EXAMPLE: DC LEVEL IN WGN

$-\infty < A < \infty$

CAN'T CHOOSE $p(A) = $ CONSTANT SINCE $\int_{-\infty}^{\infty} p(A)\,dA \to \infty$

LET $A \sim N(0, \sigma_A^2)$ AND LET $\sigma_A^2 \to \infty$ AFTER WE FIND NP TEST.

EXAMPLE: SINUSOID WITH UNKNOWN PHASE IN WGN

CHOOSE $p(\phi) = 1/2\pi \quad -\pi \le \phi \le \pi$

LEADS TO QUADRATURE M.F.

WHAT IS DIFFERENCE BETWEEN EXAMPLES?

GENERALIZED LIKELIHOOD RATIO TEST (GLRT)

NO OPTIMALITY PROPERTIES BUT WORKS WELL IN PRACTICE.

REPLACES UNKNOWN PARAMETERS BY THEIR MAXIMUM LIKELIHOOD ESTIMATOR (MLE). DECIDE $\mathcal{H}_1$ IF

$$L_G(\underline{x}) = \frac{p(\underline{x}; \hat{\underline{\theta}}_1, \mathcal{H}_1)}{p(\underline{x}; \hat{\underline{\theta}}_0, \mathcal{H}_0)} > \gamma$$

$\hat{\underline{\theta}}_1$ = MLE OF $\underline{\theta}_1$ ASSUMING $\mathcal{H}_1$ TRUE
$\hat{\underline{\theta}}_0$ = " " $\underline{\theta}_0$ " $\mathcal{H}_0$ "

EQUIVALENTLY

$$L_G(\underline{x}) = \frac{\max_{\underline{\theta}_1} p(\underline{x}; \underline{\theta}_1, \mathcal{H}_1)}{\max_{\underline{\theta}_0} p(\underline{x}; \underline{\theta}_0, \mathcal{H}_0)}$$

CALLED JUST LIKELIHOOD RATIO TEST IN STATISTICS

ASIDE - MLEs

EXAMPLE: DC LEVEL IN WGN

$$p(\underline{x}; A) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n]-A)^2}$$

TO FIND MLE:

1) REPLACE $\underline{x}$ BY OBSERVED SAMPLES

2) MAXIMIZE PDF (CALLED LIKELIHOOD FUNCTION OVER A)

~~VIEWED AS FUNCTION OF A~~

1) $\underline{x} \to \underline{x}_0$ (OBSERVED SAMPLES)

2) MAXIMIZE $p(\underline{x}_0; A)$ OR $\ln p(\underline{x}_0; A)$

$$\ln p(\underline{x}_0; A) = -\frac{N}{2}\ln 2\pi\sigma^2 - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x_0[n]-A)^2$$

$$\frac{\partial \ln p(\underline{x}_0; A)}{\partial A} = -\frac{1}{2\sigma^2}(-2)\sum_{n=0}^{N-1}(x_0[n]-A) = 0$$

$$\Rightarrow \sum_{n=0}^{N-1}(x_0[n]-A) = 0$$

$$\Rightarrow A = \hat{A} = 1/N \sum_{n=0}^{N-1} x_0[n]$$

OR DROPPING THE "0" SUBSCRIPT

$$\hat{A} = \bar{x} = \text{SAMPLE MEAN}$$

MLE ≡ SAMPLE MEAN

BACK TO GLRT

EXAMPLE : DC LEVEL IN WGN, A UNKNOWN
$-\infty < A < \infty$

GLRT SAYS TO DECIDE $\mathcal{H}_1$ IF

$$L_G(\underline{x}) = \frac{p(\underline{x}; \hat{A}, \mathcal{H}_1)}{p(\underline{x}; \mathcal{H}_0)} > \gamma \qquad \underline{\theta}_1 = A \quad \text{NO } \underline{\theta}_0$$

NO UNKNOWN PARAMETERS

$$L_G(\underline{x}) = \frac{\frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum(x[n]-\hat{A})^2}}{\frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum x^2[n]}}$$

BUT $\hat{A} = \bar{x}$

TAKE LOGS

$$\ln L_G(\underline{x}) = -\frac{1}{2\sigma^2}\left(\sum(x[n]-\bar{x})^2 - \sum x^2[n]\right)$$

