Asymptotically Optimal Detection/ Localization of LPI Signals of Emitters using Distributed Sensors

Naresh Vankayalapati and Steven Kay

Dept. of Electrical, Computer and Biomedical Engineering
University of Rhode Island

Kingston, RI 02881

ABSTRACT

The interception of low probability of intercept signals has become increasingly difficult due to the availability of waveform agile transmitters and increased spectral bandwidth. Current detection techniques based on energy detection ignore the important cross-sensor correlation information. Additionally, existing TDOA/FDOA based localization techniques pair up the sensors and perform only pair-wise processing which is highly inefficient. To circumvent this problem we show how to use multiple spatially distributed sensors to detect and localize an emitter whose waveform is completely unknown. We present the generalized likelihood ratio detector which optimally combines the multiple sensor information for improved detection. Additionally, as part of the detector the MLE for target location is available, leading to improved localization.

1. INTRODUCTION

Intercept receivers are becoming increasingly desirable due to their covert nature [1]. In addition intercept receivers enjoy a higher power density as there is only a one-way power loss at the intercept receiver as compared to the two-way loss at the transmitting active radar. On the other hand, since the angle of signal arrival and the signal itself are generally unknown to the interceptor, efficient processing techniques such as matched filtering cannot be implemented. Also, many modern active radar systems are designed with low probability of intercept (LPI) features. They incorporate physical attributes such as frequency variability, infrequent scanning, etc. to reduce the probability of interception by an interceptor and signal design attributes such as low power, wide bandwidth, etc. to decrease the probability of detection and parameter identification at the interceptor [2]. This leads to the need for implementing highly efficient detectors and localizers in the interceptor systems.

A simple radar system with a single transmitter and a single receiver both at the same physical location is called a monostatic radar. In general its performance is inferior to a multistatic radar system. An active multistatic radar system has one or more transmitters and many spatially separated receiving stations. A passive multistatic radar consists of only a network of distributed intercepting receivers. Such a system of multiple receive platforms has a higher probability of intercepting the signals of interest. Also multiple platforms provide more data samples over a given interval of time which increases the probability of detection. Each of these receive platforms can have an individual detector to process the received signal. This is called decentralized target detection. These local decisions at individual receive platforms are communicated to a central processor where they are processed to arrive at a global decision. Several such decision fusion techniques have been proposed and used over the years [3, 4]. Despite some practical advantages such as requiring low bandwidth data links between receive platforms and less processing power, the decentralized detection approach has an obvious performance loss due to the absence of cross-platform correlation information. On the other hand, centralized detection has better performance but requires more resources. Large bandwidth data links are required to transfer the received signals from all the receiving stations to the fusion center. High speed signal processors are required to process the large amount of data available at the fusion center for real-time detection. Since centralized detection relies on the cross-platform correlation information, the received signals must be spatially coherent.

When a signal is transmitted by an active device, from here on referred to as the target, it reaches each of the interceptor receiving stations after a certain amount of time, which is the propagation delay. If the target is

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moving with respect to the receiving station, then there is a Doppler shift in the frequency of the signal. These propagation delays and Doppler shifts are different at each receiving station and depend on the target location and velocity. Therefore, using these, the target location and velocity can be estimated. The amplitude and phase of the signal are also different at each receiving station. Taking all these factors into account and assuming there is no noise, the complex envelope of the signal that is received at the ith receiving station after sampling is

$$\tilde{A}_i \tilde{s}[n-n_i] \exp\left(\frac{j2\pi k_i(n-n_i)}{N}\right)$$

where $\tilde{s}[n]$ is the signal emitted by the target, $\tilde{A}_i = A_i e^{j\phi_i}$ is the change in amplitude and phase, n_i is the propagation delay and k_i is the Doppler shift. (For simplicity we assume a discrete-frequency Doppler shift of $\frac{k_i}{N}$, for some large N.) Note that we are using "$\sim$" to represent complex variables. An important difference between the detectors in the active radars and the detectors in the intercept receiver systems is that in the active case the received signal is an echo of the transmitted signal. Thus, *the signal can be modeled as known* with unknown parameters. In the intercept detection case, however, *the signal itself is unknown.* In some cases the signal is modeled as a stochastic process with unknown parameters [5]. We model the signal as deterministic and completely unknown, i.e, our signal modeling assumptions coincide with those of Stein [6].

