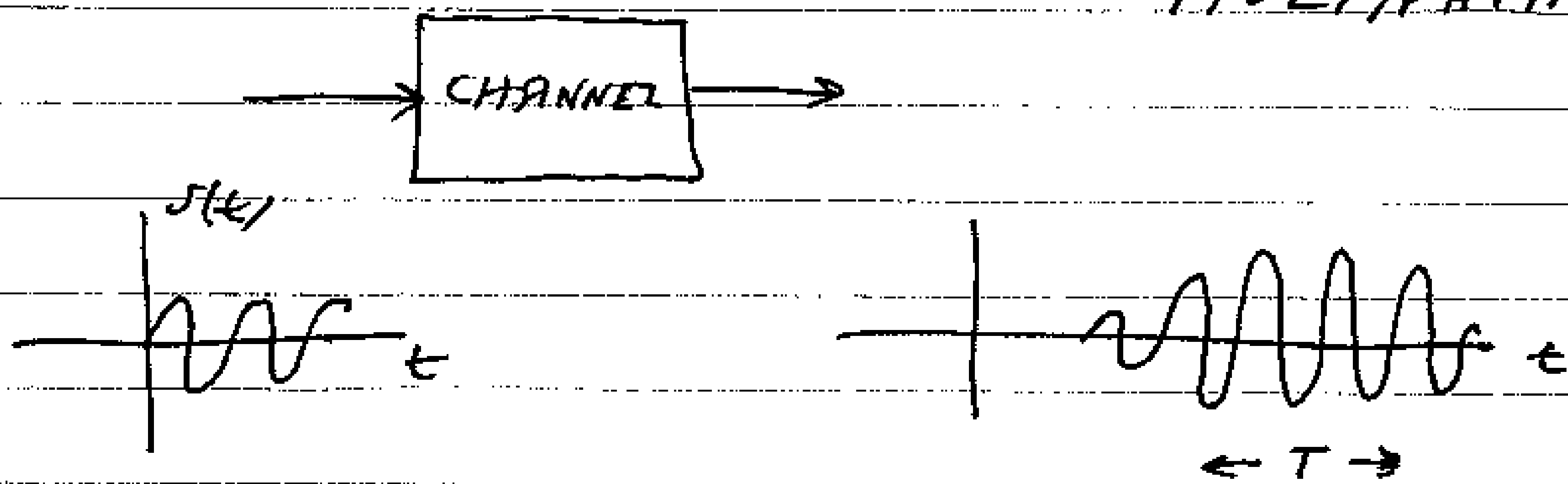
EXAMPLE: RAYLEIGH FADING SINUSOID (TIME VARYING MULTIPATH)



OVER SHORT INTERVAL T OUTPUT IS CONSTANT AMPLITUDE SINUSOID WITH RANDOM AMPLITUDE AND RANDOM PHASE

MODEL OVER T SEC INTERVAL AS

$$s[n] = A\cos(2\pi f_0 n + \phi)$$
$$= a\cos 2\pi f_0 n + b\sin 2\pi f_0 n$$

$$\underline{\theta} = \begin{bmatrix} a \\ b \end{bmatrix} \sim N(\underline{0}, \sigma_s^2 \underline{I})$$

$$\Rightarrow \quad A = \sqrt{a^2+b^2} \sim \text{RAYLEIGH PDF}$$
$$\phi = \arctan\frac{-b}{a} \sim \text{UNIFORM}$$

WITH THESE ASSUMPTIONS

$s(n)$ IS WSS GAUSSIAN RANDOM PROCESS SINCE

$$E(s(n)) = 0$$

$$E(s(n)s(n+k)) =$$

$$E\left[(a\cos 2\pi f_0 n + b\sin 2\pi f_0 n)(a\cos 2\pi f_0(n+k) + b\sin 2\pi f_0(n+k))\right]$$

$$= E(a^2)\cos 2\pi f_0 n \cos 2\pi f_0(n+k)$$

$$+ E(b^2)\sin 2\pi f_0 n \sin 2\pi f_0(n+k)$$

$$= \sigma_s^2 \cos 2\pi f_0 k = r_{ss}[k]$$

AUTOCORRELATION

$$\Rightarrow P_{ss}(f) = \frac{\sigma_s^2}{2}\left(\delta(f+f_0) + \delta(f-f_0)\right)$$

TO FIND NP DETECTOR WE HAVE BAYESIAN LINEAR MODEL WITH

$$\underline{H} = \begin{bmatrix} 1 & 0 \\ \cos 2\pi f_0 & \sin 2\pi f_0 \\ \vdots & \vdots \\ \cos 2\pi f_0(N-1) & \sin 2\pi f_0(N-1) \end{bmatrix}$$

$$\underline{\theta} = \begin{bmatrix} a \\ b \end{bmatrix} \sim N(\underline{0}, \sigma_s^2 \underline{I})$$

$$C_{\underline{\theta}} = \sigma_s^2 \underline{I}$$

DECIDE $\mathcal{H}_1$ IF

$$T(\underline{x}) = \underline{x}^T \underline{H}\,\underline{C}_\theta \underline{H}^T (\underline{H}\,\underline{C}_\theta \underline{H}^T + \sigma^2 \underline{I})^{-1}\underline{x} > \gamma'$$

$$T(\underline{x}) = \sigma_s^2\, \underline{x}^T \underline{H}\,\underline{H}^T (\sigma_s^2 \underline{H}\,\underline{H}^T + \sigma^2 \underline{I})^{-1}\underline{x}$$