$$= -\frac{1}{2\sigma^2}\left(-2\sum x[n]\bar{x} + N\bar{x}^2\right)$$

$$= -\frac{1}{2\sigma^2}\left(-2N\bar{x}^2 + N\bar{x}^2\right)$$

$$= \frac{N\bar{x}^2}{2\sigma^2}$$

OR WE DECIDE $\mathcal{H}_1$ IF

$$(\bar{x})^2 > \gamma'$$

OR $|\bar{x}| > \sqrt{\gamma'} = \gamma''$

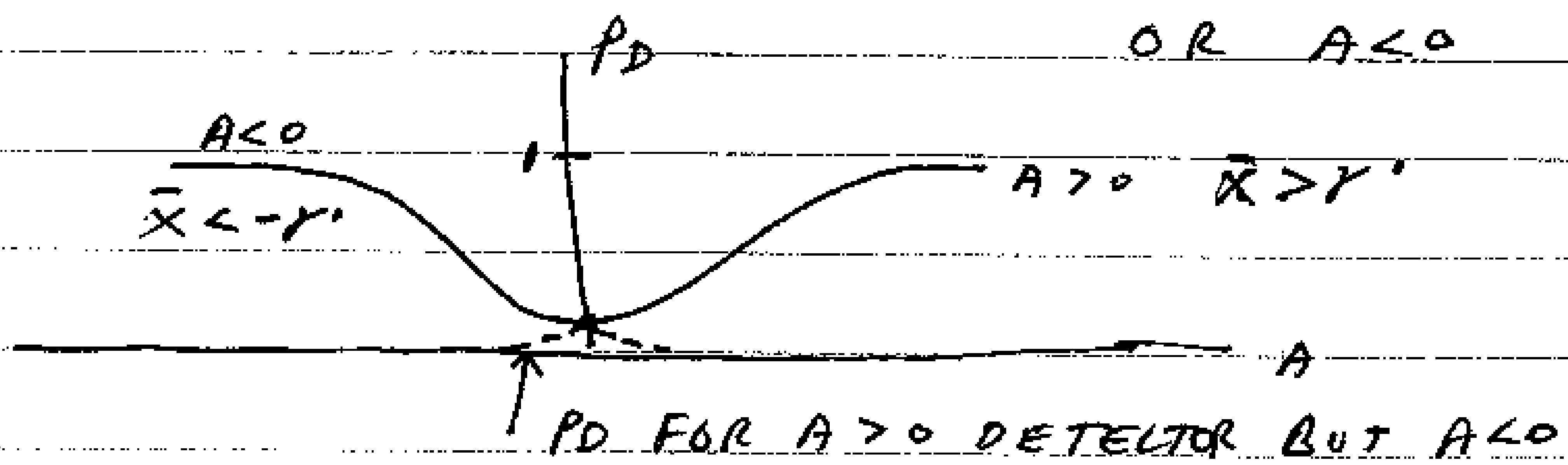
THIS IS DETECTOR WE GUESSED AT.

TO COMPARE TO NP (UNREALIZABLE ALSO CALLED CLAIRVOYANT DETECTOR):

