TO IMPLEMENT GLRT

$$L_G(\underline{x}) = \frac{p(\underline{x};\hat{A},\mathcal{H}_1)}{p(\underline{x};\mathcal{H}_0)}$$

NEED MLE $\hat{A}$. BUT

$$p(\underline{x};A,\mathcal{H}_1) = C_1^N e^{-\frac{1}{2}C_2\sum_{n=0}^{N-1}(x[n]-A)^4}$$

MUST MINIMIZE $J(A) = \sum_{n=0}^{N-1}(x[n]-A)^4$
OVER A $\Rightarrow$ HARD TO DO ANALYTICALLY.

BUT

$$T_R(\underline{x}) = \left.\frac{\partial \ln p(\underline{x};A)}{\partial A}\right|_{A=0}^{2} I^{-1}(0)$$

$$\left.\frac{\partial \ln p(\underline{x};A)}{\partial A}\right|_{A=0} = -2C_2(-1)\sum_n (x[n]-A)^3\Big|_{A=0}$$

$$= 2C_2 \sum_n x^3[n]$$

$$T_R(\underline{x}) = \frac{4C_2^2}{I(0)}\left(\sum_{n=0}^{N-1} x^3[n]\right)^2$$

OR

$$T_R(\underline{x}) = \frac{4N^2 c_2^2}{I(0)} \left(\frac{1}{N} \sum_{n=0}^{N-1} x^3[n] \right)^2$$

SEE EX. 6.9 FOR EXACT EXPRESSIONS

NOTE THAT UNDER $\mathcal{H}_0$

$$E(x^3[n]) = 0 \qquad \text{WHY?}$$

$$\Rightarrow \frac{1}{N} \sum_{n=0}^{N-1} x^3[n] \approx 0$$

UNDER $\mathcal{H}_1$ THERE WILL BE A SIGNAL CONTRIBUTION OF $\left(\frac{1}{N} \sum A^3\right)^2 = A^6$.

CASE 2: NUISANCE PARAMETERS

$$\mathcal{H}_0 : \underline{\theta}_r = \underline{\theta}_{r_0}, \ \underline{\theta}_s$$

$$\mathcal{H}_1 : \underline{\theta}_r \neq \underline{\theta}_{r_0}, \ \underline{\theta}_s$$

RAO TEST BECOMES

$$T_R(\underline{x}) = \left.\frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \underline{\theta}_r}\right|^T_{\underline{\theta}=\tilde{\underline{\theta}}} \left[\underline{I}^{-1}(\tilde{\underline{\theta}}) \right]_{\theta_r \theta_r} \cdot \left.\frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \underline{\theta}_r}\right|_{\underline{\theta}=\tilde{\underline{\theta}}}$$

WHERE $\tilde{\underline{\theta}} = [\underline{\theta}_{r_0}^T \ \hat{\underline{\theta}}_{s_0}^T]^T$

$=$ MLE OF $\underline{\theta}$ UNDER $\mathcal{H}_0$

$=$ CONSTRAINED MLE

NEED ONLY FIND MLE UNDER $\mathcal{H}_0 \Rightarrow$ NO SIGNAL PARAMETER MLE NEEDED

EXAMPLE : DC LEVEL IN WGN

$$\mathcal{H}_0 : A = 0, \ \sigma^2 > 0$$
$$\mathcal{H}_1 : A \neq 0, \ \sigma^2 > 0$$

A, σ^2 UNKNOWN, ONLY NEED MLE UNDER $\mathcal{H}_0$ OR NEED $\hat{\sigma}^2$ FOR WGN

$$\underline{\theta} = \begin{bmatrix} A \\ \sigma^2 \end{bmatrix} = \begin{bmatrix} \underline{\theta}_r \\ \underline{\theta}_s \end{bmatrix} \qquad r = 1, \ s = 1$$

$$\underline{\theta}_{r_0} = A_0 = 0$$

$$\hat{\underline{\theta}}_{s_0} = \hat{\sigma}_0^2 = \frac{1}{N} \sum_{n=0}^{N-1} x^2[n] \Rightarrow \tilde{\underline{\theta}} = \begin{bmatrix} 0 \\ \hat{\sigma}_0^2 \end{bmatrix}$$

RAO TEST BECOMES

$$T_R(\underline{x}) = \left. \frac{\partial \ln p(\underline{x}; A, \sigma^2)}{\partial A} \right|_{\substack{A=0 \\ \sigma^2 = \hat{\sigma}_0^2}} \left[\mathbf{I}^{-1}(\overset{0}{\downarrow}A, \overset{\hat{\sigma}_0^2}{\downarrow}\sigma^2) \right]_{AA}$$

$$\cdot \left. \frac{\partial \ln p(\underline{x}; A, \sigma^2)}{\partial A} \right|_{\substack{A=0 \\ \sigma^2 = \hat{\sigma}_0^2}}$$

