PARAMETERS
(GLRT STILL O.K.)

SIGNAL MUST BE KNOWN EXCEPT FOR AMPLITUDES $\Rightarrow$ LINEAR MODEL $\underline{s} = \underline{H}\underline{\theta}$ WITH $\underline{H}$ KNOWN

COLORED WSS GAUSSIAN NOISE

NOW ASSUME NOISE IS WSS WITH UNKNOWN PARAMETERS OR POWER SPECTRAL DENSITY IS NOT COMPLETELY KNOWN.

CONSIDER

1) KNOWN DETERMINISTIC SIGNAL

2) DETERMINISTIC SIGNAL WITH UNKNOWN PARAMETERS.

NEED MODEL FOR PSD SINCE WE HAVE

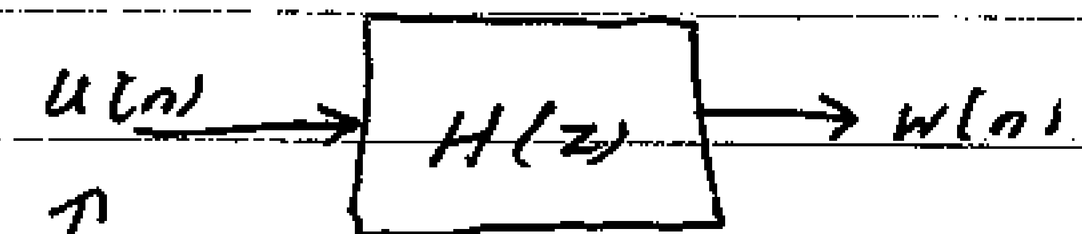
$$P_{ww}(f) = \sum_{k=-\infty}^{\infty} r_{ww}[k] e^{-j2\pi f k}$$

ACF

DEPENDS ON $\{ r_{ww}[0], r_{ww}[1], \ldots \}$

TOO MANY PARAMETERS TO ESTIMATE.

EXAMPLE : AUTOREGRESSIVE PSD MODEL



WHITE GAUSSIAN PROCESS, $\mathrm{VAR}(u[n]) = \sigma_u^2$

$$H(z) = \frac{1}{1 + a[1] z^{-1} + \ldots + a[p] z^{-p}}$$

OR $$w[n] = -\sum_{k=1}^{p} a[k] w[n-k] + u[n]$$

CALLED AUTOREGRESSIVE PROCESS OF ORDER p OR AR(p)

PSD IS

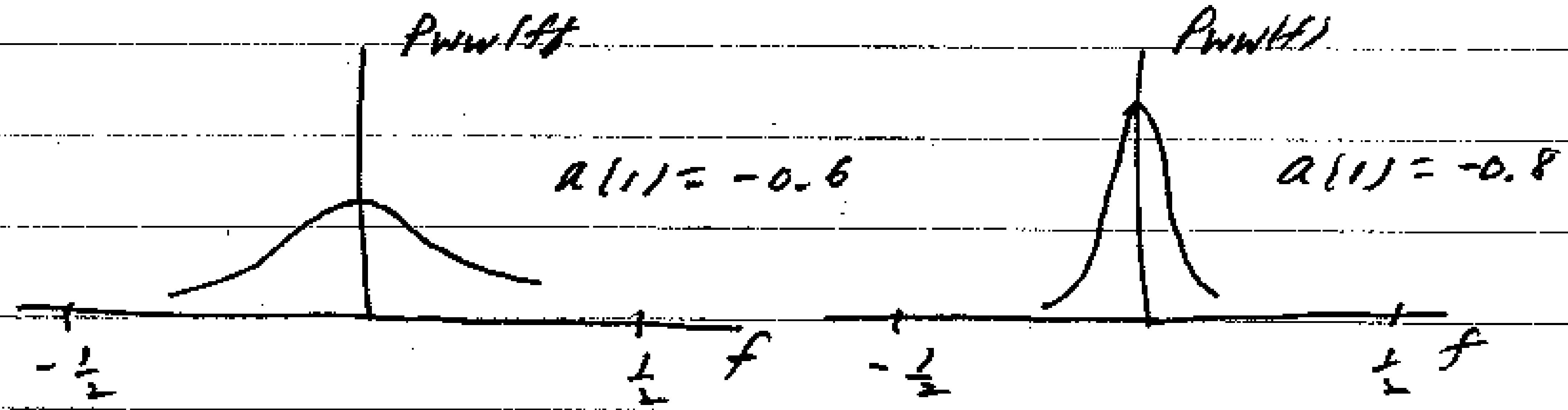
$$P_{WW}(f) = \frac{\sigma_u^2}{|1 + a(1)e^{-j2\pi f} + \dots + a(p)e^{-j2\pi f p}|^2}$$