CLAIRVOYANT DETECTOR

$$P_D = Q\left(Q^{-1}(P_{FA}) - \sqrt{d^2}\right)$$

$$d^2 = NA^2/\sigma^2 \quad \text{FOR } A>0 \text{ OR } A<0$$



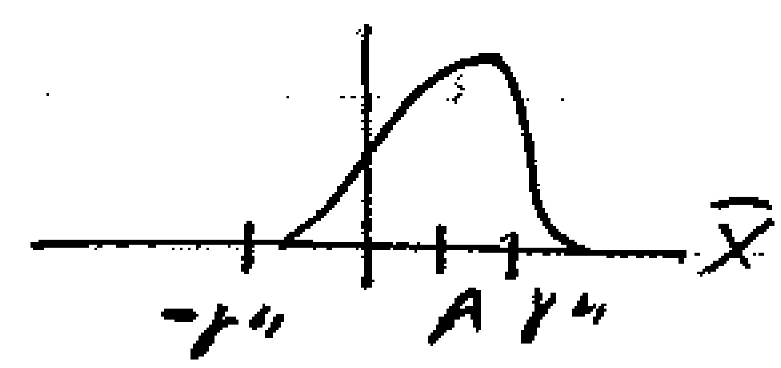
ALL OTHER DETECTORS HAVE P_D BELOW THIS UPPER BOUND

GLRT: DECIDE $\mathcal{H}_1$ IF $|\bar{x}| > \gamma''$

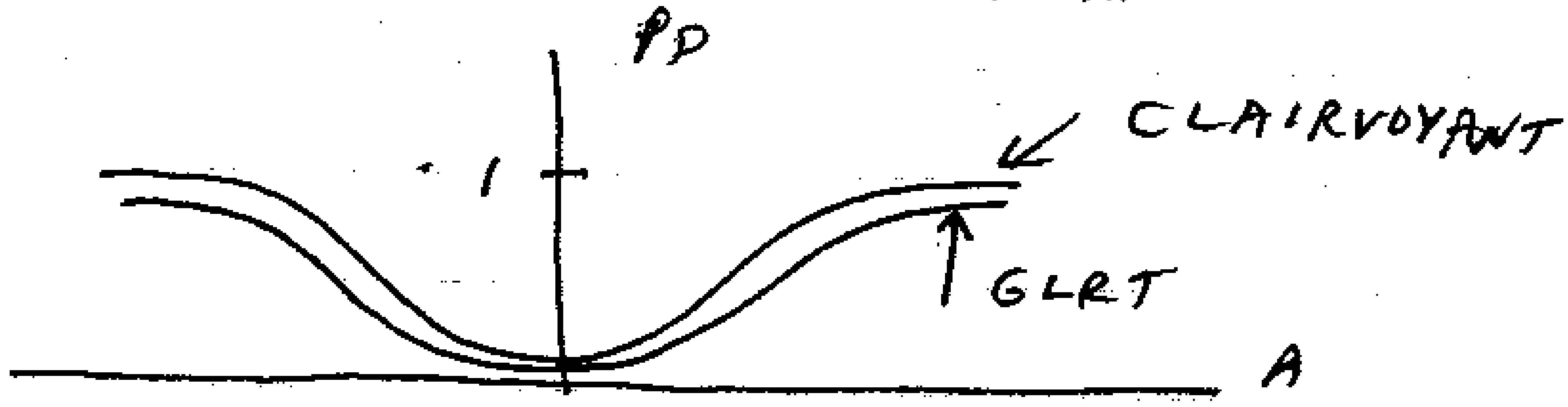
TO MAKE SURE GLRT IS IMPLEMENTABLE MUST BE ABLE TO SPECIFY γ'' INDEPENDENT OF A. POSSIBLE SINCE $p(\underline{x}; \mathcal{H}_0)$ NOT DEPENDENT ON A.

$$P_{FA} = \Pr\{ |\bar{x}| > \gamma''; \mathcal{H}_0 \} \qquad \bar{x} \sim \mathcal{N}(0, \sigma^2/N)$$
$$= 2 \Pr\{ \bar{x} > \gamma''; \mathcal{H}_0 \}$$
$$= 2 Q\left(\gamma'' / \sqrt{\sigma^2/N} \right)$$

$$P_D = \Pr\{ |\bar{x}| > \gamma''; \mathcal{H}_1 \} \qquad \bar{x} \sim \mathcal{N}(A, \sigma^2/N)$$
$$= \Pr\{ \bar{x} > \gamma''; \mathcal{H}_1 \} + \Pr\{ \bar{x} < -\gamma''; \mathcal{H}_1 \}$$



$$= Q\left(\frac{\gamma'' - A}{\sqrt{\sigma^2/N}} \right) + 1 - Q\left(\frac{-\gamma'' - A}{\sqrt{\sigma^2/N}} \right)$$
$$= Q\left(Q^{-1}(P_{FA}/2) - \frac{A}{\sqrt{\sigma^2/N}} \right)$$
$$+ Q\left(Q^{-1}(P_{FA}/2) + \frac{A}{\sqrt{\sigma^2/N}} \right)$$



SLIGHT DEGRADATION BUT MUCH

MORE ROBUST!

EXAMPLE: DC LEVEL IN WGN, A, σ^2 BOTH UNKNOWN

NO UMP TEST. CAN'T IMPLEMENT NP SINCE A UNKNOWN $\bar{x} \gtrless \gamma'$? AND σ^2 UNKNOWN $\Rightarrow$ CAN'T SET γ'!

$$\mathcal{H}_0: A = 0, \ \sigma^2 > 0$$
$$\mathcal{H}_1: A \neq 0, \ \sigma^2 > 0$$

UNKNOWN - CALLED A NUISANCE PARAMETER

GLRT DECIDES $\mathcal{H}_1$ IF

$$L_G(\underline{x}) = \frac{p(\underline{x}; \hat{A}, \hat{\sigma}_1^2, \mathcal{H}_1)}{p(\underline{x}; \hat{\sigma}_0^2, \mathcal{H}_0)} > \gamma$$

$\hat{\sigma}_0^2$ = MLE OF σ^2 UNDER $\mathcal{H}_0$
$\hat{A}, \hat{\sigma}_1^2$ = MLE OF A, σ^2 UNDER $\mathcal{H}_1$

NOTE THAT $\hat{\sigma}_1^2 \neq \hat{\sigma}_0^2$

TO FIND $\hat{\sigma}_0^2$ WE MAXIMIZE

$$p(\underline{x};\sigma^2,\mathcal{H}_0) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1} x^2[n]}$$

$$\ln p = -\frac{N}{2}\ln 2\pi - \frac{N}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\sum x^2[n]$$

$$\frac{\partial \ln p}{\partial\sigma^2} = -\frac{N}{2\sigma^2} + \frac{1}{2\sigma^4}\sum x^2[n] = 0$$

$$\Rightarrow \hat{\sigma}_0^2 = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$

$$p(\underline{x};\hat{\sigma}_0^2,\mathcal{H}_0) = \frac{1}{(2\pi\hat{\sigma}_0^2)^{N/2}} e^{-N/2}$$

