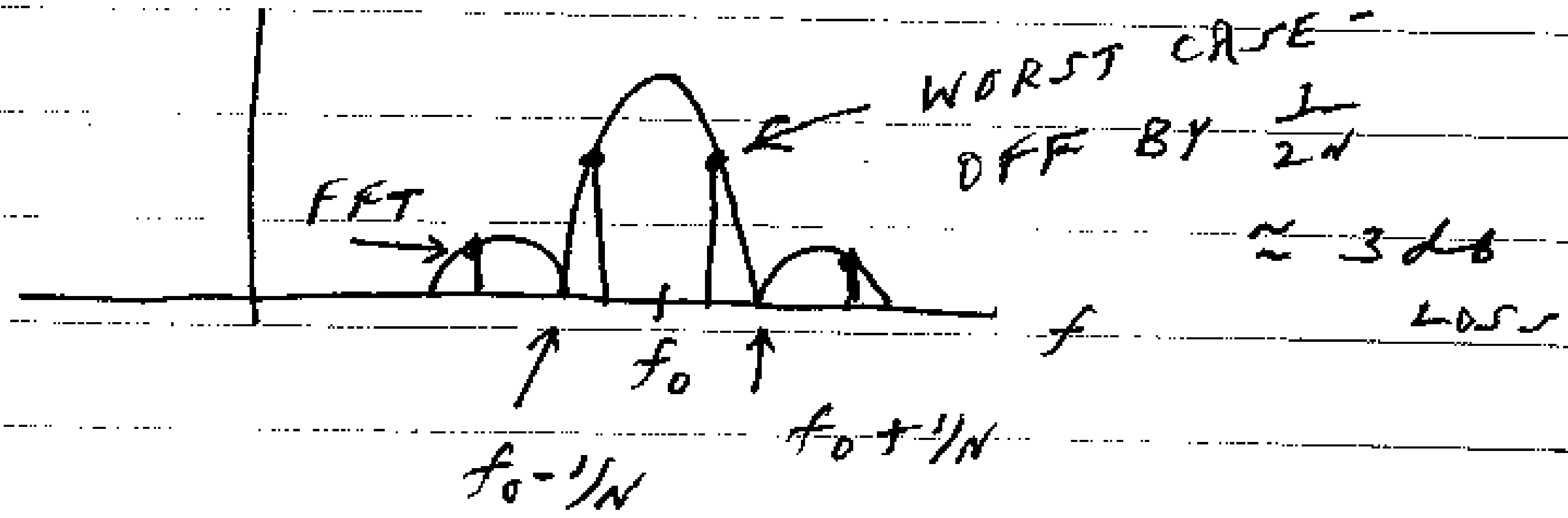
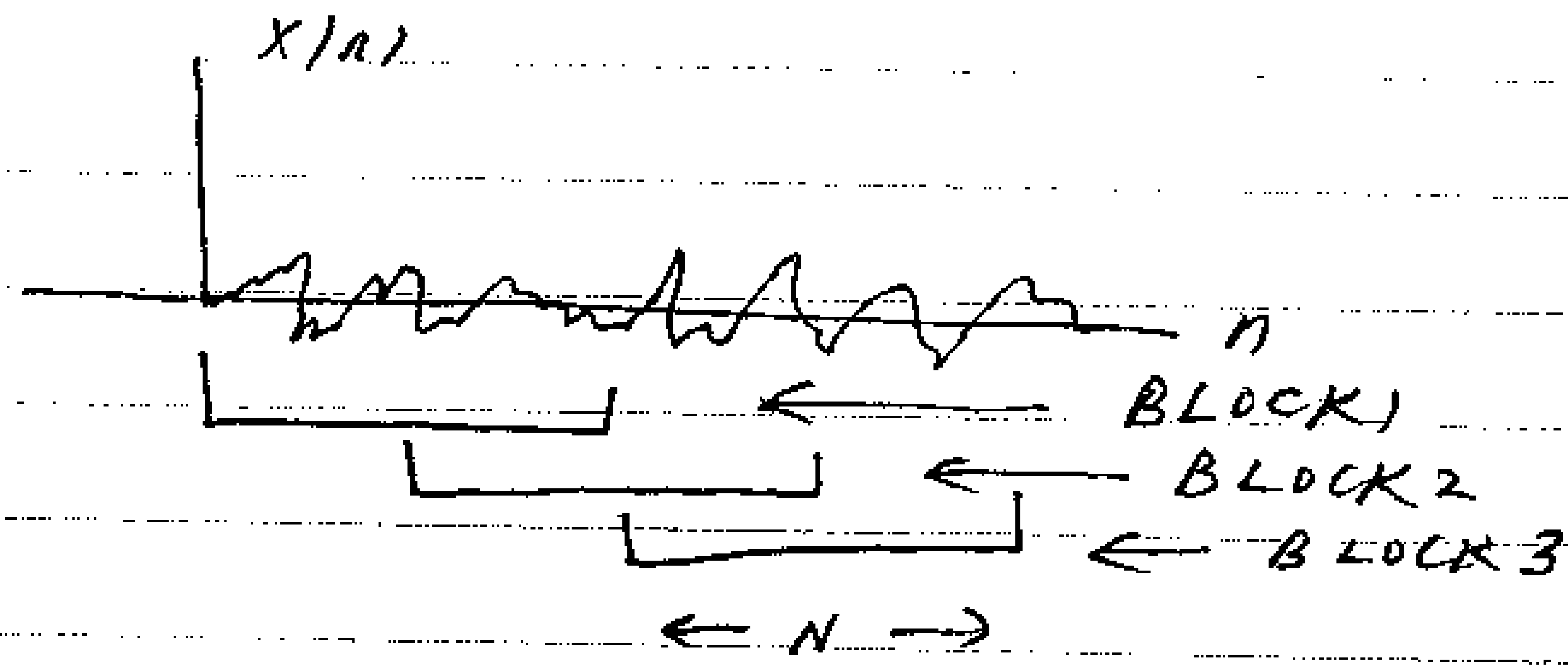