CONSIDER AR(1)

$$P_{WW}(f) = \frac{\sigma_u^2}{|1 + a(1)e^{-j2\pi f}|^2}$$

$$r_{WW}[k] = \mathcal{F}^{-1}\{P_{WW}(f)\}$$

$$= \frac{\sigma_u^2}{1 - a^2(1)}(-a(1))^{|k|}$$



$a(1)$ CONTROLS BANDWIDTH

$|a(1)| < 1$

σ_u^2 CONTROLS TOTAL POWER

WE WILL ASSUME $w[n]$ IS AR(1) WITH UNKNOWN PARAMETERS $a(1)$, σ_u^2

KNOWN DETERMINISTIC SIGNALS

$a[i]$, σ_u^2 UNKNOWN.

$$\mathcal{H}_0 : x[n] = w[n] \qquad n = 0, 1, \ldots, N-1$$
$$\mathcal{H}_1 : x[n] = s[n] + w[n] \qquad n = 0, 1, \ldots, N-1$$

$s[n]$ KNOWN

GLRT DECIDES $\mathcal{H}_1$ IF

$$\frac{p(\underline{x}; \hat{a}_1[i], \hat{\sigma}_{u_1}^2, \mathcal{H}_1)}{p(\underline{x}; \hat{a}_0[i], \hat{\sigma}_{u_0}^2, \mathcal{H}_0)} > \gamma$$

NEED MLES UNDER $\mathcal{H}_0$ AND $\mathcal{H}_1$.

CONSIDER $\mathcal{H}_0$, SINCE $w[n]$ IS WSS

AS $N \to \infty$

$$\ln p(\underline{x}) = -\frac{N}{2}\ln 2\pi - \frac{N}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\ln P_{xx}(f) + \frac{I(f)}{P_{xx}(f)} \right) df$$

SEE SECTION 2.4

$$\ln p(\underline{w}; a[i], \sigma_u^2) = -\frac{N}{2}\ln 2\pi - \frac{N}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\ln \frac{\sigma_u^2}{|A(f)|^2} + \frac{I(f)}{\sigma_u^2/|A(f)|^2} \right) df$$

WHERE $A(f) = 1 + a(1) e^{-j2\pi f}$

$$I_W(f) = \frac{1}{N} \left| \sum_{n=0}^{N-1} w(n) e^{-j2\pi f n} \right|^2$$

BUT $\int_{-\frac{1}{2}}^{\frac{1}{2}} \ln |A(f)|^2 \, df = 0$

FOR $A(f)$ MINIMUM PHASE OR

$|a(1)| < 1$

$$\ln p(\underline{w}; a(1), \sigma_u^2) = -\frac{N}{2} \ln 2\pi$$

$$-\frac{N}{2} \ln \sigma_u^2 - \frac{N}{2\sigma_u^2} \int_{-\frac{1}{2}}^{\frac{1}{2}} |A(f)|^2 I_W(f) \, df$$

DIFFERENTIATE WITH RESPECT TO σ_u^2 AND SET EQUAL TO ZERO $\Rightarrow$

$$\hat{\sigma}_u^2 = \int_{-\frac{1}{2}}^{\frac{1}{2}} |A(f)|^2 I_W(f) \, df$$

ALSO TO FIND $\hat{a}(1)$ MINIMIZE

$$\sigma(a(1)) = \int_{-\frac{1}{2}}^{\frac{1}{2}} |A(f)|^2 I_W(f) \, df$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} (1 + a(1) e^{-j2\pi f})(1 + a(1) e^{j2\pi f}) \, I_W(f) \, df$$

$$= \int ((1 + a^2(1)) + a(1) e^{-j2\pi f} + a(1) e^{j2\pi f}) \, I_W(f) \, df$$

$$= \int (1 + a^2[1] + 2a[1] \cos 2\pi f) I_W(f)\, df$$

$$= \int (1 + a^2[1] + 2a[1] e^{j2\pi f}) I_W(f)\, df$$

WHY ?

$$= (1+a^2[1])\, \mathcal{F}^{-1}\{I_W(f)\}\big|_{k=0}$$
$$+ 2a[1]\, \mathcal{F}^{-1}\{I_W(f)\}\big|_{k=1}$$