In the next section we give a detailed description of the problem and model it as a statistical hypothesis test. In Section 3 we find the MLE for target location using the linear model and the GLRT for the described hypothesis test. In Section 4 we compare the detection performance of the GLRT against some commonly used detectors and the localization performance of the MLE against the existing TDOA technique. We provide conclusions in Section 5.

2. PROBLEM DESCRIPTION AND ASSUMPTIONS

The problem we are addressing can be described as follows. We have M intercepting sensors placed at multiple locations. Each of these sensors collects N time samples in a given interval of time. The total MN samples that are collected are available for processing at a fusion center. As is usual in practice, we assume $N > M$. We further assume that the noise at each of the receiver stations is white Gaussian and also that the noise at a receiver station is independent from noise at all the other receiver stations. For simplicity we assume that the variance of noise is $\sigma^2 = 1$. In situations where the noise does not meet these criteria, the observations can be pre-whitened [7] before processing so that the above assumptions hold. Now, the task of the detector at the fusion center is to detect the presence of an unknown signal in the MN observed samples. Once the signal is detected, the estimator must estimate the location and velocity of the target.

Let $\tilde{s}[n], n = 0, 1 \cdots, N-1$ be the nth complex time sample of the transmitted signal. A time delayed and Doppler shifted version of this signal and with a different amplitude and phase reaches the sensors. Let $\tilde{A}_i = A_i e^{j\phi_i}$ be the change in the amplitude and phase. Let n_i be the time delay and k_i be the Doppler shift. Let $\tilde{w}_i[n]$ be the nth complex time sample of the additive noise at the ith receiver station. If $\tilde{x}_i[n]$ is the nth time sample of the observation at the ith receiver station then we can write

$$\tilde{x}_i[n] = \tilde{A}_i \tilde{s}[n-n_i] \exp\left(\tfrac{j2\pi k_i(n-n_i)}{N}\right) + \tilde{w}_i[n] \quad \begin{array}{l} n = 0, 1, \cdots, N-1 \\ i = 0, 1, \cdots, M-1. \end{array}$$

When there is no signal, the observation is just noise, $\tilde{x}_i[n] = \tilde{w}_i[n]$. So, the hypothesis test for the detection problem can be written as

$$\begin{array}{ll} \mathcal{H}_0 : \tilde{x}_i[n] = \tilde{w}_i[n] & \\ \mathcal{H}_1 : \tilde{x}_i[n] = \tilde{A}_i \tilde{s}[n-n_i] \exp\left(\tfrac{j2\pi k_i(n-n_i)}{N}\right) + \tilde{w}_i[n] & n = 0, 1, \cdots, N-1 \\ & i = 0, 1, \cdots, M-1. \end{array}$$

Notice that here we are modeling the received signal as unknown but deterministic with unknown parameters $\tilde{A}_i, n_i, k_i, i = 0, 1, \cdots, M-1$ and $\tilde{s}[n], n = 0, 1, \cdots, N-1$. Now, if we let $\omega = \exp(\frac{j2\pi}{N})$ and $\tilde{\mathbf{W}}$ be the $N \times N$

matrix $\tilde{\mathbf{W}} = \mathrm{diag}(\omega^0, \omega^1, \cdots, \omega^{N-1})$ and let $\mathbf{P}$ be an $N \times N$ permutation matrix defined as $[\mathbf{P}]_{ij} = 1$ if $i = j+1$ and 0 otherwise, $i = 0, 1, \cdots, N-1$, $j = 0, 1, \cdots, N-1$ and $[\mathbf{P}]_{0,N-1} = 1$, then we can write the hypothesis test in vector form as follows