WHERE

$$[\underline{I}^{-1}(\tilde{\underline{\theta}})]_{AA} = \text{AA PARTITION OF } \underline{I}^{-1}(\tilde{\underline{\theta}})$$

$$\underline{I}(\underline{\theta}) = \underline{I}(A, \sigma^2)$$

$$= \begin{bmatrix} I_{AA} & I_{A\sigma^2} \\ I_{\sigma^2 A} & I_{\sigma^2\sigma^2} \end{bmatrix}$$

$$= \begin{bmatrix} N/\sigma^2 & 0 \\ 0 & N/2\sigma^4 \end{bmatrix}$$

$$\underline{I}^{-1}(\underline{\theta}) = \begin{bmatrix} \sigma^2/N & 0 \\ 0 & \frac{2\sigma^4}{N} \end{bmatrix}$$

$$\underline{I}^{-1}(\tilde{\underline{\theta}}) = \underline{I}^{-1}(A=0, \sigma^2 = \hat{\sigma}_0^2)$$

$$= \begin{bmatrix} \hat{\sigma}_0^2/N & 0 \\ 0 & \frac{2\hat{\sigma}_0^4}{N} \end{bmatrix}$$

$$(\underline{I}^{-1}(\tilde{\underline{\theta}}))_{AA} = \hat{\sigma}_0^2/N$$

AND $$\frac{\partial \ln p(\underline{x}; A, \sigma^2)}{\partial A} = -\frac{1}{2\sigma^2}\frac{\partial}{\partial A}\sum_n (x[n]-A)^2$$

$$= \frac{1}{\sigma^2} \sum_n (x(n) - A)$$

$$\left.\frac{\partial \ln p(\underline{x}; A, \sigma^2)}{\partial A}\right|_{\substack{A=0 \\ \sigma^2 = \hat{\sigma}_0^2}} = \frac{1}{\hat{\sigma}_0^2} \sum x(n)$$

$$= \frac{N\bar{x}}{\hat{\sigma}_0^2}$$

$$T_R(\underline{x}) = \left(\frac{N\bar{x}}{\hat{\sigma}_0^2}\right)^2 \frac{\hat{\sigma}_0^2}{N}$$

$$= \frac{N\bar{x}^2}{\hat{\sigma}_0^2}$$

RECALL THAT GLRT WAS

$$2 \ln L_G(\underline{x}) = N \ln \frac{1}{1 - \frac{\bar{x}^2}{\hat{\sigma}_0^2}}$$

TO SHOW THAT THEY ARE EQUIVALENT AS $N \to \infty$ NOTE THAT AS $N \to \infty$ MUST HAVE $A \to 0$ (WHY?)

$\Rightarrow \quad \bar{x} \to 0 \qquad$ WHY?

$\bar{x}/\hat{\sigma}_0^2 \ll 1$

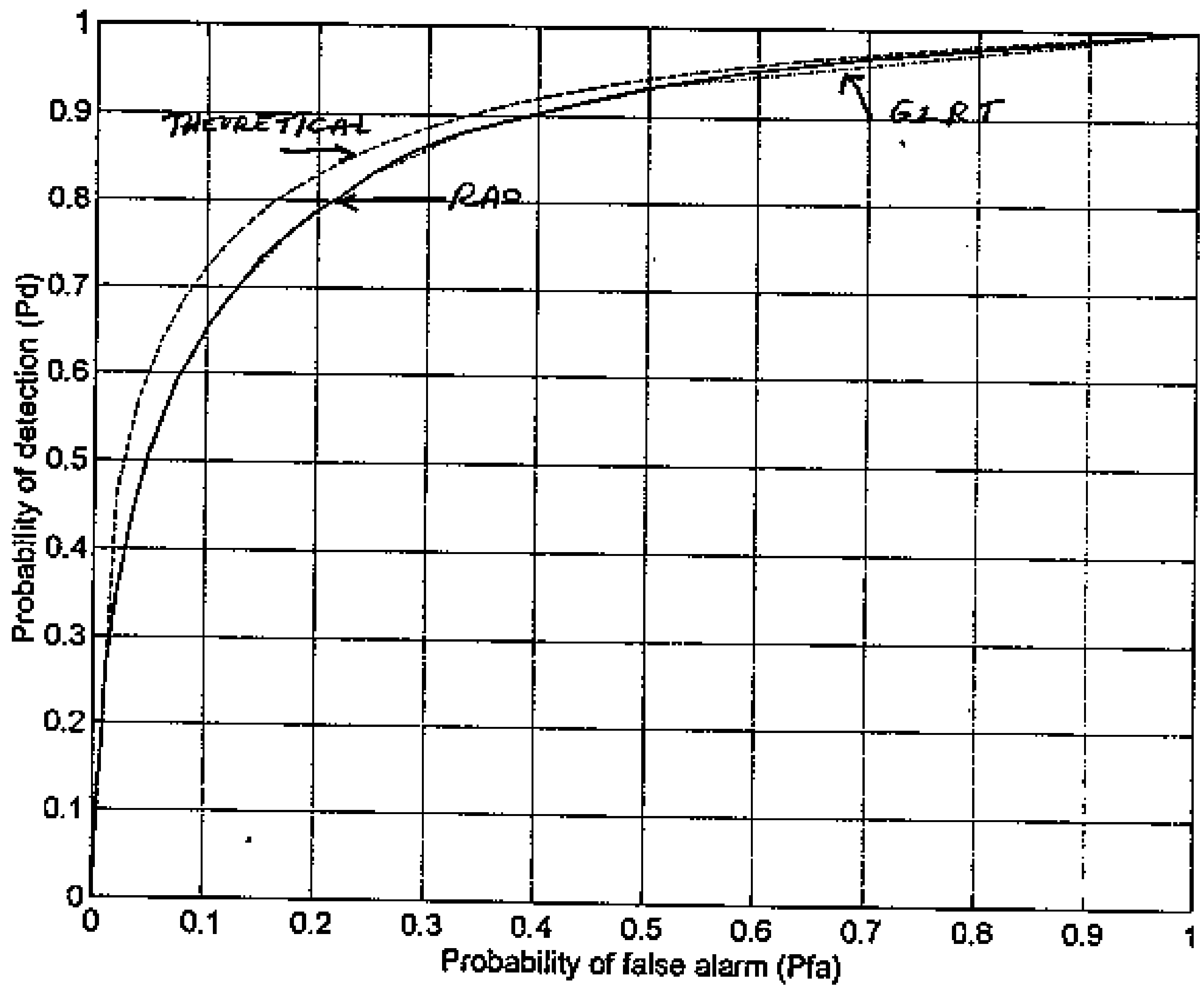
BUT $\ln \frac{1}{1-x} \simeq x$ FOR $x \ll 1$