BUT $I_W(f) = \frac{1}{N} |W(f)|^2$

$$\mathcal{F}^{-1}\{I_W(f)\} = \frac{1}{N} \sum_{n=-\infty}^{\infty} w[n]w[n+k] \quad k \geq 0$$

$$= \frac{1}{N} \sum_{n=0}^{N-k-1} w[n]w[n+k]$$

SINCE $w[n] = 0$ FOR $n < 0,\ n > N-1$

OR $\mathcal{F}^{-1}\{I_W(f)\} = \hat{r}_{ww}[k]$

(BIASED ACF ESTIMATOR)

$$J(a[1]) = (1 + a^2[1])\, \hat{r}_{ww}[0]$$
$$+ 2a[1]\, \hat{r}_{ww}[1]$$

$$\Rightarrow \hat{a}[1] = -\frac{\hat{r}_{ww}[1]}{\hat{r}_{ww}[0]}$$

ALSO

$$\hat{\sigma}_u^2 = \hat{r}_{ww}(0) + \hat{a}(1)\,\hat{r}_{ww}(1)$$

FOLLOWS FROM $\hat{\sigma}_u^2 = J(\hat{a}(1))$

NOW

$$\ln p(\underline{w};\hat{a}_0(1),\hat{\sigma}_{u_0}^2,\mathcal{H}_0) =$$

$$-\frac{N}{2}\ln 2\pi - \frac{N}{2}\ln\hat{\sigma}_{u_0}^2 - \frac{N}{2\hat{\sigma}_{u_0}^2}\underbrace{\int |\hat{A}(f)|^2 I_w(f)\,df}_{\hat{\sigma}_{u_0}^2}$$

$$= -\frac{N}{2}\ln 2\pi - \frac{N}{2}\ln\hat{\sigma}_{u_0}^2 - \frac{N}{2}$$

UNDER $\mathcal{H}_1$ WE HAVE SAME RESULT IF WE REPLACE $w[n]$ BY $x[n]-s[n]$

OR

$$\hat{a}_0(1) = -\frac{\hat{r}_{xx}(1)}{\hat{r}_{xx}(0)}$$

$$\hat{a}_1(1) = -\frac{\hat{r}_{x-s,x-s}(1)}{\hat{r}_{x-s,x-s}(0)}$$

$$\hat{\sigma}_{u_0}^2 = \int |\hat{A}_0(f)|^2 I_x(f)\,df$$

$$\hat{\sigma}_{u_1}^2 = \int |\hat{A}_1(f)|^2 I_{x-s}(f)\,df$$

NOW GLRT DECIDES $\mathcal{H}_1$ IF

$$\underbrace{2\ln \frac{p(\underline{x};\hat{a}_1(1),\hat{\sigma}_{u_1}^2,\mathcal{H}_1)}{p(\underline{x};\hat{a}_0(1),\hat{\sigma}_{u_0}^2,\mathcal{H}_0)}} > 2\ln\gamma$$

$$2\left(-\frac{N}{2}\ln 2\pi - \frac{N}{2}\ln\hat{\sigma}_{u_1}^2 - N/2 + \frac{N}{2}\ln 2\pi + \frac{N}{2}\ln\hat{\sigma}_{u_0}^2 + \frac{N}{2}\right)$$

$$= N\ln\frac{\hat{\sigma}_{u_0}^2}{\hat{\sigma}_{u_1}^2}$$

$$= N\ln\frac{\int|\hat{A}_0(f)|^2 I_x(f)\,df}{\int|\hat{A}_1(f)|^2 I_{x-s}(f)\,df}$$

BUT $\int |A(f)|^2 I(f)\,df$

$$= \frac{1}{N}\int |A(f)X(f)|^2\,df$$

$$= \frac{1}{N} \sum_{n=-\infty}^{\infty} \mathcal{F}^{-1}\{A(f)\,X(f)\}_n^2 \quad \text{PARSEVAL}$$

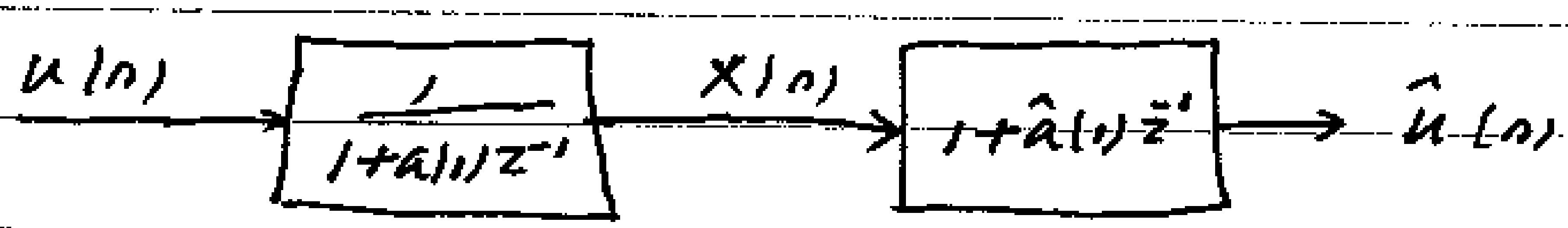
$$\approx \frac{1}{N} \sum_{n=1}^{N-1} \left(x(n) + a(1)\,x(n-1) \right)^2$$

(IGNORE $n=0$ AND $n=N$ TERMS)