ASSUME f_0 IS MULTIPLE OF $1/N$ FOR ANALYSIS. ALSO, IN PRACTICE CANNOT COMPUTE FFT FOR EACH ASSUMED n_0. USUALLY USE 50% OVERLAP OR

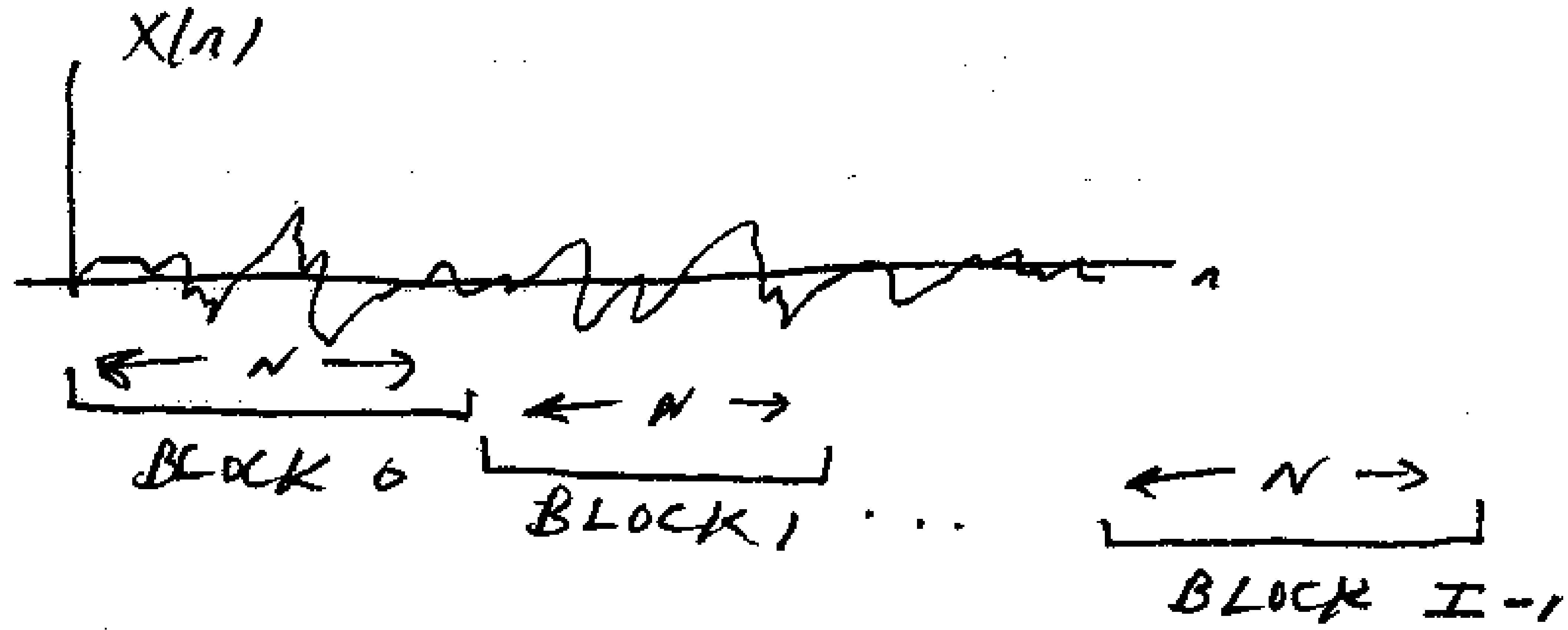


SOME LOSS RELATIVE TO SHIFTING BLOCK FOR EACH SAMPLE. WORST CASE LOSS IS ABOUT 1.5 dB DUE TO LOSS IN SIGNAL ENERGY BY 1/4.