USE MATRIX INVERSION LEMMA

$$(\underbrace{\underline{A}}_{\sigma^2 \underline{I}} + \underbrace{\underline{B}}_{\sigma_s^2 \underline{H}}\underbrace{\underline{C}}_{\underline{I}}\underbrace{\underline{D}}_{\underline{H}^T})^{-1} = \underline{A}^{-1} - \underline{A}^{-1}\underline{B}(\underline{D}\,\underline{A}^{-1}\underline{B} + \underline{C}^{-1})^{-1}\underline{D}\,\underline{A}^{-1}$$

$$T(\underline{x}) = \sigma_s^2\, \underline{x}^T \underline{H}\,\underline{H}^T \left[\frac{1}{\sigma^2}\underline{I} - \frac{1}{\sigma^4}\sigma_s^2 \underline{H}\left(\frac{\sigma_s^2 \underline{H}^T\underline{H}}{\sigma^2} + \underline{I}\right)^{-1} \cdot \underline{H}^T\right]\underline{x}$$

$$\text{BUT} \quad \underline{H}^T\underline{H} = \begin{bmatrix} \sum \cos^2 2\pi f_0 n & \sum \cos 2\pi f_0 n \sin 2\pi f_0 n \\ & \sum \sin^2 2\pi f_0 n \end{bmatrix}$$

$$\approx \begin{bmatrix} N/2 & 0 \\ 0 & N/2 \end{bmatrix} = \frac{N}{2}\underline{I}$$

$$T(\underline{x}) = \sigma_s^2\, \underline{x}^T \underline{H}\,\underline{H}^T \left[\frac{1}{\sigma^2}\underline{I} - \frac{\sigma_s^2}{\sigma^4}\frac{1}{\frac{N\sigma_s^2}{2\sigma^2}+1}\underline{H}\,\underline{H}^T\right]\underline{x}$$

$$= \frac{\sigma_s^2}{\sigma^2}\underline{x}^T\underline{H}\,\underline{H}^T\underline{x} - \frac{\sigma_s^4}{\sigma^4}\frac{1}{\frac{N\sigma_s^2}{2\sigma^2}+1}\underline{x}^T\underline{H}\underbrace{\underline{H}^T\underline{H}}_{\frac{N}{2}\underline{I}}\underline{H}^T\underline{x}$$

$$= \frac{N\sigma_s^2}{\frac{N\sigma_s^2}{2}+\sigma^2} \frac{1}{N} \underline{x}^T \underline{H}\underline{H}^T \underline{x}$$

OR DECIDE $\mathcal{H}_1$ IF

$$T'(\underline{x}) = \frac{1}{N} \underline{x}^T \underline{H}\underline{H}^T \underline{x} > \gamma''$$

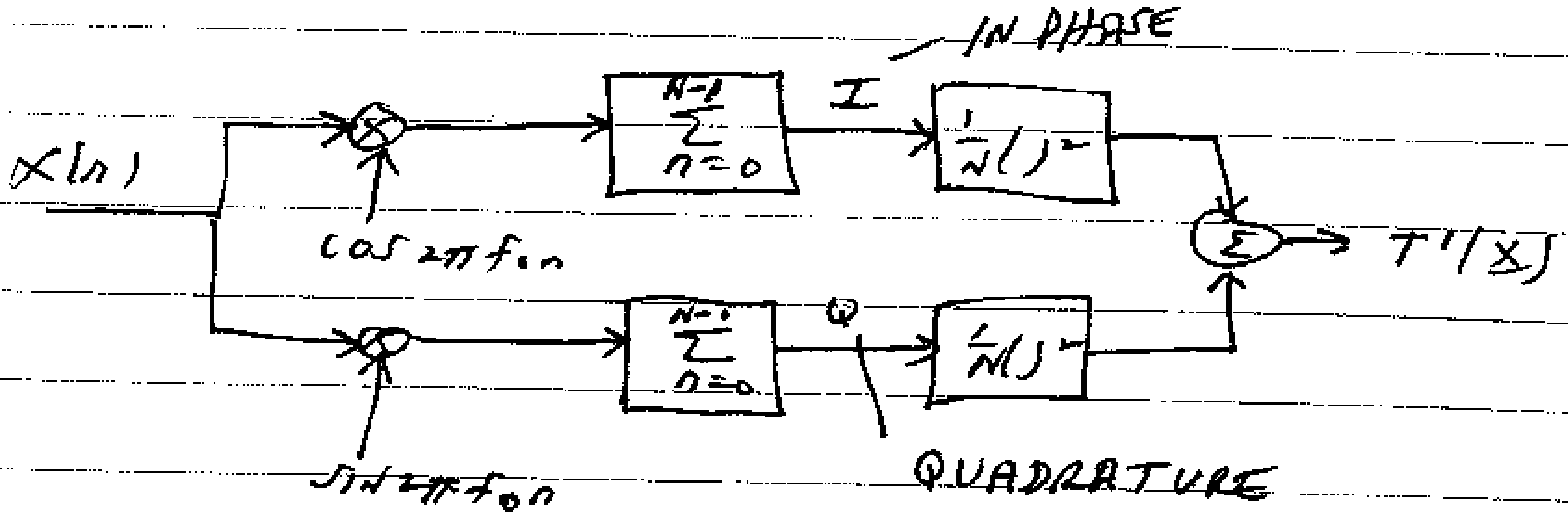
$$T'(\underline{x}) = \frac{1}{N} \| \underline{H}^T \underline{x} \|^2$$

BUT $\underline{H}^T \underline{x} = \begin{bmatrix} 1 & \cos 2\pi f_0 & \ldots & \cos 2\pi f_0(N-1) \\ 0 & \sin 2\pi f_0 & \ldots & \sin 2\pi f_0(N-1) \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix}$

$$= \begin{bmatrix} \sum_n x(n) \cos 2\pi f_0 n \\ \sum_n x(n) \sin 2\pi f_0 n \end{bmatrix}$$