$$\Rightarrow 2 \ln L_G(\underline{x}) = N \ln \frac{1}{1 - \frac{\bar{x}^2}{\hat{\sigma}_0^2}}$$

$$\simeq N \frac{\bar{x}^2}{\hat{\sigma}_0^2} = T_R(\underline{x})$$

FOR FINITE DATA RECORDS

$N = 10$



```
%   fig67new.m
%
    lambda=5;sig2=1;N=30;A=sqrt(lambda*sig2/N);
    nreal=5000;
    for i=1:nreal
    x0=randn(N,1);x1=x0+A;
    y0=(mean(x0))^2/(x0'*x0/N);
    y1=(mean(x1))^2/(x1'*x1/N);
    T0R(i,1)=N*y0;T1R(i,1)=N*y1;
    T0(i,1)=N*log(1/(1-y0));
    T1(i,1)=N*log(1/(1-y1));
    end
    for i=1:51
    Pfaa(i,1)=(i-1)/50;
    u=Qinv(Pfaa(i)/2);
    Pda(i,1)=Q(u-sqrt(lambda))+Q(u+sqrt(lambda));
    end
    [Pfag,Pdg]=rocurve(T0,T1,51);
    [PfaR,PdR]=rocurve(T0R,T1R,51);
    plot(Pfag,Pdg,'-.',PfaR,PdR,'-',Pfaa,Pda,'--')
    xlabel('Probability of false alarm (Pfa)')
    ylabel('Probability of detection (Pd)')
    grid
    print
```

CHAPTER 7 - DETERMINISTIC SIGNALS - UNKNOWN PARAMETERS

LACK OF SIGNAL KNOWLEDGE DEGRADES DETECTOR PERFORMANCE. COMPARE SIGNAL KNOWN VS SIGNAL UNKNOWN CASES.

1) SIGNAL KNOWN $\Rightarrow$ MATCHED FILTER
2) SIGNAL UNKNOWN $\Rightarrow$ USE GLRT
$\Rightarrow$ ENERGY DETECTOR

$$\mathcal{H}_0 : x(n) = w(n) \qquad n = 0, 1, \ldots, N-1$$
$$\mathcal{H}_1 : x(n) = s(n) + w(n) \qquad n = 0, 1, \ldots, N-1$$

WGN WITH KNOWN VARIANCE σ^2

1) $s(n)$ DETERMINISTIC AND KNOWN
$\Rightarrow$ USE $T(\underline{x}) = \sum_{n=0}^{N-1} x(n) s(n)$

$$P_D = Q\left(Q^{-1}(P_{FA}) - \sqrt{\mathcal{E}/\sigma^2}\right)$$

2) $s(n)$ DETERMINISTIC AND UNKNOWN

USE A GLRT.

$$L_G(\underline{x}) = \frac{p(\underline{x}; \hat{s}[0], \hat{s}[1], \ldots, \hat{s}[N-1], \mathcal{H}_1)}{p(\underline{x}; \mathcal{H}_0)}$$

TO FIND MLE OF $s[n]$ MAXIMIZE

$$p(\underline{x}; s[0], s[1], \ldots, s[N-1], \mathcal{H}_1) =$$

$$\frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2} \sum_n (x[n]-s[n])^2}$$

OVER $s[0], s[1], \ldots, s[N-1]$

$$\Rightarrow \hat{s}[n] = x[n]$$

$$L_G(\underline{x}) = \frac{\frac{1}{(2\pi\sigma^2)^{N/2}}}{\frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum x^2[n]}}$$

$$\ln L_G(\underline{x}) = \frac{1}{2\sigma^2} \sum_n x^2[n]$$

GLRT DECIDES $\mathcal{H}_1$ IF

$$T(\underline{x}) = \sum_{n=0}^{N-1} x^2[n] > \gamma'$$

QUADRATIC OR ENERGY DETECTOR (NONCOHERENT)

CAN'T FIND PERFORMANCE FROM PREVIOUS WORK SINCE WE HAD $s[n]$ = RANDOM PROCESS.