ASSUME FOR PURPOSES OF ANALYSIS WE USE <u>NONOVERLAPPING</u> BLOCKS AND SIGNAL LIES ENTIRELY IN ONE BLOCK $\Rightarrow$ BLOCKS ARE INDE-

PENDENT. CAN SHOW THIS IS REASONABLE APPROXIMATION TO 50% OVERLAP CASE.

PROBLEM:



$$\mathcal{H}_0: \quad x_i(n) = w_i(n) \qquad n = 0, 1, \ldots, N-1; \quad i = 0, 1, \ldots, I-1$$

$$\mathcal{H}_1: \quad x_i(n) = \begin{cases} w_i(n) & n = 0, 1, \ldots, N-1; \quad i = 0, 1, \ldots, i_0 - 1 \\ A\cos(2\pi \underset{\uparrow\, k_0/N}{f_0} n + \phi) + w_i(n) & n = 0, 1, \ldots, N-1; \quad i = i_0 \\ w_i(n) & n = 0, 1, \ldots, N-1; \quad i = i_0 + 1, \ldots, I-1 \end{cases}$$

$x_i(n)$ IS i^{th} BLOCK

ALL $w_i(n)$'S ARE INDEPENDENT (NONOVERLAPPING)

GLRT DECIDES $\mathcal{H}_1$ IF

$$T(\underline{x}) = \max_{i,k} \frac{1}{N} \left| \sum_{n=0}^{N-1} x_i[n] e^{-j\frac{2\pi}{N}kn} \right|^2 > \gamma'$$

$$= \max_{i,k} \frac{1}{N} |X_i(k)|^2 > \gamma'$$

UNDER $\mathcal{H}_1$ SIGNAL HAS FREQUENCY $f_0 = k_0/N$ (k_0^{th} DFT BIN) AND CONTAINED IN BLOCK i_0.

k_0 = DOPPLER BIN OF SIGNAL

i_0 = RANGE BIN OF SIGNAL

$\Rightarrow \frac{1}{N}|X_i(k)|^2$ = RANGE-DOPPLER MAP

DETECTION PERFORMANCE

NEED PDF OF $X(k)$ $k = 1, 2, \ldots, \frac{N}{2}-1$ FOR WGN OR

$$X(k) = \sum_{n=0}^{N-1} w[n] e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} w[n] \cos\frac{2\pi}{N}kn - j\sum_{n=0}^{N-1} w[n] \sin\frac{2\pi}{N}kn$$

$$= U[k] + jV[k]$$

BUT $\{U(1), U(2), \ldots, U(\frac{N}{2}-1), V(1), V(2), \ldots, V[\frac{N}{2}-1]\}$

ARE ALL GAUSSIAN AND IID WITH MEAN ZERO AND VARIANCE $N\sigma^2/2$.

$$\begin{pmatrix} \underline{U} \\ \underline{V} \end{pmatrix} = \underbrace{\begin{bmatrix} \underline{c}_1^T \\ \vdots \\ \underline{c}_{N/2-1}^T \\ \underline{s}_1^T \\ \vdots \\ \underline{s}_{N/2-1}^T \end{bmatrix}}_{\underline{H}} \underline{W}$$

$\underline{H}$ $2(N/2-1)\times N$

$$\underline{c}_i = [\,1 \quad \cos\tfrac{2\pi}{N}i \;\ldots\; \cos\tfrac{2\pi}{N}i(N-1)\,]^T$$

$$\underline{s}_i = [\,0 \; -\sin\tfrac{2\pi}{N}i \;\ldots -\sin\tfrac{2\pi}{N}i(N-1)\,]^T$$

$$\underline{c}_i^T\underline{c}_j = N/2\,\delta_{ij}$$
$$\underline{s}_i^T\underline{s}_j = N/2\,\delta_{ij}$$
$$\underline{c}_i^T\underline{s}_j = 0 \quad \text{ALL } i,j$$

$\Rightarrow$ $\underline{H}$ IS SCALED "ORTHOGONAL" MATRIX AND $\underline{H}\,\underline{H}^T = \frac{N}{2}\underline{I}$.

$$\begin{pmatrix} \underline{U} \\ \underline{V} \end{pmatrix} = \underline{H}\underline{W}$$

$$E\left[\begin{bmatrix} \underline{U} \\ \underline{V} \end{bmatrix}\begin{bmatrix} \underline{U} \\ \underline{V} \end{bmatrix}^T\right] = \underline{H}E(\underline{W}\underline{W}^T)\underline{H}^T = \underline{H}\,\sigma^2\underline{I}\,\underline{H}^T$$

$$= \frac{N\sigma^2}{2}\underline{I}.$$

LET $T_{i,k}(\underline{x}) = \dfrac{|X_i(k)|^2}{N\sigma^2/2} = \dfrac{U_i^2(k) + V_i^2(k)}{N\sigma^2/2}$

BUT FOR A GIVEN i ALL
$U_i(k), V_i(k)$ ARE INDEPENDENT
($\mathrm{COV}(U_i[k], V_i[l]) = 0$ ALL k, l
$\mathrm{COV}(U_i[k], U_i[l]) = 0$ $k \neq l$
$\mathrm{COV}(V_i[k], V_i[l]) = 0$ $k \neq l$)

AND THEY ARE INDEPENDENT FROM BLOCK TO BLOCK. WHY?