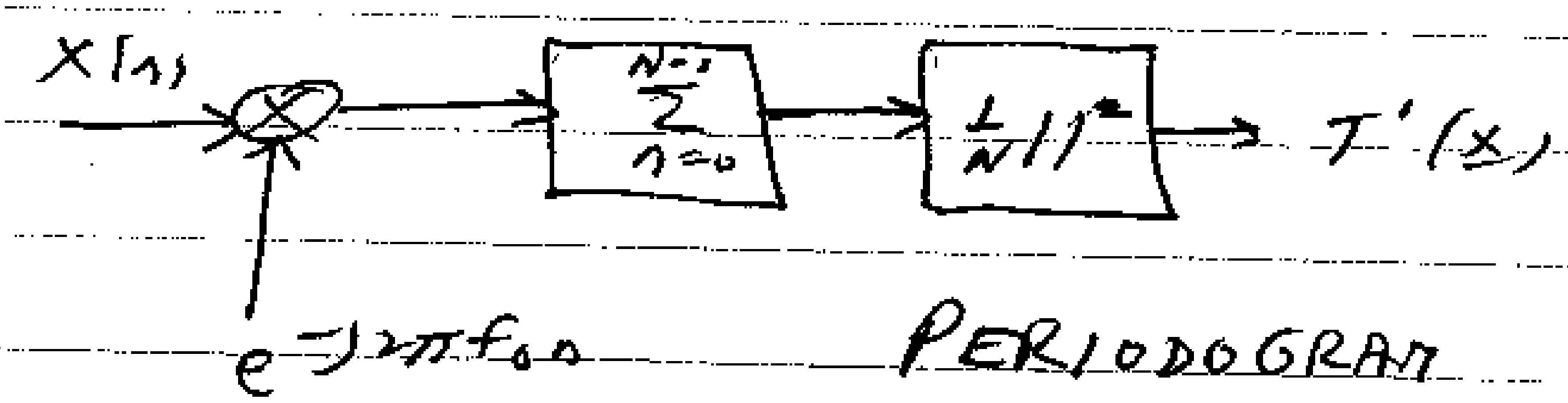
$$T'(\underline{x}) = \frac{1}{N} \left[\left(\sum_n x(n) \cos 2\pi f_0 n \right)^2 + \left(\sum_n x(n) \sin 2\pi f_0 n \right)^2 \right]$$

$$= \frac{1}{N} \Big| \underbrace{\sum_{n=0}^{N-1} x(n) e^{-j2\pi f_0 n}}_{\text{FOURIER TRANSFORM FOR } f=f_0} \Big|^2$$



CALLED A QUADRATURE OR INCOHERENT MF

HOW DOES IT ACCOMODATE UNKNOWN PHASE?



PERIODOGRAM (FFT OR NARROWBAND DETECTOR)

PERFORMANCE OF QUADRATURE MF

LET $\xi_1 = \sum_n x(n) \cos 2\pi f_0 n$

$\xi_2 = \sum_n x(n) \sin 2\pi f_0 n$

ξ_1, ξ_2 ARE JOINTLY GAUSSIAN SINCE

$\underline{z} = \underline{H}^T \underline{x}$ AND

$\underline{x} \sim N(\underline{0}, \sigma^2 \underline{I}) \quad \mathcal{H}_0$

$\quad N(\underline{0}, C_s + \sigma^2 \underline{I}) \quad \mathcal{H}_1$

WHERE $C_s = E(\underline{H}\underline{\theta}\,\underline{\theta}^T\underline{H})$

$= \underline{H}\underline{C}_\theta\underline{H}^T = \sigma_s^2 \underline{H}\underline{H}^T$

UNDER $\mathcal{H}_0$

$$\underline{z} \sim N(\underline{0}, \underbrace{\underline{H}^T \underline{C}_x \underline{H}}_{\underline{H}^T \sigma^2 I \underline{H}})$$

$$\approx \frac{N\sigma^2}{2}\underline{I} \qquad \underline{H}^T\underline{H} \approx \frac{N}{2}\underline{I}$$

$$= \sigma_0^2 \underline{I}$$

$\Rightarrow$ z_1, z_2 ARE UNCORRELATED AND HENCE INDEPENDENT

UNDER $\mathcal{H}_1$

$$\underline{z} \sim N(\underline{0}, \underbrace{\underline{H}^T C_x \underline{H}}_{\underline{H}^T(C_s + \sigma^2 \underline{I})\underline{H}})$$

BUT $C_s = \sigma_s^2 \underline{H}\underline{H}^T \qquad (\underline{s} = \underline{H}\underline{\theta},$

$\qquad \underline{C}_\theta = \sigma_s^2 \underline{I})$

$$\underline{H}^T(\underline{C}_s + \sigma^2 \underline{I})\underline{H} = \underline{H}^T \sigma_s^2 \underline{H}\underline{H}^T\underline{H} + \sigma^2 \underline{H}^T\underline{H}$$

$$\approx \sigma_s^2 (N/2)^2 \underline{I} + \sigma^2 \frac{N}{2} \underline{I}$$

$$= \frac{N}{2}\left(\frac{N}{2}\sigma_s^2 + \sigma^2\right)\underline{I} = \sigma_1^2 \underline{I}$$

$\Rightarrow$ z_1, z_2 ARE INDEPENDENT

BUT $T'(\underline{x}) = \frac{1}{N}(z_1^2 + z_2^2)$

$$= \frac{\sigma_0^2}{N}\underbrace{\left(\frac{z_1^2}{\sigma_0^2} + \frac{z_2^2}{\sigma_0^2}\right)}_{\sim \chi_2^2 \text{ UNDER } \mathcal{H}_0}$$

$$P_{FA} = P_r\{T'(\underline{x}) > \gamma'';\mathcal{H}_0\}$$

$$= P_r\{\underbrace{\frac{N T'(\underline{x})}{\sigma_0^2}}_{\chi_2^2} > \frac{N\gamma''}{\sigma_0^2};\mathcal{H}_0\}$$

$$= e^{-\frac{1}{2}N\gamma''/\sigma_0^2}$$

SIMILARLY

$$P_D = P_r\{T'(\underline{x}) > \gamma'';\mathcal{H}_1\}$$

$$= P_r\{\frac{N T'(\underline{x})}{\sigma_1^2} > \frac{N\gamma''}{\sigma_1^2};\mathcal{H}_1\}$$