$$\mathcal{H}_0 : \tilde{\mathbf{s}} = \mathbf{0}$$
$$\mathcal{H}_1 : \tilde{\mathbf{s}} \neq \mathbf{0}$$

where

$$\tilde{\mathbf{x}}_i = \tilde{A}_i \mathbf{P}^{n_i} \tilde{\mathbf{W}}^{k_i} \tilde{\mathbf{s}} + \tilde{\mathbf{w}}_i \quad i = 0, 1, \cdots, M-1$$

and $\tilde{\mathbf{x}}_i = \begin{bmatrix} \tilde{x}_i[0] & \tilde{x}_i[1] & \cdots & \tilde{x}_i[N-1] \end{bmatrix}^T$, $\tilde{\mathbf{s}} = \begin{bmatrix} \tilde{s}[0] & \tilde{s}[1] & \cdots & \tilde{s}[N-1] \end{bmatrix}^T$ and $\tilde{\mathbf{w}}_i = \begin{bmatrix} \tilde{w}_i[0] & \tilde{w}_i[1] & \cdots & \tilde{w}_i[N-1] \end{bmatrix}^T$. Notice that the permutation matrix $\mathbf{P}$ circularly shifts $\tilde{\mathbf{s}}$ and so the above hypothesis test is only valid when the discrete time delays are relatively small compared to N. Now let $\tilde{\mathbf{x}} = \begin{bmatrix} \tilde{\mathbf{x}}_0^T & \tilde{\mathbf{x}}_1^T & \cdots & \tilde{\mathbf{x}}_{M-1}^T \end{bmatrix}^T$, $\tilde{\mathbf{w}} = \begin{bmatrix} \tilde{\mathbf{w}}_0^T & \tilde{\mathbf{w}}_1^T & \cdots & \tilde{\mathbf{w}}_{M-1}^T \end{bmatrix}^T$, $\tilde{\mathbf{A}} = \begin{bmatrix} \tilde{A}_0 & \tilde{A}_1 & \cdots & \tilde{A}_{M-1} \end{bmatrix}^T$, $\mathbf{n} = \begin{bmatrix} n_0 & n_1 & \cdots & n_{M-1} \end{bmatrix}^T$, $\mathbf{k} = \begin{bmatrix} k_0 & k_1 & \cdots & k_{M-1} \end{bmatrix}^T$ and $\tilde{\mathbf{H}}(\tilde{\mathbf{A}}, \mathbf{n}, \mathbf{k}) = \begin{bmatrix} (\tilde{A}_0 \mathbf{P}^{n_0} \tilde{\mathbf{W}}^{k_0})^T & (\tilde{A}_1 \mathbf{P}^{n_1} \tilde{\mathbf{W}}^{k_1})^T & \cdots & (\tilde{A}_{M-1} \mathbf{P}^{n_{M-1}} \tilde{\mathbf{W}}^{k_{M-1}})^T \end{bmatrix}^T$. The hypothesis test can be written as follows

$$\mathcal{H}_0 : \tilde{\mathbf{s}} = \mathbf{0}$$
$$\mathcal{H}_1 : \tilde{\mathbf{s}} \neq \mathbf{0}$$

where

$$\tilde{\mathbf{x}} = \tilde{\mathbf{H}}(\tilde{\mathbf{A}}, \mathbf{n}, \mathbf{k}) \tilde{\mathbf{s}} + \tilde{\mathbf{w}}$$

and $\tilde{\mathbf{w}}$ has a complex normal distribution with zero mean and the identity matrix as the covariance matrix, i.e, $\tilde{\mathbf{w}} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_{MN})$, $\mathbf{I}_{MN}$ is an $MN \times MN$ identity matrix. Here $\tilde{\mathbf{A}}$ is $M \times 1$, $\tilde{\mathbf{s}}$ is $N \times 1$, $\mathbf{n}$ is $M \times 1$, $\mathbf{k}$ is $M \times 1$, and are all assumed unknown.