UNDER $\mathcal{H}_1$ SHOW THAT

$$\hat{A} = \bar{x}$$

$$\hat{\sigma}_1^2 = \frac{1}{N}\sum_{n=0}^{N-1}(x[n]-\bar{x})^2$$

$$\Rightarrow p(\underline{x};\hat{A},\hat{\sigma}_1^2,\mathcal{H}_1) =$$

$$\frac{1}{(2\pi\hat{\sigma}_1^2)^{N/2}} e^{-N/2}$$

$$L_G(\underline{x}) = \left(\frac{\hat{\sigma}_0^2}{\hat{\sigma}_1^2}\right)^{N/2}$$

GLRT FITS DATA WITH SIGNAL $\hat{A}=\bar{x}$ UNDER $\mathcal{H}_1$ AND FINDS RESIDUAL POWER

$\hat{\sigma}_1^2$ AND COMPARES TO CASE OF NO FIT OR $\hat{\sigma}_0^2$. WHEN SIGNAL IS PRESENT $\hat{\sigma}_1^2 << \hat{\sigma}_0^2$

$\Rightarrow \quad L_G(\underline{x}) >> 1.$

ALTERNATIVELY,

$$
\begin{aligned}
\hat{\sigma}_1^2 &= \frac{1}{N} \sum (x[n] - \bar{x})^2 \\
&= \frac{1}{N} \sum (x^2[n] - 2\bar{x}\, x[n] + \bar{x}^2) \\
&= \frac{1}{N} \sum x^2[n] - 2\bar{x}^2 + \bar{x}^2 \\
&= \frac{1}{N} \sum x^2[n] - \bar{x}^2 \\
&= \hat{\sigma}_0^2 - \bar{x}^2
\end{aligned}
$$

$$
\begin{aligned}
2 \ln L_G(\underline{x}) &= N \ln \frac{\hat{\sigma}_1^2 + \bar{x}^2}{\hat{\sigma}_1^2} \\
&= N \ln \left(1 + \frac{\bar{x}^2}{\hat{\sigma}_1^2}\right)
\end{aligned}
$$

EQUIVALENT TEST IS

$$T(\underline{x}) = \frac{\bar{x}^2}{\hat{\sigma}_1^2} > \gamma''$$

WE SEE THAT $\bar{x}$ HAS BEEN NORMALIZED $\Rightarrow$ INVARIANT TO σ^2 UNDER $\mathcal{H}_0$ $\Rightarrow$ CAN FIND γ''

$$T(\underline{x}) = \frac{\left(\frac{1}{N}\sum_n w[n]\right)^2}{\frac{1}{N}\sum_n (w[n]-\bar{w})^2} \quad \text{UNDER } \mathcal{H}_0$$

$$= \frac{\left(\frac{1}{N}\sum \sigma u[n]\right)^2}{\frac{1}{N}\sum (\sigma u[n]-\sigma\bar{u})^2} \quad u[n] \sim N(0,1)$$

$$= \frac{\left(\frac{1}{N}\sum u[n]\right)^2}{\frac{1}{N}\sum (u[n]-\bar{u})^2}$$

PDF OF T DOES NOT DEPEND ON σ^2 UNDER $\mathcal{H}_0$. $\Rightarrow$ GLRT CAN BE IMPLEMENTED

LARGE DATA RECORD PERFORMANCE OF GLRT

RESULTS VALID IF:

1) N IS LARGE $\Rightarrow$ SIGNAL WEAK

2) MLE ATTAINS ASYMPTOTIC PDF $\hat{\underline{\theta}} \sim \mathcal{N}(\underline{\theta}, \underline{I}^{-1}(\underline{\theta}))$

$\underline{I}(\underline{\theta})$ = FISHER INFORMATION MATRIX

NOTE: RECALL DC LEVEL IN WGN

$$P_D = Q(Q^{-1}(P_{FA}) - \sqrt{d^2})$$

$$d^2 = NA^2/\sigma^2$$

FOR GIVEN (P_{FA}, P_D) NEED GIVEN d^2. AS $N \to \infty$ CAN ATTAIN GIVEN (P_{FA}, P_D) WITH $A \to 0$ (WEAK SIGNAL).

GENERAL RESULTS

GIVEN $p(\underline{x}; \underline{\theta})$ $\quad \underline{\theta} = \begin{bmatrix} \underline{\theta}_r \\ \underline{\theta}_s \end{bmatrix} = \begin{bmatrix} r \times 1 \\ s \times 1 \end{bmatrix}$

$\underline{\theta}_r$ IS TO BE TESTED

$\underline{\theta}_s$ IS A NUISANCE PARAMETER

EXAMPLE: DC LEVEL IN WGN

$$\mathcal{H}_0: A = 0, \quad \sigma^2 > 0$$

$$\mathcal{H}_1: A \neq 0, \quad \sigma^2 > 0$$

$\uparrow$ UNKNOWN

$$\underline{\theta} = \begin{pmatrix} A \\ \sigma^2 \end{pmatrix} \Rightarrow \quad \underline{\theta}_r = A \qquad r=1$$

$$\underline{\theta}_s = \sigma^2 \qquad s=1$$

IN GENERAL WE TEST

$$\mathcal{H}_0 : \underline{\theta}_r = \underline{\theta}_{r_0}, \ \underline{\theta}_s$$

$$\mathcal{H}_1 : \underline{\theta}_r \neq \underline{\theta}_{r_0}, \ \underline{\theta}_s$$