$\Rightarrow$ $T_{i,k}(\underline{x}) \sim \chi_2^2$ UNDER $\mathcal{H}_0$
ALL i, k AND INDEPENDENT

$$P_{FA} = \Pr\left\{ \max_{i,k} \frac{\sigma^2}{2} T_{i,k}(\underline{x}) > \gamma' ; \mathcal{H}_0 \right\}$$

$$= \Pr\left\{ \max_{i,k} T_{i,k}(\underline{x}) > \frac{2\gamma'}{\sigma^2} ; \mathcal{H}_0 \right\}$$

$$= 1 - \Pr\left\{ \max_{i,k} T_{i,k}(\underline{x}) < \frac{2\gamma'}{\sigma^2} ; \mathcal{H}_0 \right\}$$

$$= 1 - \Pr\left\{ T_{i,k}(\underline{x}) < \frac{2\gamma'}{\sigma^2} \text{ FOR ALL } i, k ; \mathcal{H}_0 \right\}$$

$\uparrow$ IID

$$= 1 - \prod_{i,k} \Pr\left\{ T_{i,k}(\underline{x}) < \frac{2\gamma'}{\sigma^2} ; \mathcal{H}_0 \right\} \quad \text{INDEPENDENCE}$$

$$= 1 - \Pr\left\{ T_{i,k}(\underline{x}) < \frac{2\gamma'}{\sigma^2} ; \mathcal{H}_0 \right\}^L \quad \text{IDENTICALLY DISTRIBUTED}$$

WHERE $L = I(N/2 - 1)$ = NUMBER OF

RANGE AND DOPPLER BINS SEARCHED OVER FOR MAXIMUM.

BUT $P_r\{\underbrace{T_{i,k}(\underline{x})}_{\chi_2^2} < \frac{2\gamma'}{\sigma^2}; \mathcal{H}_0\}$

$$= \int_0^{2\gamma'/\sigma^2} \frac{1}{2} e^{-\frac{1}{2}u}\, du$$

$$= 1 - e^{-\gamma'/\sigma^2}$$

$$\Rightarrow P_{FA} = 1 - \left(1 - e^{-\gamma'/\sigma^2}\right)^L$$

USUALLY WANT P_{FA} SMALL $\Rightarrow$
$e^{-\gamma'/\sigma^2} \ll 1$. BUT
$(1-x)^L \simeq 1 - Lx$ FOR $x \ll 1$

$$\Rightarrow P_{FA} \simeq 1 - \left(1 - Le^{-\gamma'/\sigma^2}\right)$$
$$= Le^{-\gamma'/\sigma^2}$$

BUT $e^{-\gamma'/\sigma^2} = P_r\{T_{i,k}(\underline{x}) > \frac{2\gamma'}{\sigma^2}; \mathcal{H}_0\}$

= PROBABILITY THAT
$\frac{1}{N}|X_i(k)|^2 > \gamma'$
= PROBABILITY OF FALSE ALARM
IN A SINGLE BIN. $= P_{FA}(BIN)$

$\therefore \quad P_{FA} = L\, P_{FA}(\text{BIN})$

$\quad\quad \propto$ NUMBER OF BINS SEARCHED.

TO FIND P_D MUST DEFINE A DETECTION. DETECTION OCCURS WHEN $\max\limits_{i,k} \frac{1}{N} |X_i(k)|^2 > \gamma'$ AND

MAXIMUM OCCURS FOR CORRECT RANGE-DOPPLER BIN. (NO DETECTION IF NOISE CAUSES THRESHOLD EXCEEDANCE AND IS LARGER THAN SIGNAL BIN OUTPUT)

$$P_D = P_r \left\{ \frac{\sigma^2}{2} T_{i_0, k_0}(\underline{x}) > \gamma' ; \mathcal{H}_1 \right\}$$

$\uparrow$ SIGNAL BIN

$$= P_r \left\{ T_{i_0,k_0}(\underline{x}) > \frac{2\gamma'}{\sigma^2} ; \mathcal{H}_1 \right\}$$