TO FIND PERFORMANCE ASSUME N IS LARGE AND APPLY CENTRAL LIMIT THEOREM ($x[n]$ ARE INDEPENDENT BUT NOT IDENTICALLY DISTRIBUTED)

ASIDE: NONCENTRAL χ^2 PDF

IF $x_1, x_2, \ldots, x_N$ ARE INDEPENDENT WITH $x_i \sim N(\mu_i, 1)$, THEN

$$y = \sum_{1}^{N} x_i^2 \sim \chi'^2_N(\lambda)$$

← NONCENTRALITY PARAMETER

↑ DEGREES OF FREEDOM

$$\lambda = \sum_{i=1}^{N} \mu_i^2$$

SEE CHAPTER 2

$E(y) = \lambda + N$

$VAR(y) = 4\lambda + 2N$

IF $\mu_i = 0$ ALL $i \Rightarrow \lambda = 0$

$\Rightarrow$ GET χ^2_N = CENTRAL χ^2 PDF

TO GET FIRST TWO MOMENTS:

$$T'(\underline{x}) = \frac{T(\underline{x})}{\sigma^2} \sim \begin{cases} \chi^2_N & \mathcal{H}_0 \\ \chi'^2_N(\lambda) & \mathcal{H}_1 \end{cases}$$

WHERE $\lambda = \sum_n \frac{s^2(n)}{\sigma^2} = \mathcal{E}/\sigma^2$

$$\Rightarrow \quad E(T;\mathcal{H}_0) = E(\sigma^2 \chi^2_N) = N\sigma^2$$

$$E(T;\mathcal{H}_1) = E(\sigma^2 \chi'^2_N(\lambda)) = \sigma^2(\lambda + N)$$

$$\mathrm{VAR}(T;\mathcal{H}_0) = \sigma^4 2N$$

$$\mathrm{VAR}(T;\mathcal{H}_1) = \sigma^4(4\lambda + 2N)$$

$$P_{FA} = Q\left(\frac{\gamma' - N\sigma^2}{\sqrt{2N\sigma^4}}\right)$$

$$P_D = Q\left(\frac{\gamma' - (\lambda + N)\sigma^2}{\sqrt{\sigma^4(4\lambda + 2N)}}\right)$$

$$\gamma' = N\sigma^2 + \sqrt{2N\sigma^4}\, Q^{-1}(P_{FA})$$

$$P_D = Q\left(\frac{\sqrt{2N}\, Q^{-1}(P_{FA}) - \lambda}{\sqrt{4\lambda + 2N}}\right)$$

AND FOR $\lambda/N \ll 1$ CAN BE SHOWN TO BE (SEE APPENDIX 7A)

$$P_D = Q\left(Q^{-1}(P_{FA}) - \sqrt{\frac{\lambda^2}{2N}}\right)$$

$\uparrow d_{ED}^2$

$$d_{ED}^2 = \frac{(\mathcal{E}/\sigma^2)^2}{2N}$$

FOR $\frac{d}{N} = \frac{\mathcal{E}}{N\sigma^2} \ll 1$

OR $\frac{\mathcal{E}}{\sigma^2} \ll N$

LOW SNR CASE

FOR MATCHED FILTER

$$d_{MF}^2 = \mathcal{E}/\sigma^2$$

$$\Rightarrow \text{LOSS IN dB} = 10\log_{10} \frac{d_{MF}^2}{d_{ED}^2} \text{ dB}$$

$$= 10\log_{10} \frac{2N}{\mathcal{E}/\sigma^2} \text{ dB}$$

THIS IS LOSS DUE TO LACK OF SIGNAL KNOWLEDGE.