3. MLE AND GLRT DETECTOR

An estimator that, on the average, yields the true value of the unknown parameter is called an unbiased estimator [8]. An unbiased estimator that minimizes the variance is a minimum variance unbiased (MVU) estimator. The Cramer-Rao lower bound (CRLB) places a lower bound on the variance of the unbiased estimator. An estimator that is unbiased and attains the CRLB is said to be efficient. MVU estimators are difficult to find in many practical problems. So, other approximately efficient estimators are used. Maximum likelihood estimators (MLE) are the most popular approximately efficient estimators used for practical problems. MLEs are asymptotically efficient and easy to find. The MLE for a vector of unknown parameters $\boldsymbol{\theta}$ is defined to be the value $\hat{\boldsymbol{\theta}}$ that maximizes the likelihood function $p(\mathbf{x}; \boldsymbol{\theta})$ over the allowable domain for $\boldsymbol{\theta}$.

In hypothesis testing problems where the probability density functions (PDFs) under both the hypotheses are completely known the Neyman-Pearson (NP) detector is the uniformly most powerful (UMP) detector [9]. When there are unknown parameters in the PDFs, the performance of the NP detector depends on the true value of these parameters. There are two common ways to deal with these unknown parameters. They can be modeled as random variables with some PDF and then integrated out or they can be modeled as unknown but deterministic and replaced with their MLEs. In the GLRT the unknown parameters are replaced by their MLEs under the different hypotheses. Asymptotically, the GLRT is the uniformly most powerful test among all tests that are invariant [9]. The likelihood ratio for the previously described hypotheses test is given by

$$L(\tilde{\mathbf{x}}) = \frac{p(\tilde{\mathbf{x}}; \tilde{\mathbf{s}}, \tilde{\mathbf{A}}, \mathbf{n}, \mathbf{k}, \mathcal{H}_1)}{p(\tilde{\mathbf{x}}; \mathcal{H}_0)} \tag{1}$$

If we replace the unknown parameters $\tilde{\mathbf{A}}$, $\tilde{\mathbf{s}}$, $\mathbf{n}$ and $\mathbf{k}$ with their respective MLEs $\hat{\tilde{\mathbf{A}}}$, $\hat{\tilde{\mathbf{s}}}$, $\hat{\mathbf{n}}$ and $\hat{\mathbf{k}}$ then the GLRT decides $\mathcal{H}_1$ if

$$L_G(\tilde{\mathbf{x}}) = \frac{p(\tilde{\mathbf{x}}; \hat{\tilde{\mathbf{s}}}, \hat{\tilde{\mathbf{A}}}, \hat{\mathbf{n}}, \hat{\mathbf{k}}, \mathcal{H}_1)}{p(\tilde{\mathbf{x}}; \mathcal{H}_0)} > \gamma \tag{2}$$

The GLRT test statistic can be shown to be [10]

$$T(\tilde{\mathbf{x}}) = \max_{\mathbf{n}, \mathbf{k}} \lambda_{\max}(\tilde{\mathbf{B}}(\mathbf{n}, \mathbf{k})) \tag{3}$$

where $\lambda_{\max}$ is the maximum eigenvalue and $\tilde{\mathbf{B}}$ is the $M \times M$ complex cross-ambiguity matrix (CAM) given by

$$[\tilde{\mathbf{B}}]_{ij} = \tilde{\mathbf{x}}_i^H \mathbf{P}^{n_i} \tilde{\mathbf{W}}^{k_i} (\tilde{\mathbf{W}}^{k_j})^H (\mathbf{P}^{n_j})^H \tilde{\mathbf{x}}_j \tag{4}$$