$\frac{\sigma^2}{2} T_{i_0,k_0}(\underline{x}) = I(f)$ IS JUST PERIODOGRAM. ALREADY SHOWED THAT

$$P_D = P_r \{ I(f_0) > \gamma' ; \mathcal{H}_1 \}$$

(UNKNOWN A, ϕ AND KNOWN f_0)

$$= P_r\left\{ \underbrace{\frac{I(f_0)}{\sigma^2/2}}_{\chi_2'^2(\lambda)} > \frac{\gamma'}{\sigma^2/2} ; H_1 \right\}$$

$\lambda = NA^2/2\sigma^2$

$$= Q_{\chi_2'^2(\lambda)}\left(2\gamma'/\sigma^2\right)$$

SEE PG 17 IN BOOK (CHAPTER 7)

SINCE $P_{FA} = L\, e^{-\gamma'/\sigma^2}$

$$\gamma'/\sigma^2 = \ln L/P_{FA}$$

$$\therefore P_D = Q_{\chi_2'^2(\lambda)}\left(2 \ln \frac{L}{P_{FA}}\right)$$

WHERE $\lambda = \dfrac{NA^2}{2\sigma^2}$

<u>EXAMPLE</u> : SPECS: $P_{FA} = 10^{-4}$

$P_D = 0.5$

WISH TO DETECT TARGET OUT TO R = 5000 YDS. USING 1 PING PER 10 SECONDS. INPUT SNR = $\dfrac{A^2}{2\sigma^2} \geq 0.1$ (−10 dB).

OBSERVATION INTERVAL MUST BE TWICE MAX. ROUND TRIP DELAY

$$T = 2R/c = \frac{2(5000)(3)}{5000} = 6 \text{ SEC.}$$

IF SIGNAL HAS FREQUENCY OF $F_T = 30{,}000$ HZ AND DOPPLER CAUSES

A ± 500 HZ SHIFT, THEN BANDWIDTH IS 1000 HZ ⇒ SAMPLE AT 2000 HZ.

BUT $I(N/2-1) = L$

AND NI = NUMBER OF SAMPLES = 12000

$$L \simeq \tfrac{1}{2} NI = 6000$$

$$P_D = Q_{\chi_2'^2(\lambda)}\left(2 \ln L/P_{FA}\right)$$

$$= Q_{\chi_2'^2(\lambda)}\left(2 \ln \frac{6000}{10^{-4}}\right)$$

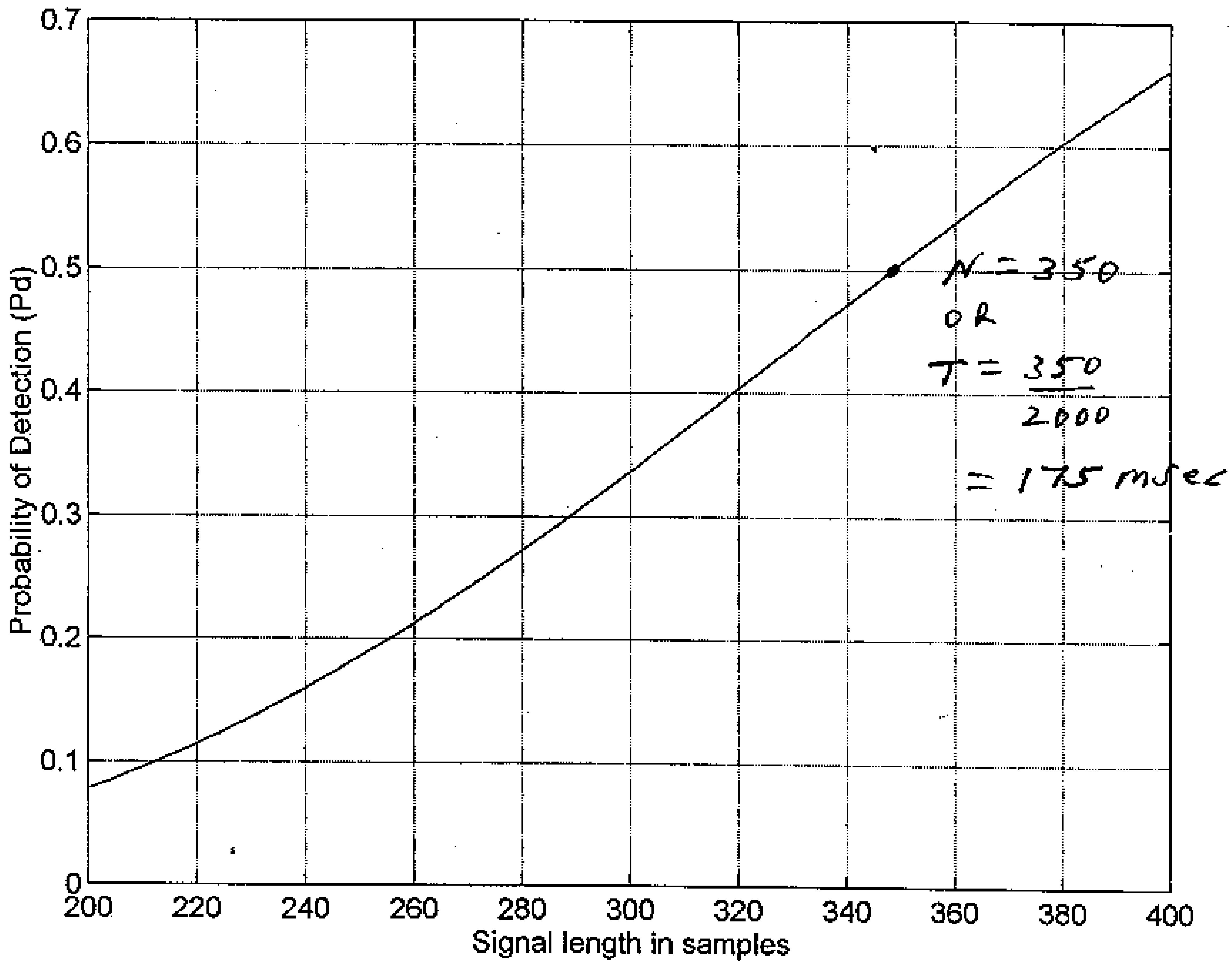
WHERE $\lambda = \frac{NA^2}{2\sigma^2} = 0.1N$