$$= e^{-\frac{1}{2}N\gamma''/\sigma_1^2}$$

BUT $-\frac{1}{2} N \gamma'' = \sigma_0^2 \ln P_{FA}$

$$\Rightarrow P_D = e^{\frac{\sigma_0^2}{\sigma_1^2} \ln P_{FA}}$$

$$= e^{\ln P_{FA}^{\sigma_0^2/\sigma_1^2}}$$

$$P_D = P_{FA}^{\sigma_0^2/\sigma_1^2}$$

$$\sigma_1^2/\sigma_0^2 = \frac{\frac{N}{2}\left(\frac{N}{2}\sigma_s^2 + \sigma^2\right)}{\frac{N}{2}\sigma^2} = \frac{\frac{N}{2}\sigma_s^2}{\sigma^2} + 1$$

$$= 1 + \bar{\eta}/2$$

a^2+b^2

WHERE $\bar{\eta} = \frac{N\sigma_s^2}{\sigma^2} = \frac{N E(A^2/2)}{\sigma^2}$

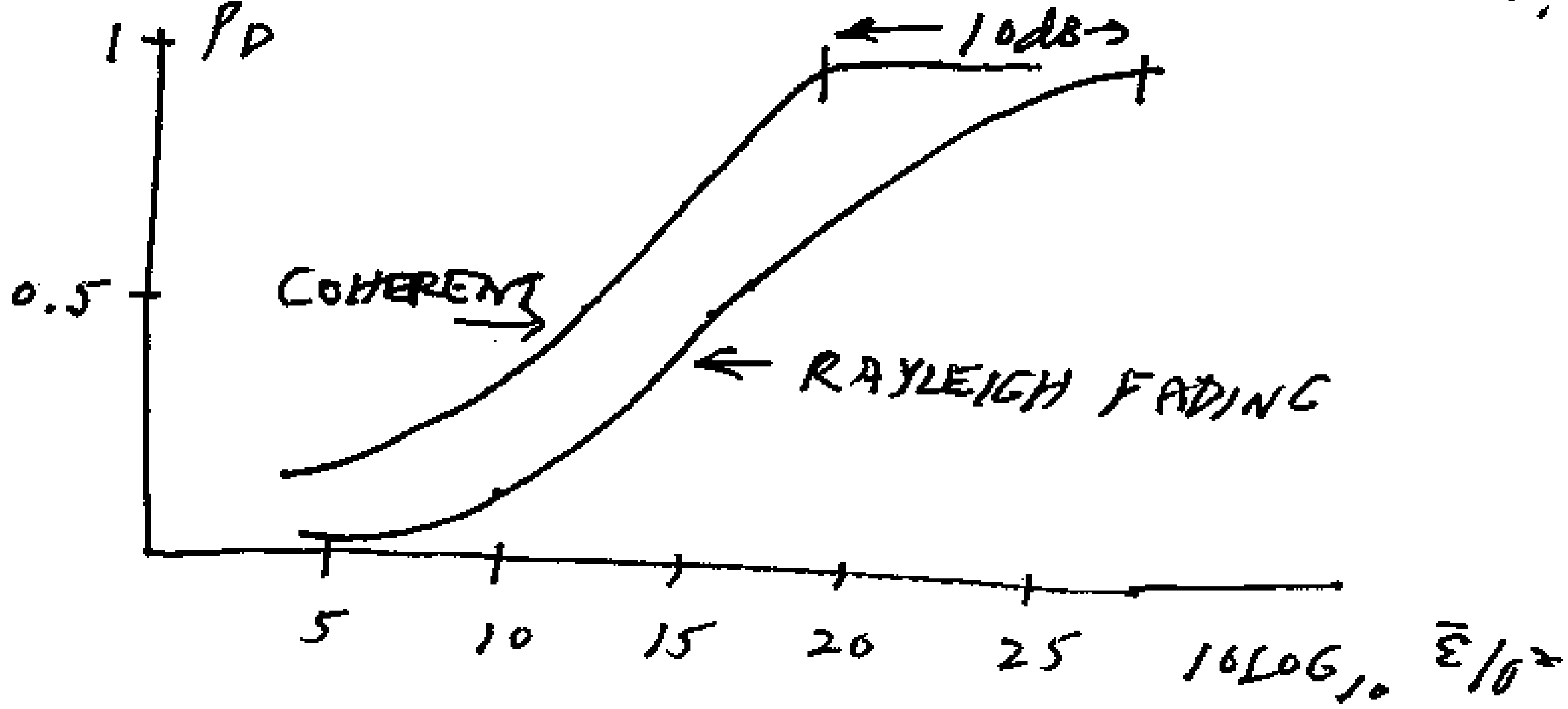
= AVERAGE ENERGY-TO-NOISE RATIO

$$\therefore P_D = P_{FA}^{\frac{1}{1+\bar{\eta}/2}}$$

P_D INCREASES SLOWLY WITH $\bar{\eta}$

COMPARE VS. COHERENT CHANNEL - NO DISTORTION OF SIGNAL

COHERENT CHANNEL ⇒ MATCHED FILTER
INCOHERENT CHANNEL ⇒ QUADRATURE M.F.



$P_{FA} = 10^{-5}$

SEE FIG 5.7 AND 4.5

SEE ALSO FIG. 5.10 FOR FSK COMPARISON

PROBLEM IS THAT RAYLEIGH FADING CAUSES AMPLITUDE TO BE SMALL WITH HIGH PROBABILITY.

SOLUTION: DIVERSITY TECHNIQUES

LARGE DATA RECORDS - WSS PROCESSES

SEE CHAPTER 2. CONSIDER $x[n]$ A ZERO MEAN WSS GAUSSIAN RANDOM PROCESS WITH AUTOCORRELATION $r_{xx}(k) = E(x[n]x[n+k])$.