EXAMPLE : DC LEVEL IN WGN

(DC LEVEL SIGNAL IS UNKNOWN)

$\mathcal{E} = NA^2 \Rightarrow$ FOR $\frac{\mathcal{E}}{\sigma^2} \ll N$

$\frac{NA^2}{\sigma^2} \ll N$ OR $\frac{A^2}{\sigma^2} \ll 1$

LOW INPUT SNR CASE

COMPARE REQUIRED INPUT SNR
FOR GIVEN DETECTION PERFORMANCE

LET $\eta = A^2/\sigma^2$ = INPUT SNR

$$d^2_{MF} = \frac{NA^2}{\sigma^2} = N\eta_{MF}$$

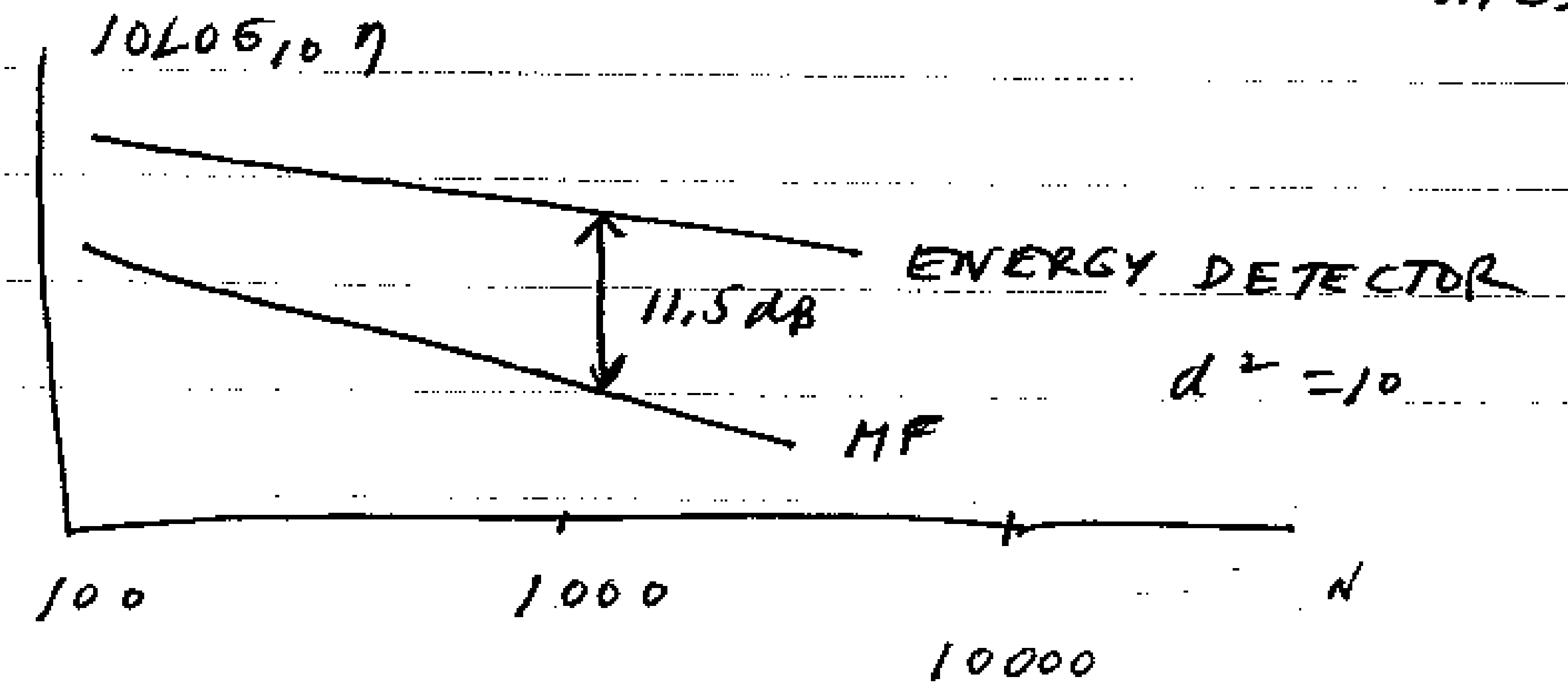
$$d^2_{ED} = \frac{\left(\frac{NA^2}{\sigma^2}\right)^2}{2N} = N\frac{A^4}{2\sigma^4}$$

$$= \frac{N}{2}\eta^2_{ED}$$

FOR SAME PERFORMANCE $d^2_{MF} = d^2_{ED}$
$= d^2$

$\Rightarrow$ $\eta_{MF} = d^2/N \propto 1/N$ (3 dB FOR N DOUBLING)

$\eta_{ED} = \sqrt{\frac{2d^2}{N}} \propto 1/\sqrt{N}$ (1.5 dB FOR N DOUBLING)



MATCHED FILTER COHERENTLY COMBINES DATA WHILE ENERGY DETECTOR INCOHERENTLY COMBINES DATA

GLRT FOR COMMON PROBLEMS

1) UNKNOWN AMPLITUDE

$$\mathcal{H}_0 : x(n) = w(n)$$

$$\mathcal{H}_1 : x(n) = \underset{\text{UNKNOWN } -\infty < A < \infty}{A}\,\underset{\text{KNOWN}}{s(n)} + w(n)$$

NO UMP TEST SINCE A CAN BE POSITIVE OR NEGATIVE

$$L_G(\underline{x}) = \frac{p(\underline{x}; \hat{A}, \mathcal{H}_1)}{p(\underline{x}; \mathcal{H}_0)}$$

$$p(\underline{x}; \underline{A}, \mathcal{H}_1) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum_n (x(n) - As(n))^2}$$