and for which H is conjugate transpose. Since $\tilde{\mathbf{B}}$ is positive definite, the maximum eigenvalue is real and positive. Therefore the GLRT decides that a signal is present if the maximum eigenvalue of the CAM, when also maximized over time delay and Doppler, is greater than a threshold. It should be noticed that the diagonal elements in the CAM are the energy terms at each of the receivers, i.e, $\tilde{\mathbf{x}}_i^H \tilde{\mathbf{x}}_i$, and the off-diagonal terms are the complex cross-ambiguity functions (CAF) between all the pairs of sensors. For two finite length discrete time complex signals $\tilde{x}_0[n]$ and $\tilde{x}_1[n]$, $n = 0, 1, \cdots, N-1$ the energy in each of the signals is given by $\mathcal{E}_i = \sum_{n=0}^{N-1} |\tilde{x}_i[n]|^2, i = 0, 1$ respectively and the CAF is a two dimensional function of differential delay (Δn) and differential Doppler shift (Δk) between the signals and is defined as

$$CAF(\Delta n, \Delta k) = \sum_{n=0}^{N-1} \tilde{x}_0[n] \tilde{x}_1^*[n + \Delta n] \exp\left(\frac{j2\pi \Delta k n}{N}\right) \tag{5}$$

where $*$ represents complex conjugate. It is important to note that the CAM contains CAF terms for all possible sensor pair combinations and not just a selected set of pairs. So, the CAM can also be written as $[\tilde{\mathbf{B}}]_{ij} = \mathcal{E}_i$ if $i = j$ and $[\tilde{\mathbf{B}}]_{ij} = CAF_{ij}$ if $i \neq j$ where CAF_{ij} is the CAF of the observations at sensor i and sensor j. Another interesting result is that when the correlation information is zero, all the off diagonal terms become zero and so the maximum eigenvalue is simply the maximum of the energies of the sensors. Hence the GLRT simply reduces to a type of energy detector.

For given locations and velocities of the sensors, the delays and Doppler shifts are functions of the target location and velocity. Therefore, the delay and Doppler shift vectors can be written as

$$\mathbf{n} = \mathbf{f}(x_T, y_T, z_T)$$

$$\mathbf{k} = \mathbf{g}(x_T, y_T, z_T, v_{x_T}, v_{y_T}, v_{z_T})$$

where $\mathbf{f}$ and $\mathbf{g}$ are M-dimensional functions. Using the invariance property of the MLE, we have

$$\hat{\mathbf{n}} = \mathbf{f}(\hat{x}_T, \hat{y}_T, \hat{z}_T)$$

$$\hat{\mathbf{k}} = \mathbf{g}(\hat{x}_T, \hat{y}_T, \hat{z}_T, \hat{v}_{x_T}, \hat{v}_{y_T}, \hat{v}_{z_T})$$

where $(\hat{x}_T, \hat{y}_T, \hat{z}_T)$ and $(\hat{v}_{x_T}, \hat{v}_{y_T}, \hat{v}_{z_T})$ are the MLEs of target position and velocity. Putting these in eq (1) we have

$$T(\tilde{\mathbf{x}}) = \max_{x_T, y_T, z_T, v_{x_T}, v_{y_T}, v_{z_T}} \lambda_{\max}(\tilde{\mathbf{B}}(x_T, y_T, z_T, v_{x_T}, v_{y_T}, v_{z_T})) \tag{6}$$

Therefore, *we have simultaneously estimated the target location and velocity as well.*