$$P_D = Q_{\chi_2'^2(0.1N)}(35.82) = 0.5$$

USE CHIPR2.M WITH

P_D = CHIPR2(2, 0.1N, 35.82, 0.001)

↑ ν ↑ λ ↑ x ↑ ERROR


Probability of Detection (Pd)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
200
220
240
260
280
300
320
340
360
380
400
Signal length in samples
N = 350
OR
T = 350/2000
= 175 msec

SKIP CHAPTER 8.

CHAPTER 9 - UNKNOWN NOISE PARAMETERS

NOW CONSIDER CASE WHERE PDF UNDER $\mathcal{H}_0$ IS NOT COMPLETELY KNOWN.

RECALL THAT TO FIND THRESHOLD:

$$P_{FA} = P_r\{T(\underline{x}) > \gamma'; \mathcal{H}_0\}$$
$$= \int_{\gamma'}^{\infty} p(T; \mathcal{H}_0)\, dT = \alpha$$

IF $p(\underline{x}; \mathcal{H}_0)$ NOT COMPLETELY KNOWN, THEN $p(T; \mathcal{H}_0)$ NOT " " $\Rightarrow$ CAN'T FIND γ'!

EXAMPLE: DC LEVEL IN WGN, $A > 0$ KNOWN

DECIDE $\mathcal{H}_1$ IF

$$T(\underline{x}) = \bar{x} > \frac{\sigma^2}{NA} \ln \gamma + \frac{A}{2} = \gamma'$$

$$T(\underline{x}) \sim \mathcal{N}(0, \sigma^2/N) \quad \text{UNDER } \mathcal{H}_0$$

$$\Rightarrow P_{FA} = Q\left(\frac{\gamma'}{\sqrt{\sigma^2/N}}\right)$$

$$\gamma' = \sqrt{\sigma^2/N}\ Q^{-1}(P_{FA})$$

IF σ^2 IS UNKNOWN, CAN'T SET THRESHOLD.

POSSIBLE APPROACH IS

$$\hat{\gamma}' = \sqrt{\frac{\hat{\sigma}^2}{N}}\ Q^{-1}(P_{FA})$$

$$\hat{\sigma}^2 = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$ "ESTIMATE AND PLUG"

PROBLEMS:

1) $\hat{\sigma}^2$ WILL BE BIASED UNDER $\mathcal{H}_1$

$E(\hat{\sigma}^2) = \sigma^2 + A^2$

2) PDF NO LONGER CORRECT SINCE

$$\bar{x} > \sqrt{\frac{\hat{\sigma}^2}{N}}\ Q^{-1}(P_{FA})$$

$$\frac{\bar{x}}{\sqrt{\hat{\sigma}^2/N}} > Q^{-1}(P_{FA})$$

↑ NOT $\mathcal{N}(0,1)$

ANOTHER APPROACH USES EXTRA NOISE SAMPLES $w_R(n)$ $n = 0, 1, \ldots, N-1$ (IF AVAILABLE)

DECIDE $\mathcal{H}_1$ IF

$$\frac{\frac{1}{N} \sum_n x(n)}{\sqrt{\hat{\sigma}^2 / N}} > \gamma''$$

WHERE $\hat{\sigma}^2 = \frac{1}{N} \sum_{n=0}^{N-1} w_R^2(n)$

$\Rightarrow$ NO BIAS, PDF UNDER $\mathcal{H}_0$ CAN BE FOUND (STUDENT t PDF) AND DOES NOT DEPEND ON σ^2.