TO FIND MLE OF $\hat{A}$ MINIMIZE

$$J(A) = \sum_n (x(n) - As(n))^2$$

$$\Rightarrow \quad \hat{A} = \sum_{n=0}^{N-1} x(n)s(n) \Big/ \sum_{n=0}^{N-1} s^2(n)$$

$\hat{A} = ?$ IF $s(n) = 1$?

$$\ln L_G(\underset{\sim}{x}) = -\frac{1}{2\sigma^2}\Big(\sum (x(n) - \hat{A}s(n))^2 - \sum x^2(n)\Big)$$

$$= -\frac{1}{2\sigma^2}\Big(-2\hat{A}\sum x(n)s(n) + \hat{A}^2\sum s^2(n)\Big)$$

$$= \frac{\hat{A}}{\sigma^2}\underbrace{\sum x(n)s(n)}_{\hat{A}\sum s^2(n)} - \frac{1}{2\sigma^2}\hat{A}^2\sum s^2(n)$$

$$= \hat{A}^2\,\frac{\sum_n s^2(n)}{2\sigma^2} > \ln\gamma$$

OR $$\hat{A}^2 > \frac{2\sigma^2 \ln\gamma}{\sum_n s^2(n)}$$

OR $$|\hat{A}| > \sqrt{\frac{2\sigma^2\ln\gamma}{\sum_n s^2(n)}}$$

OR $$\Big(\sum_{n=0}^{N-1} x(n)s(n)\Big)^2 > \gamma'$$

REPLICA-CORRELATOR + SQUARER TO ACCOUNT FOR LACK OF KNOWLEDGE OF SIGN OF A.

PERFORMANCE GIVEN IN TEXT -
ONLY SLIGHT DEGRADATION
(SEE FIG 7.3)

2) UNKNOWN ARRIVAL TIME

GIVEN A SIGNAL $s(n)$ $n = 0, 1, \ldots, M-1$
THAT ARRIVES AT $n = n_0$ OR $s(n - n_0)$
WE WISH TO DETECT IT WITHOUT
KNOWING n_0.

$$\mathcal{H}_0 : x(n) = w(n) \qquad n = 0, 1, \ldots, N-1$$
$$\mathcal{H}_1 : x(n) = s(n - n_0) + w(n) \qquad "$$

$s(n)$ IS KNOWN AND $w(n)$ IS WGN.
ASSUME ENTIRE SIGNAL IS INCLUDED
IN OBSERVATION INTERVAL $[0, N-1]$

A GLRT DECIDES $\mathcal{H}_1$ IF

$$\frac{p(\underline{x}; \hat{n}_0, \mathcal{H}_1)}{p(\underline{x}; \mathcal{H}_0)} > \gamma$$

NEED MLE OF n_0

$$p(\underline{x};n_0) = \prod_{n=0}^{n_0-1} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}x^2[n]}$$

$$\cdot \prod_{n=n_0}^{n_0+M-1} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x[n]-s[n-n_0])^2}$$

$$\cdot \prod_{n=n_0+M}^{N-1} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}x^2[n]}$$

TO MAXIMIZE OVER n_0 NEED TO MAXIMIZE

$$p(\underline{x};n_0) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1} x^2[n]}$$

$$\cdot \prod_{n=n_0}^{n_0+M-1} e^{-\frac{1}{2\sigma^2}(-2x[n]s[n-n_0] + s^2[n-n_0])}$$

OR MAXIMIZE

$$e^{-\frac{1}{2\sigma^2}\sum_{n=n_0}^{n_0+M-1}(-2x[n]s[n-n_0] + s^2[n-n_0])}$$

OR MINIMIZE

$$\sum_{n=n_0}^{n_0+M-1} (-2x[n]s[n-n_0] + s^2[n-n_0])$$

BUT $\displaystyle\sum_{n=n_0}^{n_0+M-1} s^2[n-n_0] = \sum_{n=0}^{M-1} s^2[n] = \mathcal{E}$