COVARIANCE MATRIX HAS SPECIAL PROPERTIES.

EXAMPLE : $N=4$ $\underline{x} = [x[0]\, x[1]\, x[2]\, x[3]]^T$

$$[\underline{C}]_{ij} = E[x[i-1]x[j-1]] \quad i = 1,2,3,4$$
$$= r_{xx}[i-j] \quad j = 1,2,3,4$$

$$\underline{C} = \begin{bmatrix} r_{xx}[0] & r_{xx}[-1] & r_{xx}[-2] & r_{xx}[-3] \\ r_{xx}[1] & r_{xx}[0] & r_{xx}[-1] & r_{xx}[-2] \\ r_{xx}[2] & r_{xx}[1] & r_{xx}[0] & r_{xx}[-1] \\ r_{xx}[3] & r_{xx}[2] & r_{xx}[1] & r_{xx}[0] \end{bmatrix}$$

$r_{xx}[-k] = r_{xx}[k] \Rightarrow \underline{C}$ IS SYMMETRIC
SAME ELEMENT ALONG EACH NW-SE DIAGONAL $\Rightarrow$ TOEPLITZ

IMPORTANT RESULT: AS $N \to \infty$, EIGENVALUES OF $\underline{C}$ ARE

$$\lambda_i = P_{xx}(f_i) \qquad f_i = i/N$$

↑ PSD $\qquad i = 0, 1, \ldots, N-1$

AND CORRESPONDING EIGENVECTORS ARE

$$\underline{v}_i = \frac{1}{\sqrt{N}} \left[1 \quad e^{j2\pi f_i} \quad e^{j4\pi f_i} \quad \ldots \quad e^{j2\pi(N-1)f_i} \right]^T$$

↑ "DFT" VECTORS

<u>EXAMPLE</u>: $r_{xx}(k) = 0 \quad k \geq 2$

(MA(1) PROCESS)

$$\underset{\text{WGN } \sigma_u^2}{u[n]} \longrightarrow \boxed{B(z)} \longrightarrow x[n]$$

$$r_{xx}(0) = \sigma_u^2 \left(1 + b^2(1)\right) \qquad B(z) = 1 + b(1) z^{-1}$$

$$r_{xx}(1) = \sigma_u^2 b(1)$$

$$r_{xx}(k) = 0 \qquad k \geq 2$$

$$\begin{bmatrix} r[0] & r[-1] & 0 & 0 & \cdots & 0 \\ r[1] & r[0] & r[-1] & 0 & \cdots & 0 \\ 0 & r[1] & r[0] & r[-1] & \cdots & 0 \\ & & \ddots & \ddots & \ddots & \\ & \underline{0} & & & & r[-1] \\ & & & 0 & r[1] & r[0] \end{bmatrix} \begin{bmatrix} 1 \\ e^{j2\pi f_i} \\ \vdots \\ e^{j2\pi(N-1)f_i} \end{bmatrix} = P(f_i) \begin{bmatrix} 1 \\ e^{j2\pi f_i} \\ \vdots \\ e^{j2\pi(N-1)f_i} \end{bmatrix}$$

EXCEPT FOR FIRST AND LAST ROWS,

ROW 2 : $r(1) + r(0)\, e^{j2\pi f_i} + r(-1)\, e^{j4\pi f_i}$

$$= e^{j2\pi f_i}\Big(\underbrace{r(1)\, e^{-j2\pi f_i} + r(0) + r(-1)\, e^{j2\pi f_i}}_{P_{xx}(f_i)} \Big)$$

ETC.

NOTE THAT $\lambda_{N-i} = \lambda_i$ AND
$\underline{v}_{N-i} = \underline{v}_i^{*} \Rightarrow \underline{C}$ WILL BE REAL

$\left(P(f_{N-i}) = P(f_i) \right)$

NOW WE HAVE APPROXIMATE EIGENDECOMPOSITION OF $\underline{C}$ AS

$$\underline{V}^H \underline{C}\, \underline{V} = \underline{\Lambda} \qquad \underline{V} = [\, \underline{v}_0\ \underline{v}_1 \ldots \underline{v}_{N-1}]$$

$(H \equiv *T)$

$$\underline{\Lambda} = \begin{bmatrix} \lambda_0 & & & \underline{0} \\ & \lambda_1 & & \\ & & \ddots & \\ \underline{0} & & & \lambda_{N-1} \end{bmatrix}$$

THUS,

$$\mathrm{DET}(\underline{C}) = \prod_{i=0}^{N-1} \lambda_i \approx \prod_{i=0}^{N-1} P_{xx}(f_i)$$

$$\underline{C}^{-1} = \sum_{i=0}^{N-1} \frac{1}{\lambda_i} \underline{v}_i \underline{v}_i^H \approx \sum_{i=0}^{N-1} \frac{1}{P_{xx}(f_i)} \underline{v}_i \underline{v}_i^H$$

CAN NOW APPROXIMATE GAUSSIAN PDF FOR WSS RANDOM PROCESSES (FOR LARGE N)

$$p(\underline{x}) = \frac{1}{(2\pi)^{N/2} \det^{1/2}(\underline{C})} e^{-\frac{1}{2}\underline{x}^T \underline{C}^{-1} \underline{x}}$$

$$\ln p(\underline{x}) = -\frac{N}{2} \ln 2\pi - \frac{1}{2} \ln \det(\underline{C}) - \frac{1}{2} \underline{x}^T \underline{C}^{-1} \underline{x}$$

$$\simeq -\frac{N}{2} \ln 2\pi - \frac{1}{2} \ln \prod_{0}^{N-1} P_{xx}(f_i) - \frac{1}{2} \underline{x}^T \sum_{0}^{N-1} \frac{1}{P_{xx}(f_i)} \underline{V}_i \underline{V}_i^H \underline{x}$$

$$= -\frac{N}{2} \ln 2\pi - \frac{1}{2} \sum_{0}^{N-1} \ln P_{xx}(f_i) - \frac{1}{2} \sum_{0}^{N-1} \frac{|\underline{V}_i^H \underline{x}|^2}{P_{xx}(f_i)}$$