($\underline{\theta}_{r_0} = 0$ IN EXAMPLE)

GLRT IS TO DECIDE $\mathcal{H}_1$ IF

$$L_G(\underline{x}) = \frac{p(\underline{x}; \hat{\underline{\theta}}_{r_1}, \hat{\underline{\theta}}_{s_1}, \mathcal{H}_1)}{p(\underline{x}; \underline{\theta}_{r_0}, \hat{\underline{\theta}}_{s_0}, \mathcal{H}_0)} > \gamma$$

$\hat{\underline{\theta}}_{r_1}, \hat{\underline{\theta}}_{s_1}$ = MLE OF $\underline{\theta}_r, \underline{\theta}_s$ UNDER $\mathcal{H}_1$

$\hat{\underline{\theta}}_{s_0}$ = MLE OF $\underline{\theta}_s$ UNDER $\mathcal{H}_0$

($\underline{\theta}_r = \underline{\theta}_{r_0}$ IS CONSTRAINT)

THEN AS $N \to \infty$

$$2 \ln L_G(\underline{x}) \sim \chi^2_r \quad \text{UNDER } \mathcal{H}_0$$

$$\chi'^2_r(\lambda) \quad \text{UNDER } \mathcal{H}_1$$

$\chi'^2_r(\lambda)$ DENOTES A NONCENTRAL χ^2 PDF WITH r DEGREES OF FREEDOM AND NONCENTRALITY PARAMETER λ (SEE CHAPTER 2)

NOTE THAT $2 \ln L_G(\underline{x}) \sim \chi^2_r$ UNDER $\mathcal{H}_0$ ALLOWS US TO FIND THRESHOLD

SEE BOOK FOR DEFINITION OF λ. (6.24)

EXAMPLE: DC LEVEL IN WGN,
A UNKNOWN, σ^2 KNOWN

$$\mathcal{H}_0 : A = 0$$
$$\mathcal{H}_1 : A \neq 0$$

$\underline{\theta}_r = A$, NO $\underline{\theta}_s$, $r = 1$

$$\Rightarrow 2 \ln L_G(\underline{x}) \overset{a}{\sim} \chi^2_1 \quad \text{UNDER } \mathcal{H}_0$$

$\leftarrow$ ASYMPTOTIC

BUT FROM PREVIOUS WORK

$$\ln L_G(\underline{x}) = \frac{N\bar{x}^2}{2\sigma^2}$$

$$2 \ln L_G(\underline{x}) = \frac{N\bar{x}^2}{\sigma^2}$$

UNDER $\mathcal{H}_0$ $\bar{x} \sim N(0, \sigma^2/N)$

$$\Rightarrow 2 \ln L_G(\underline{x}) = \Big(\underbrace{\frac{\bar{x}}{\sqrt{\sigma^2/N}}}_{\uparrow N(0,1)}\Big)^2 \sim \chi_1^2$$

ASYMPTOTIC PDF HOLDS EXACTLY - FOR ANY N.

<u>EXAMPLE</u>: DC LEVEL IN WGN

A UNKNOWN, σ^2 UNKNOWN

$$\mathcal{H}_0: A = 0, \sigma^2 > 0$$
$$\mathcal{H}_1: A \neq 0, \sigma^2 > 0$$

$$\underline{\theta_r} = A, \quad \underline{\theta_s} = \sigma^2 \quad r = 1, s = 1$$

$$\Rightarrow 2 \ln L_G(\underline{x}) \overset{a}{\sim} \chi_1^2$$

SAME AS WHEN σ^2 IS KNOWN - NUISANCE PARAMETERS DO NOT AFFECT THRESHOLD BUT DO

DECREASE λ AND HENCE P_D.

FROM PREVIOUS WORK

$$2 \ln L_G(\mathbf{x}) = N \ln \frac{1}{1 - \frac{\bar{x}^2}{\hat{\sigma}_0^2}}$$

$$\stackrel{a}{\sim} \chi_1^2$$

NOW THE PDF IS ONLY APPROXIMATE
$\Rightarrow$ THRESHOLD ONLY APPROXIMATE
BUT LESS ERROR AS $N \to \infty$.