DOES NOT DEPEND ON n_0

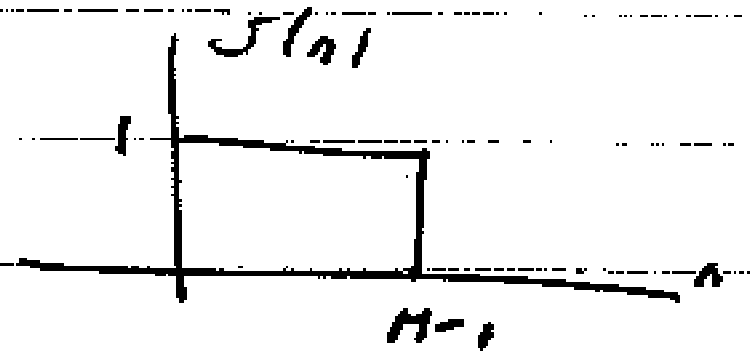
$\Rightarrow$ MAXIMIZE OVER n_0

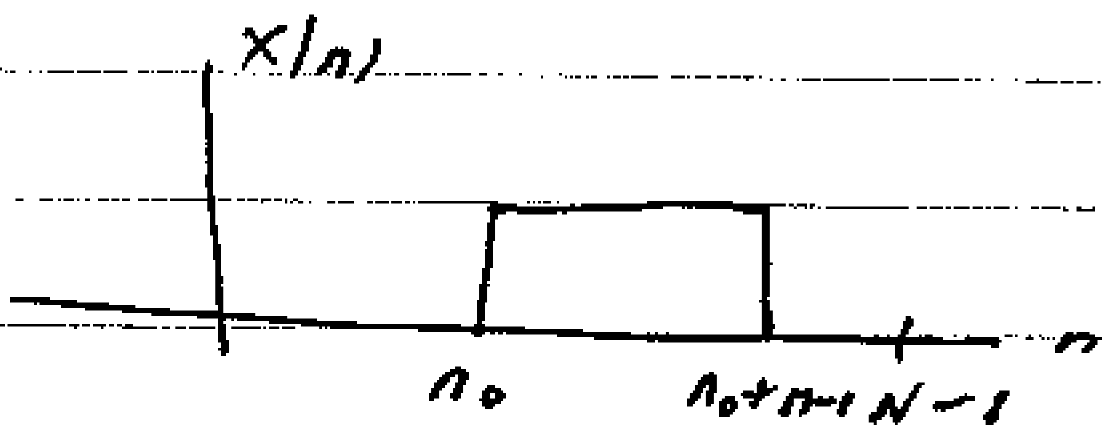
$$\sum_{n=n_0}^{n_0+M-1} x[n]\, s[n-n_0]$$

THIS IS A CORRELATOR THAT CORRELATES AGAINST ALL POSSIBLE SIGNALS - DIFFERENT ARRIVAL TIMES.

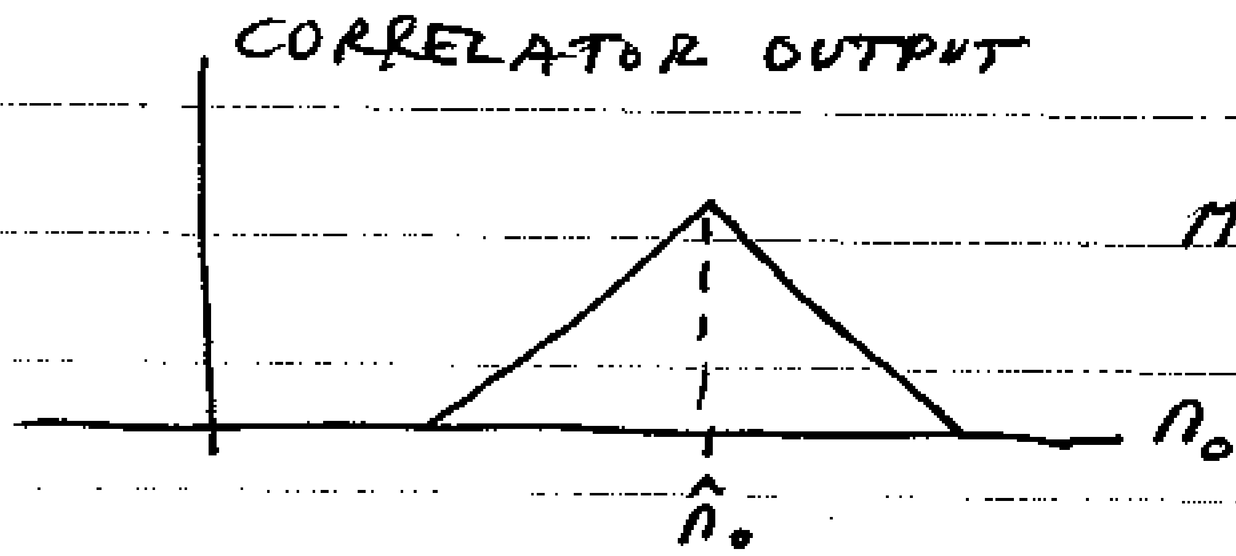
$\Rightarrow$ MLE $\equiv$ "RUNNING" CORRELATOR

EXAMPLE :





NO NOISE



MLE IS WHERE PEAK OCCURS

$$\frac{p(\underline{x};\hat{n}_0, \mathcal{H}_1)}{p(\underline{x};\mathcal{H}_0)} = \prod_{n=\hat{n}_0}^{\hat{n}_0+M-1} e^{-\frac{1}{2\sigma^2}\left(-2x[n]s[n-\hat{n}_0] + s^2[n-\hat{n}_0]\right)}$$

TAKE LOGS ⇒ WE DECIDE $\mathcal{H}_1$ IF

$$-\frac{1}{2\sigma^2}\sum_{n=\hat{n}_0}^{\hat{n}_0+M-1}\left(-2x[n]s[n-\hat{n}_0]+s^2[n-\hat{n}_0]\right) > \ln\gamma$$