BUT $|\underline{V}_i^H \underline{x}|^2 = \frac{1}{N} \left| \sum_{n=0}^{N-1} x(n) e^{-j2\pi f_i n} \right|^2$

$$= I(f_i) = \text{PERIODOGRAM}$$

$$\ln p(\underline{x}) \simeq -\frac{N}{2} \ln 2\pi - \frac{N}{2} \sum_{i=0}^{N-1} \ln P_{xx}(i/N) \frac{1}{N}$$

$$- \frac{N}{2} \sum_{i=0}^{N-1} \frac{I(i/N)}{P_{xx}(i/N)} \frac{1}{N}$$

OR $-\frac{1}{2}$ TO $\frac{1}{2}$

$$\simeq -\frac{N}{2} \ln 2\pi - \frac{N}{2} \int_0^1 \ln P_{xx}(f) \, df$$

$$- \frac{N}{2} \int_0^1 \frac{I(f)}{P_{xx}(f)} \, df$$

FINALLY,

$$\ln p(\underline{x}) \approx -\frac{N}{2}\ln 2\pi - \frac{N}{2}\int_{-\frac{1}{2}}^{\frac{1}{2}}\left(\ln P_{xx}(f) + \frac{I(f)}{P_{xx}(f)}\right)df$$

IMPORTANT RESULT!

ESTIMATOR - CORRELATOR FOR WSS SIGNAL

WISH TO DETECT $s[n]$ WHICH IS ZERO MEAN WSS RANDOM PROCESS WITH PSD $P_{ss}(f)$ IN WGN. ASSUME N IS LARGE.

DECIDE $\mathcal{H}_1$ IF

$$l(\underline{x}) = \ln p(\underline{x};\mathcal{H}_1) - \ln p(\underline{x};\mathcal{H}_0) > \gamma'$$

BUT $P_{xx}(f) = \begin{cases} \sigma^2 & \mathcal{H}_0 \\ P_{ss}(f) + \sigma^2 & \mathcal{H}_1 \end{cases}$

$$l(\underline{x}) = -\frac{N}{2}\int \ln(P_{ss}(f) + \sigma^2) + \frac{I(f)}{P_{ss}(f)+\sigma^2}\,df$$
$$+\frac{N}{2}\int \ln \sigma^2 + \frac{I(f)}{\sigma^2}\,df$$

$$= -\frac{N}{2}\int \ln\left(\frac{P_{ss}(f)}{\sigma^2}+1\right)df$$

$$+\frac{N}{2}\int I(f)\underbrace{\left(\frac{1}{\sigma^2}-\frac{1}{P_{ss}(f)+\sigma^2}\right)}_{\frac{P_{ss}(f)}{\sigma^2(P_{ss}(f)+\sigma^2)}}df$$

DECIDE $\mathcal{H}_1$ IF

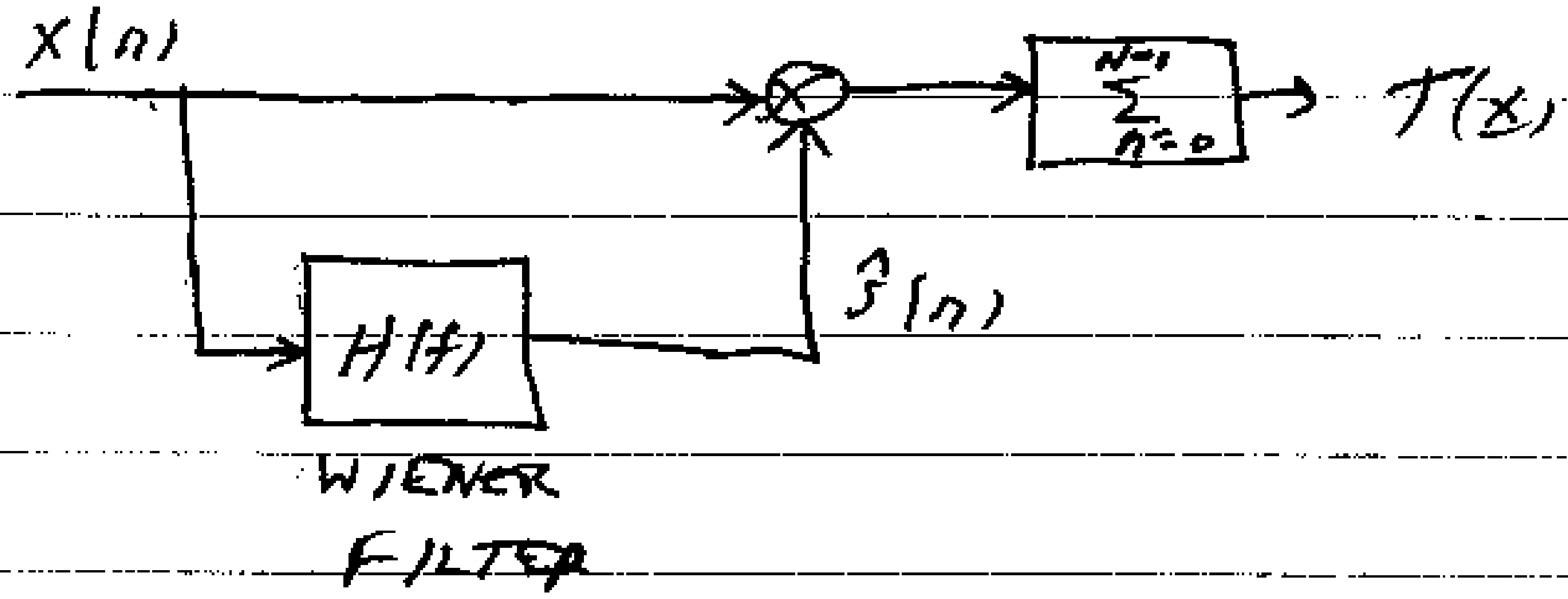
$$T(\underline{x}) = N\int_{-\frac{1}{2}}^{\frac{1}{2}}\underbrace{\frac{P_{ss}(f)}{P_{ss}(f)+\sigma^2}}_{H(f)} I(f)\,df > \gamma''$$