TO FIND THRESHOLD:

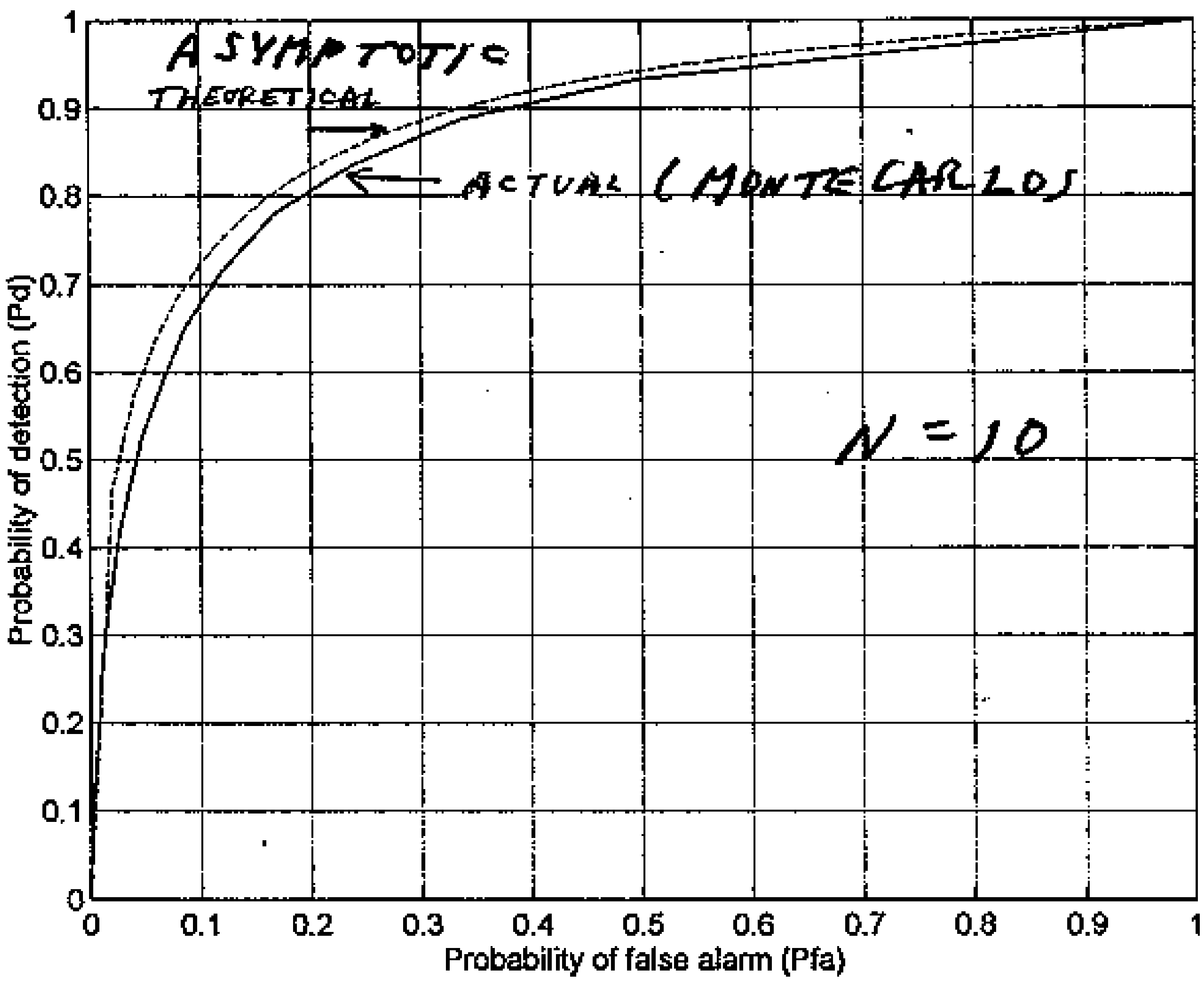
$$P_{FA} = P_r\{2 \ln L_G(\mathbf{x}) > \gamma' ; \mathcal{H}_0\}$$
$$= P_r\{\chi_1^2 > \gamma' ; \mathcal{H}_0\}$$