$\sqrt{\hat{\sigma}^2 / N}$ CALLED NORMALIZATION FACTOR

IF THRESHOLD MAY BE SET INDEPENDENT OF UNKNOWN PARAMETER $\Rightarrow$ CONSTANT FALSE ALARM RATE (CFAR) DETECTOR (MEETS NP CONDITIONS)

NOW CONSIDER GLRT APPROACH.

EXAMPLE : KNOWN DC LEVEL IN WGN

σ^2 IS UNKNOWN

GLRT DECIDES $\mathcal{H}_1$ IF

$$L_G(\underline{x}) = \frac{p(\underline{x}; \hat{\sigma}_1^2, \mathcal{H}_1)}{p(\underline{x}; \hat{\sigma}_0^2, \mathcal{H}_0)} > \gamma$$

NOTE DIFFERENT MLES UNDER $\mathcal{H}_0$ $\mathcal{H}_1$

(DIFFERENT THAN "ESTIMATE AND PLUG")

EASILY SHOWN THAT

$$\hat{\sigma}_0^2 = \frac{1}{N} \sum_n x^2[n]$$

$$\hat{\sigma}_1^2 = \frac{1}{N} \sum_n (x[n]-A)^2$$

$$L_G(\underline{x}) = \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2}\right)^{N/2}$$

TO SHOW THAT IT IS NOT CFAR :

UNDER $\mathcal{H}_0$ $x[n] = \sigma u[n]$ $\quad u[n] \sim \mathcal{N}(0,1)$

$$L_G(\underline{x})^{2/N} = \frac{\frac{1}{N} \sum_n \sigma^2 u^2[n]}{\frac{1}{N} \sum_n (\sigma u[n]-A)^2}$$

$$= \frac{\frac{1}{N}\sum u^2(n)}{\frac{1}{N}\sum (u(n)-A/\sigma)^2}$$

DEPENDS ON σ^2. A SUFFICIENT CONDITION FOR STATISTIC TO BE CFAR IS SCALE INVARIANCE OR

$$T(\sigma \underline{x}) = T(\underline{x}) \quad \text{ALL } \sigma > 0$$

GLRT MAY NOT RESULT IN CFAR DETECTOR.

EXAMPLE: UNKNOWN DC LEVEL IN WGN WITH UNKNOWN σ^2

SEE EXAMPLE 6.5. GLRT DECIDES $\mathcal{H}_1$ IF

$$T(\underline{x}) = 2 \ln L_G(\underline{x}) = N \ln \frac{1}{1 - \frac{\bar{x}^2}{\hat{\sigma}_0^2}} > \gamma'$$

SHOWN TO BE CFAR SINCE IT IS SCALE INVARIANT. ALSO,

$$T(\underline{x}) \overset{a}{\sim} \chi_1^2 \quad \text{UNDER } \mathcal{H}_0$$

ASYMPTOTICALLY CAN EASILY FIND THRESHOLD FROM PDF

IN GENERAL, IF WE HAVE

$\mathcal{H}_0 : \underline{\theta}_r = \underline{\theta}_{r_0}, \underline{\theta}_s$

$\mathcal{H}_1 : \underline{\theta}_r \neq \underline{\theta}_{r_0}, \underline{\theta}_s$

WHERE $\underline{\theta}_s$ ARE NUISANCE PARAMETERS OR <u>NOISE</u> PARAMETERS

$\Rightarrow 2 \ln L_G(\underline{x}) \overset{a}{\sim} \chi^2_r$

$\Rightarrow$ ASYMPTOTICALLY CFAR

IN PREVIOUS EXAMPLE

$\underline{\theta}_r = A, \underline{\theta}_s = \sigma^2$

GLRT FOR WGN - σ^2 UNKNOWN

CASE 1: KNOWN DETERMINISTIC SIGNAL

CASE 2: DETERMINISTIC SIGNAL WITH UNKNOWN PARAMETERS

CASE 1: $\mathcal{H}_0 : x(n) = w(n) \quad n = 0, 1, \ldots, N-1$
$\mathcal{H}_1 : x(n) = s(n) + w(n) \quad "$

$w(n)$ IS WGN WITH UNKNOWN σ^2,
$s(n)$ IS KNOWN

GLRT DECIDES $\mathcal{H}_1$ IF

$$L_G(\underline{x}) = \frac{p(\underline{x}; \hat{\sigma}_1^2)}{p(\underline{x}; \hat{\sigma}_0^2)} > \gamma$$

$$\hat{\sigma}_0^2 = \text{MLE UNDER } \mathcal{H}_0 = \frac{1}{N} \sum_{0}^{N-1} x^2(n)$$
$$\hat{\sigma}_1^2 = \text{MLE UNDER } \mathcal{H}_1 = \frac{1}{N} \sum_{0}^{N-1} (x(n) - s(n))^2$$

$$\Rightarrow L_G(\underline{x}) = \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2} \right)^{N/2}$$