$H(f)$ IS WIENER FILTER FOR LARGE (LIKE $C_s(C_s+\sigma^2 I)^{-1}$)
DATA RECORD. NONCAUSAL FILTER

$$T(\underline{x}) = \int_{-\frac{1}{2}}^{\frac{1}{2}}\underbrace{H(f)X(f)}_{\hat{S}(f)}X^*(f)\,df$$

ESTIMATOR - CORRELATOR

$$\approx \sum_{n=0}^{N-1}\hat{s}[n]x[n]$$



$$H(f) = \frac{P_{ss}(f)/\sigma^2}{P_{ss}(f)/\sigma^2 + 1}$$

$$\sim \frac{SNR(f)}{SNR(f)+1}$$

ACCENTUATES BANDS WHERE SNR IS HIGH $0 < H(f) < 1$ AND $H(f)$ IS REAL

CHAPTER 6 - DECISION THEORY II

SEE SUMMARY. NOW WE DEAL WITH UNKNOWN PARAMETERS. MAY STILL BE ABLE TO USE NP.

EXAMPLE: DC LEVEL WITH UNKNOWN A ($A > 0$)

TRY NP TEST.

DEPENDENCE ON A SHOWN

$$\frac{p(\underline{x};A,\mathcal{H}_1)}{p(\underline{x};\mathcal{H}_0)} = \frac{\frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n]-A)^2}}{\frac{1}{(2\pi\sigma^2)^{N/2}} e^{-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}x^2[n]}}$$

$$= e^{-\frac{1}{2\sigma^2}\left[\sum (x[n]-A)^2 - \sum x^2[n]\right]} > \gamma$$

TAKE LOGS

$$-\frac{1}{2\sigma^2}\left(-2A\sum x[n] + NA^2\right) > \ln\gamma$$

$$A\sum x[n] > \sigma^2 \ln\gamma + NA^2/2$$

SINCE WE KNOW THAT $A>0$ (BUT NOT ITS VALUE), DECIDE $\mathcal{H}_1$ IF

$$T(\underline{x}) = \frac{1}{N}\sum_{n=0}^{N-1} x[n] > \frac{\sigma^2}{NA}\ln\gamma + \frac{A}{2} = \gamma'$$

WE HAVE THE TEST STATISTIC. HOW ABOUT γ'?

UNDER $\mathcal{H}_0$ $T \sim \mathcal{N}(0, \sigma^2/N)$

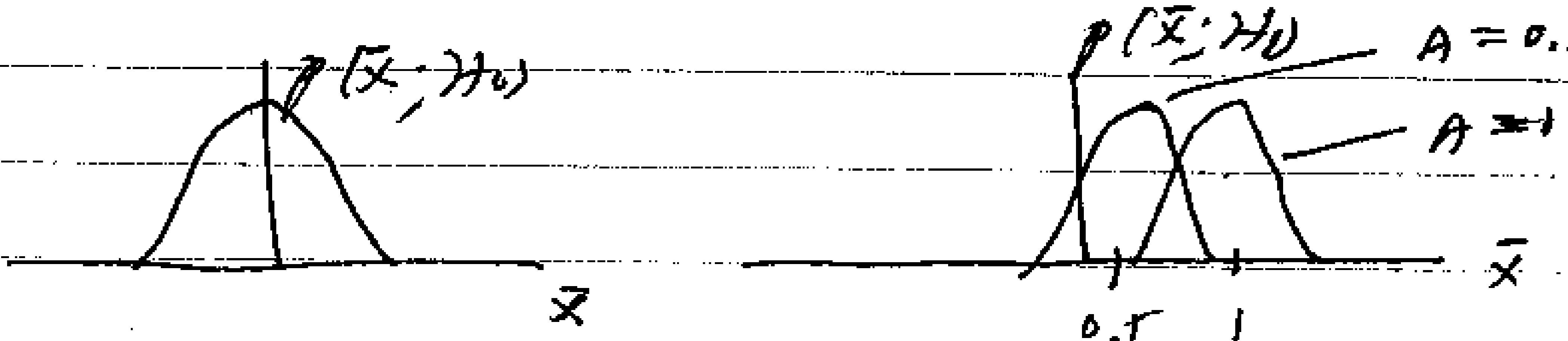
$$\Rightarrow \quad P_{FA} = Q\left(\gamma'/\sqrt{\sigma^2/N}\right)$$

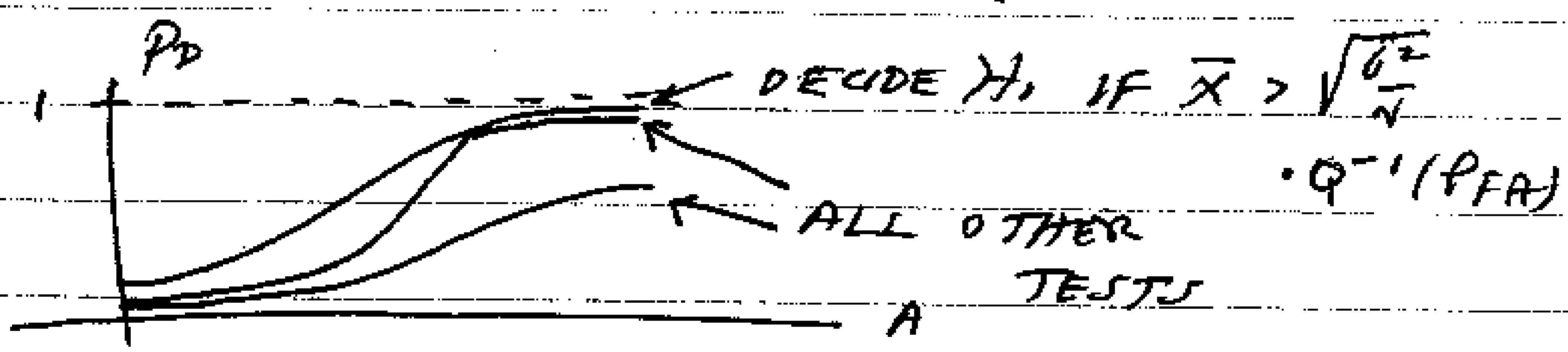
$$\gamma' = \sqrt{\sigma^2/N}\; Q^{-1}(P_{FA})$$