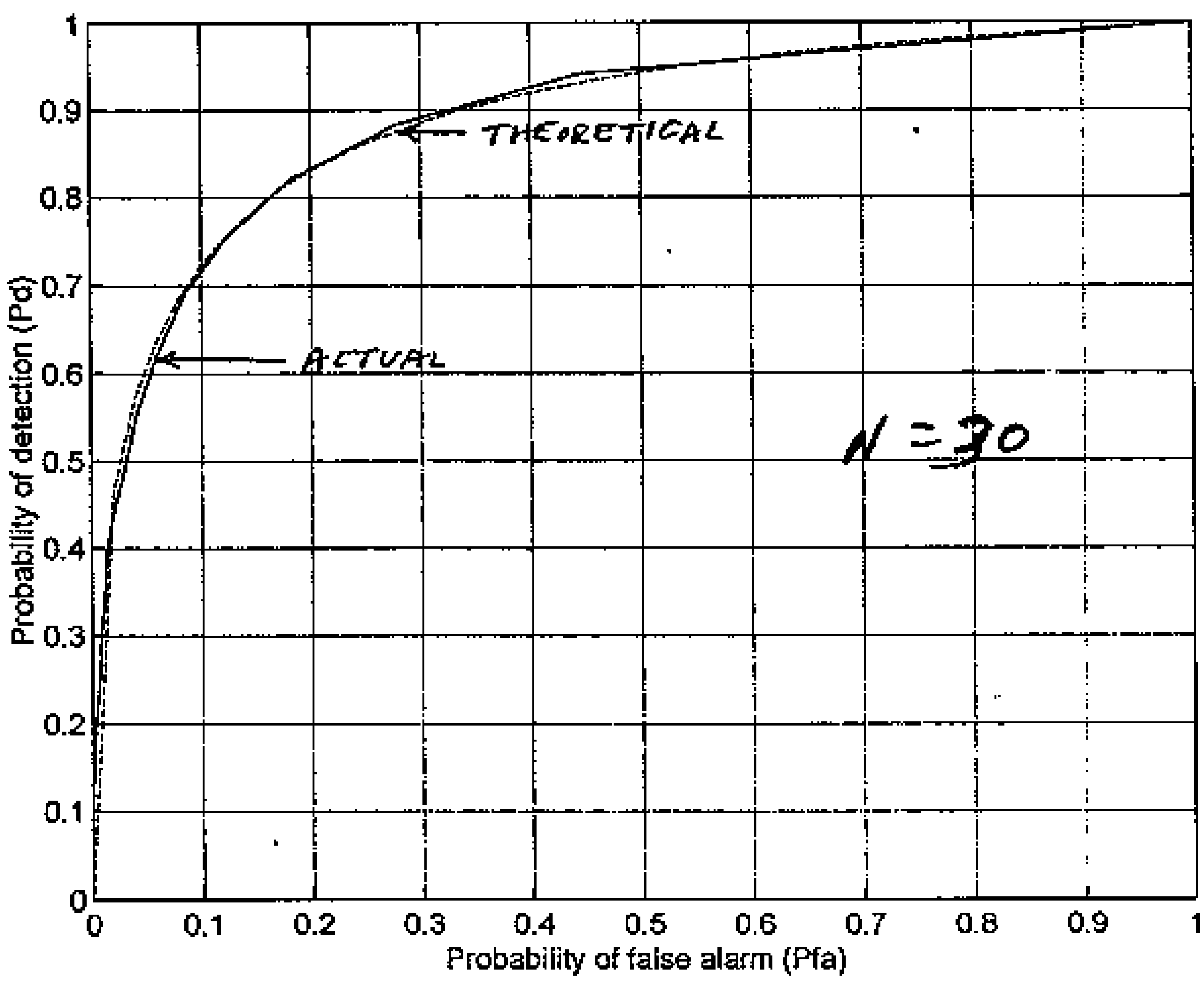
$$= P_r\{z^2 > \gamma' ; \mathcal{H}_0\} \qquad z \sim \mathcal{N}(0,1)$$
$$= P_r\{z > \sqrt{\gamma'} ; \mathcal{H}_0\}$$
$$+ P_r\{z < -\sqrt{\gamma'} ; \mathcal{H}_0\}$$

$$= 2Q(\sqrt{\gamma'})$$

TO FIND ACTUAL P_{FA}, P_D (FOR FINITE N)

WE RESORT TO MONTE CARLO COMPUTER SIMULATIONS. SEE FIG. 6.5





```
%   fig65new.m
%
    lambda=5;sig2=1;N=30;A=sqrt(lambda*sig2/N);
    nreal=5000;
    for i=1:nreal
    x0=randn(N,1);x1=x0+A;
    y0=(mean(x0))^2/(x0'*x0/N);
    y1=(mean(x1))^2/(x1'*x1/N);
    T0(i,1)=N*log(1/(1-y0));
    T1(i,1)=N*log(1/(1-y1));
    end
    for i=1:51
    Pfaa(i,1)=(i-1)/50;
    u=Qinv(Pfaa(i)/2);
    Pda(i,1)=Q(u-sqrt(lambda))+Q(u+sqrt(lambda));
    end
    [Pfa,Pd]=rocurve(T0,T1,51);
    plot(Pfa,Pd,'-',Pfaa,Pda,'--')
    xlabel('Probability of false alarm (Pfa)')
    ylabel('Probability of detection (Pd)')
    grid
    print
```

σ^2

PARAMETERS

DATA UNDER $\mathcal{H}_0, \mathcal{H}_1$

$T(\underline{x})$ UNDER $\mathcal{H}_0$

$T(\underline{x})$ UNDER $\mathcal{H}_1$

THEORETICAL

ACTUAL

SQRT(SIG2)*

RAO TEST

ASYMPTOTICALLY EQUIVALENT TO GLRT ⇒ SAME P_{FA}, P_D BUT DIFFERENT TEST STATISTIC

CASE 1: NO NUISANCE PARAMETERS

$$\mathcal{H}_0 : \underline{\theta} = \underline{\theta}_0 \qquad \mathcal{H}_0 : A = 0$$
$$\mathcal{H}_1 : \underline{\theta} \neq \underline{\theta}_0 \qquad \mathcal{H}_1 : A \neq 0$$