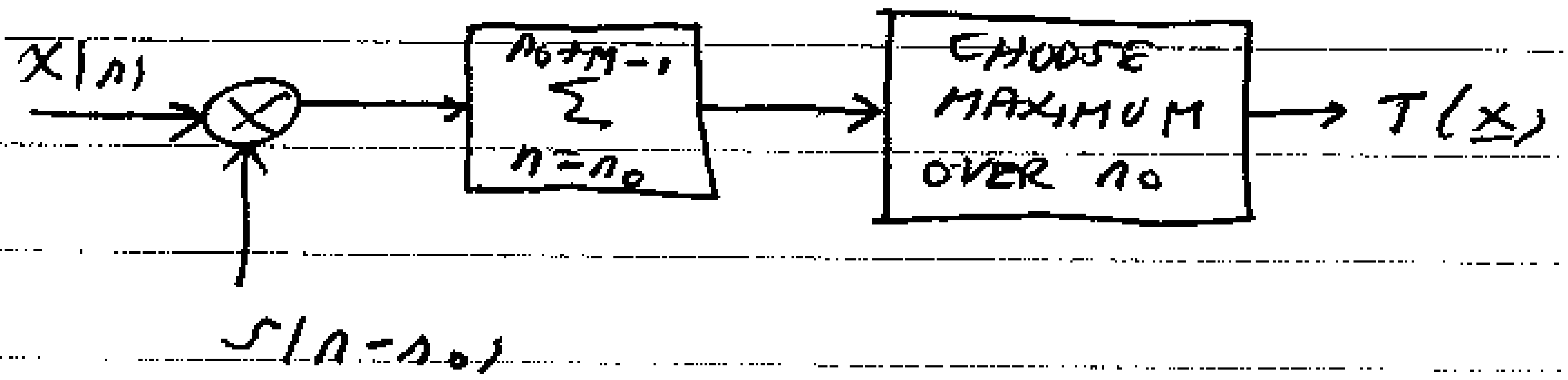
BUT $\displaystyle\sum_{n=\hat{n}_0}^{\hat{n}_0+M-1} s^2[n-\hat{n}_0] = \mathcal{E}$

$$\sum_{n=\hat{n}_0}^{\hat{n}_0+M-1} x[n]s[n-\hat{n}_0] > \sigma^2\ln\gamma + \mathcal{E}/2 = \gamma'$$

OR FINALLY SINCE $\hat{n}_0$ IS VALUE THAT MAXIMIZES $\displaystyle\sum_{n=n_0}^{n_0+M-1} x[n]s[n-n_0]$

WE DECIDE $\mathcal{H}_1$ IF

$$\max_{n_0}\sum_{n=n_0}^{n_0+M-1} x[n]s[n-n_0] > \gamma'$$



NOTE THAT IF WE EXCEED THRESHOLD AND DECLARE $\mathcal{H}_1$ (SIGNAL PRESENT), THEN ALSO HAVE MLE OF ARRIVAL TIME OR $\hat{n}_0$.

PFA, PD HARD TO FIND ANALYTICALLY
NEED PDF OF MAXIMUM OF CORRELATED
(DUE TO OVERLAP FOR n_0, n_0+1, ETC.)
GAUSSIAN RANDOM VARIABLES.

SINUSOIDAL DETECTION

IMPORTANT PRACTICAL PROBLEM. WILL CONSIDER

$$\mathcal{H}_0 : x[n] = w[n] \qquad n = 0, 1, \ldots, N-1$$
$$\mathcal{H}_1 : x[n] = A\cos(2\pi f_0 n + \phi) + w[n] \qquad "$$

$w[n]$ IS WGN AND SIGNAL IS DETERMINISTIC WITH UNKNOWN PARAMETERS

CASES:

1) A UNKNOWN
2) A, ϕ UNKNOWN
3) A, ϕ, f_0 UNKNOWN
* 4) A, ϕ, f_0 AND ARRIVAL TIME UNKNOWN

* DEFER FOR NOW

1) A UNKNOWN

FROM PREVIOUS WORK (SLIDES 153, 154)

DECIDE $\mathcal{H}_1$ IF

$$\left(\sum_{n=0}^{N-1} x[n] s[n] \right)^2 > \gamma'$$

$$\left(\sum_{n} x[n] \cos(2\pi f_0 n + \phi) \right)^2 > \gamma'$$

NOTE THAT MLE OF A IS

$$\hat{A} = \frac{\sum x[n] s[n]}{\sum s^2[n]}$$

$$= \frac{\sum x[n] \cos(2\pi f_0 n + \phi)}{\sum \cos^2(2\pi f_0 n + \phi)}$$

$$\approx \frac{2}{N} \sum_{n=0}^{N-1} x[n] \cos(2\pi f_0 n + \phi)$$

2) A, ϕ UNKNOWN

$$s[n] = A \cos(2\pi f_0 n + \phi)$$

ASSUME $A > 0$, ELSE CAN'T IDENTIFY A, ϕ UNIQUELY SINCE