ALTERNATIVELY, CONSIDER

$$T(\underline{x}) = \frac{N}{2} \left(L_G(\underline{x})^{2/N} - 1 \right)$$

$$= \frac{N}{2} \left(\frac{\hat{\sigma}_0^2 - \hat{\sigma}_1^2}{\hat{\sigma}_1^2} \right)$$

$$= \frac{\sum_n x^2(n) - \sum_n (x(n) - s(n))^2}{2 \hat{\sigma}_1^2}$$

$$= \frac{\sum_{n=0}^{N-1} x(n)s(n) - \frac{1}{2}\sum_{n=0}^{N-1} s^2(n)}{\hat{\sigma}_1^2}$$

WHEN σ^2 IS KNOWN, AN NP TEST IS

$$\frac{\sum_{n=0}^{N-1} x(n)s(n) - \frac{1}{2}\sum_{n=0}^{N-1} s^2(n)}{\sigma^2} > \ln \gamma$$

NORMALIZING FACTOR IS AN ESTIMATE.

GLRT NOT CFAR (CAN BE SHOWN)

CASE 2 : ASSUME LINEAR MODEL FOR SIGNAL

$$\underline{x} = \underset{\substack{\uparrow \\ \text{UNKNOWN SIGNAL PARAMETERS}}}{H\underline{\theta}} + \underline{w} \leftarrow N(\underline{0}, \underset{\substack{\uparrow \\ \text{UNKNOWN}}}{\sigma^2} \underline{I})$$

GLRT FOR LINEAR MODEL WITH σ^2 UNKNOWN

EXAMPLE: UNKNOWN DC LEVEL IN WGN WITH UNKNOWN σ^2

IN SECTION 7.7 ASSUMED σ^2 KNOWN.

THEOREM 9.1 - $\underline{x} = \underline{H}\underline{\theta} + \underline{w}$, $\underline{H}$ IS $N \times p$ AND KNOWN, $\underline{\theta}$ IS $p \times 1$ AND UNKNOWN, $\underline{w} \sim N(\underline{0}, \sigma^2 \underline{I})$ WITH σ^2 UNKNOWN. CONSIDER

$$\mathcal{H}_0: \underline{A}\underline{\theta} = \underline{b}, \quad \sigma^2 > 0$$

$$\mathcal{H}_1: \underline{A}\underline{\theta} \neq \underline{b}, \quad \sigma^2 > 0$$

$$\underset{r \times p}{\uparrow} \qquad r \leq p$$

GLRT DECIDES $\mathcal{H}_1$ IF

$$T(\underline{x}) = \frac{N-p}{r}\left(L_G(\underline{x})^{2/N} - 1\right)$$

$$= \frac{N-p}{r} \frac{(\underline{A}\hat{\underline{\theta}}_1 - \underline{b})^T \left[\underline{A}(\underline{H}^T\underline{H})^{-1}\underline{A}^T\right]^{-1}(\underline{A}\hat{\underline{\theta}}_1 - \underline{b})}{\underline{x}^T(\underline{I} - \underline{H}(\underline{H}^T\underline{H})^{-1}\underline{H}^T)\underline{x}} > \gamma'$$

WHERE $\hat{\underline{\theta}}_1 = (\underline{H}^T\underline{H})^{-1}\underline{H}^T\underline{x}$ = MLE OF $\underline{\theta}$

UNDER $\mathcal{H}_1$ (UNRESTRICTED MLE)
DETECTION PERFORMANCE IS
(EXACTLY - FINITE DATA RECORDS)

$$P_{FA} = Q_{F_{r,N-p}}(\gamma')$$

$$P_D = Q_{F'_{r,N-p}(\lambda)}(\gamma')$$

$F_{r,N-p}$ = CENTRAL F PDF WITH r NUMERATOR DEGREES OF FREEDOM AND $N-p$ DENOMINATOR DEGREES OF FREEDOM

$F'_{r,N-p}(\lambda)$ = NONCENTRAL F WITH NONCENTRALITY PARAMETER λ.

$$\lambda = \frac{(\underline{A}\underline{\theta}_1 - \underline{b})^T [\underline{A}(\underline{H}^T\underline{H})^{-1}\underline{A}^T]^{-1}(\underline{A}\underline{\theta}_1 - \underline{b})}{\sigma^2}$$

WHERE $\underline{\theta}_1$ IS VALUE OF $\underline{\theta}$ UNDER $\mathcal{H}_1$.

GLRT IS SAME AS WHEN σ^2 KNOWN EXCEPT DENOMINATOR, WHICH WAS σ^2, REPLACED BY ESTIMATE OF σ^2 OR