WITHOUT KNOWING A WE CAN STILL FIND NP TEST. THIS IS BECAUSE PDF OF $T(\underline{x})$ UNDER $\mathcal{H}_0$ DOES NOT DEPEND ON A.

UNDER $\mathcal{H}_1$ $\quad T(\underline{x}) \sim \mathcal{N}(A, \sigma^2/N)$

$\Rightarrow$ P_D WILL DEPEND ON A





SINCE NP IS OPTIMUM, ALL OTHER TESTS ARE POORER

NP TEST IS SAID TO BE

UNIFORMLY MOST POWERFUL (UMP)

OVER ALL $A > 0$

LARGEST P_D

UMP TESTS SELDOM EXIST ☹

IF A CAN BE NEGATIVE AS WELL
NP TEST BECOMES

$$\bar{x} > \sqrt{\sigma^2/N}\, Q^{-1}(P_{FA}) \qquad A > 0$$

$$< -\sqrt{\sigma^2/N}\, Q^{-1}(P_{FA}) \qquad A < 0$$

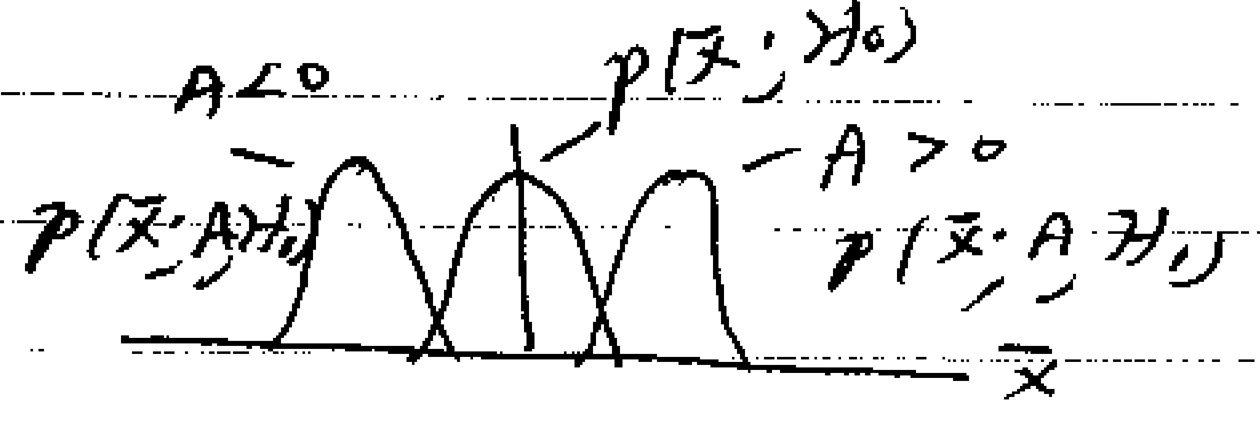
SINCE A IS UNKNOWN, NO SINGLE
TEST IS OPTIMAL $\Rightarrow$ NO UMP TEST.

$$\left.\begin{array}{l} \mathcal{H}_0 : A = 0 \\ \mathcal{H}_1 : A > 0 \end{array}\right\} \text{ONE-SIDED} \Rightarrow \text{UMP MAY EXIST}$$

$$\left.\begin{array}{l} \mathcal{H}_0 : A = 0 \\ \mathcal{H}_1 : A \neq 0 \end{array}\right\} \text{TWO-SIDED} \Rightarrow \text{NO UMP EXISTS}$$

FOR THIS EXAMPLE MIGHT TRY
TO DECIDE $\mathcal{H}_1$ IF

$$|\bar{x}| > \gamma''$$



BUT NO OPTIMALITY PROPERTIES.

BAYESIAN APPROACH

ASSUME WE HAVE UNKNOWN PARAMETERS $\underline{\theta}_i$ UNDER $\mathcal{H}_i$.

IF WE HAVE SOME PRIOR KNOWLEDGE EMBODIED IN $p(\underline{\theta}_0)$, $p(\underline{\theta}_1)$, THEN WE CAN "INTEGRATE OUT" UNKNOWN PARAMETERS.

PRIOR

$$p(\underline{x}; \mathcal{H}_0) = \int p(\underline{x} \mid \underline{\theta}_0; \mathcal{H}_0)\, p(\underline{\theta}_0)\, d\underline{\theta}_0$$

$$p(\underline{x}; \mathcal{H}_1) = \int p(\underline{x} \mid \underline{\theta}_1; \mathcal{H}_1)\, p(\underline{\theta}_1)\, d\underline{\theta}_1$$

PRIOR

REQUIRES MULTIDIMENSIONAL INTEGRATION. ALSO IF PRIOR PDFS ARE INCORRECT, RESULTS CAN BE POOR.

SEE EX. 6.3

WHEN NO PRIOR KNOWLEDGE IS AVAILABLE CAN STILL USE BAYESIAN IF $p(\underline{\theta}_i)$ CHOSEN TO BE "FLAT".