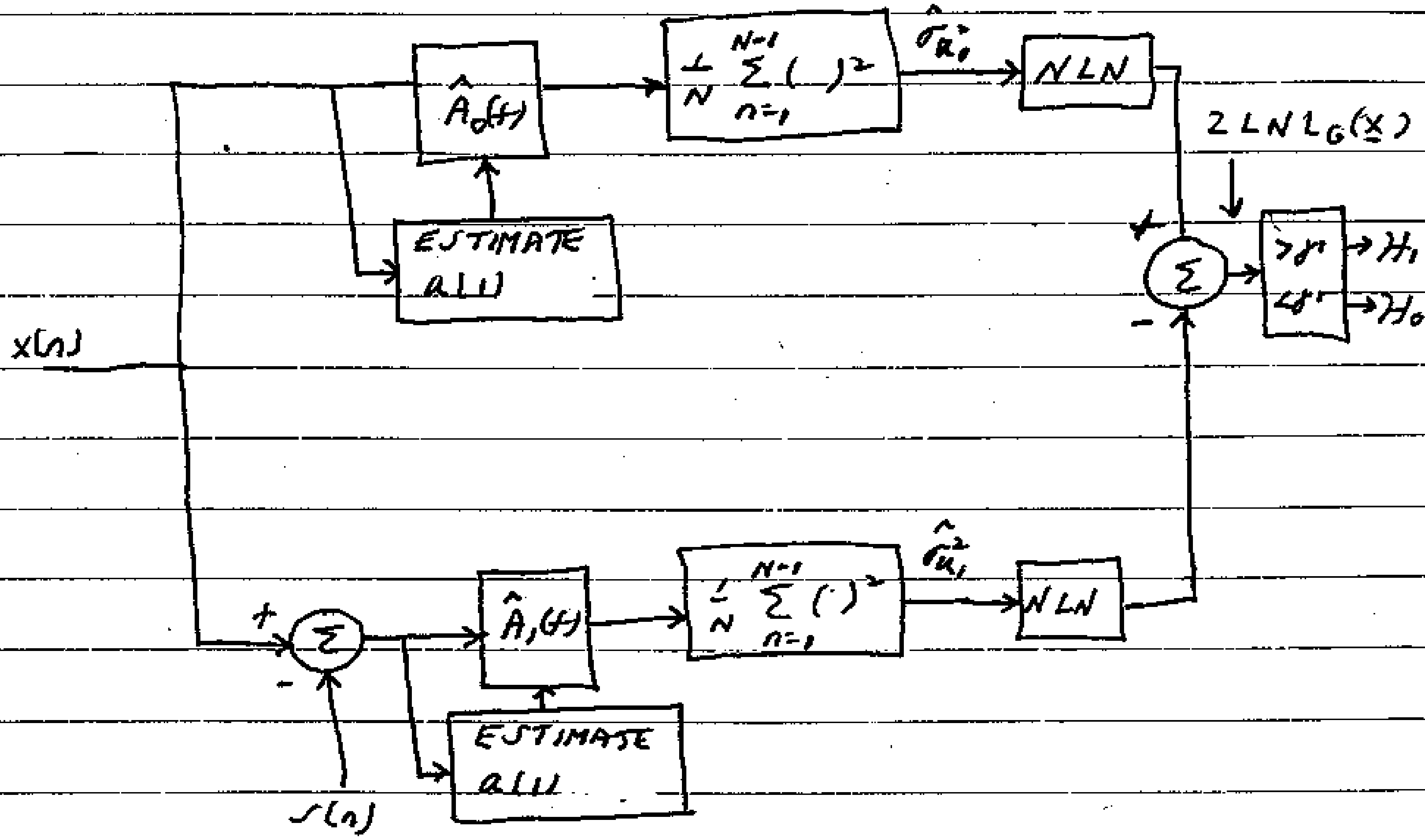
GLRT DECIDES $\mathcal{H}_1$ IF

$$N \ln \frac{\frac{1}{N} \sum_{n=1}^{N-1} \left(x(n) + \hat{a}_0(1)\,x(n-1) \right)^2}{\frac{1}{N} \sum_{n=1}^{N-1} \left[(x(n)-s(n)) + \hat{a}_1(1)\,(x(n-1)-s(n-1)) \right]^2} > 2 \ln \gamma$$

NOTE THAT EFFECT OF FIR FILTER IS TO PREWHITEN NOISE



SEE ALSO RAO TEST - THEOREM 9.2



$$\hat{A}_0(f) = 1 + \hat{a}_0[1]\, e^{-j2\pi f}$$
$$\hat{A}_1(f) = 1 + \hat{a}_1[1]\, e^{-j2\pi f}$$

FIGURE 9.4 – GLRT FOR KNOWN DETERMINISTIC SIGNAL IN COLORED AUTOREGRESSIVE NOISE WITH UNKNOWN PARAMETERS