σ^2 KNOWN

RAO TEST DECIDES $\mathcal{H}_1$ IF

$$T_R(\underline{x}) = \left.\frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \underline{\theta}}\right|^T_{\underline{\theta}=\underline{\theta}_0} \underline{I}^{-1}(\underline{\theta}_0) \left.\frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \underline{\theta}}\right|_{\underline{\theta}=\underline{\theta}_0} > \gamma$$

$$\frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \underline{\theta}} = \begin{bmatrix} \frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \theta_1} \\ \vdots \\ \frac{\partial \ln p(\underline{x};\underline{\theta})}{\partial \theta_p} \end{bmatrix} = \text{GRADIENT}$$

$\underline{I}(\underline{\theta})$ = FISHER INFORMATION MATRIX.

NOTE: NO MLES REQUIRED.

ASIDE: FISHER INFORMATION -

QUANTIFIES INFORMATION IN DATA ABOUT UNKNOWN PARAMETER θ

ALSO AS $N \to \infty$, $VAR(\hat{\theta}) \to I^{-1}(\theta)$.

$\uparrow$ MLE

$$I(\theta) = -E\left[\frac{\partial^2 \ln p(\underline{x};\theta)}{\partial \theta^2}\right]$$

EXAMPLE: DC LEVEL IN WGN

A UNKNOWN, σ^2 KNOWN

$$p(\underline{x};A) = \frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n]-A)^2}$$

$$\frac{\partial \ln p(\underline{x};A)}{\partial A} = -\frac{1}{2\sigma^2}(2)(-1)\sum_n (x[n]-A)$$

$$= \frac{1}{\sigma^2}\sum_n (x[n]-A)$$

$$\frac{\partial^2 \ln p(\underline{x};A)}{\partial A^2} = \frac{1}{\sigma^2}\sum -1 = -N/\sigma^2$$

DOESN'T DEPEND ON $\underline{x}$ (NOT TRUE IN GENERAL)

$$\Rightarrow I(A) = -E\left[\frac{\partial^2 \ln p(\underline{x};A)}{\partial A^2}\right] = N/\sigma^2$$

NOTE $VAR(\hat{A}) = VAR(\bar{x}) = I^{-1}(A)$
$= \sigma^2/N$

HOLDS FOR FINITE DATA RECORDS
FOR THIS EXAMPLE.

BACK TO RAO TEST

$$\mathcal{H}_0 : A = 0$$
$$\mathcal{H}_1 : A \neq 0 \quad \Rightarrow \underline{\theta} = A, \ \underline{\theta}_0 = 0$$

$$T_R(\underline{x}) = \left.\frac{\partial \ln p(\underline{x}; A)}{\partial A}\right|_{A=0} I^{-1}(0) \cdot \left.\frac{\partial \ln p(\underline{x}; A)}{\partial A}\right|_{A=0}$$

$$\frac{\partial \ln p(\underline{x}; A)}{\partial A} = \frac{1}{\sigma^2} \sum_n (x[n] - A)$$

$$\left.\frac{\partial \ln p(\underline{x}; A)}{\partial A}\right|_{A=0} = \frac{N\bar{x}}{\sigma^2}$$

$$I^{-1}(0) = \sigma^2/N$$

$$\Rightarrow T_R(\underline{x}) = \left(\frac{N\bar{x}}{\sigma^2}\right)^2 \sigma^2/N = \frac{N\bar{x}^2}{\sigma^2}$$

DEPENDS ON MLE BUT WE DIDN'T NEED TO FIND $\bar{x}$ EXPLICITLY.

CLAIM: SAME RESULT AS GLRT

RECALL

$$2 \ln L_G(\underline{x}) = \frac{N\bar{x}^2}{\sigma^2}$$

IN GENERAL

$$T_R(\underline{x}) \overset{\text{AS } N \to \infty}{\approx} 2 \ln L_G(\underline{x})$$

RAO TEST ESPECIALLY VALUABLE FOR NONGAUSSIAN NOISE.

EXAMPLE: DC LEVEL IN NONGAUSSIAN NOISE

$$\mathcal{H}_0 : x[n] = w[n] \qquad n = 0, 1, \ldots, N-1$$

$$\mathcal{H}_1 : x[n] = \underset{\uparrow \text{UNKNOWN}}{A} + w[n] \qquad n = 0, 1, \ldots, N-1$$

$w[0], w[1], \ldots, w[N-1]$ ARE ZERO MEAN, INDEPENDENT AND EACH HAS PDF

$$p(w) = c_1 e^{-\frac{1}{2} c_2 w^4} \qquad -\infty < w < \infty$$

(GENERALIZED GAUSSIAN)