4. PERFORMANCE OF THE GLRT AND THE MLE

4.1 Detection

We compared the performance of the GLRT against two commonly used detectors. One detector computes the energy of the observations individually at each sensor and when the energy at any one sensor exceeds a predetermined threshold, the target is declared as present. In mathematical terms, if $\mathcal{E}_i, i = 0, 1, \cdots, M-1$ are the energies of the observations at the M sensors, then the detector decides that a target is present if $\max\{\mathcal{E}_0, \mathcal{E}_1, \cdots, \mathcal{E}_{M-1}\} > \gamma_{\mathcal{E}}$. This is called the maximum energy detector. It can be noticed that this detector is based solely on the energy information. The other detector is one that is based solely on the cross-sensor correlation information. Here a sensor is fixed as the reference sensor and the CAFs are computed between observations from the reference sensor and all the other sensors. When the maximum magnitude over possible delay and Doppler shifts of any one of these CAFs exceeds a threshold, the target is declared as present. In mathematical terms, assume sensor 0 is the reference sensor and $\Delta n_i = (n_i - n_0)$ and $\Delta k_i = (k_i - k_0)$ are the difference in delay and Doppler at sensor i and sensor 0 respectively. Then if

$$|CAF_i| = \max_{\Delta n_i,\ \Delta k_i} \left| \sum_{n=0}^{N-1} \tilde{x}_0[n]\tilde{x}_i^*[n+\Delta n_i] \exp\left(\frac{j2\pi\Delta k_i n}{N}\right) \right|, \quad i = 0, 1, \cdots, M-1$$

are the maximum magnitudes over delay and Doppler of the CAFs between sensor 0 and all the other sensors, then a target is declared as present if $\max\{|CAF_1|, |CAF_2|, \cdots, |CAF_{M-1}|\} > \gamma_{CAF}$. This is referred to as pair-wise maximum CAF detector.

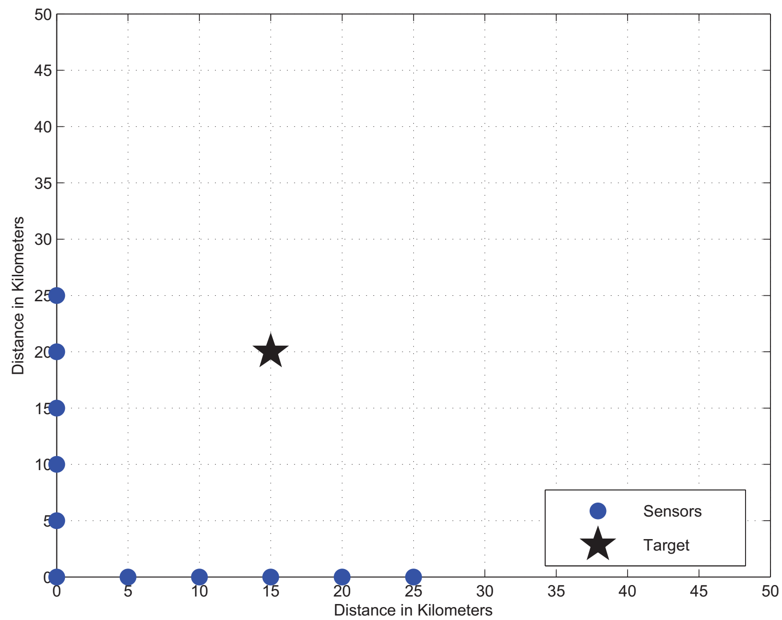

Figure 1. Physical placement of the sensors and the target position used for simulation.

For the purpose of simulation a set of 11 sensors were placed in a configuration as shown in Figure 1. To simplify the computations we assumed that the target is stationary and so the Doppler shift is zero. However, delays are incorporated. As most LPI signals have noise-like properties we used a realization of a Gaussian random process for the signal but once generated was fixed throughout the simulation. White Gaussian noise was also used as the additive noise at the sensors. We collected 32 samples at each of the sensors. The signal-to-noise ratio (SNR), which is the ratio of the energy in the signal to the noise power at each sensor, averaged over all the sensors was about 6.5 dB, i.e., if $\mathcal{E}_{si} = |\tilde{A}_i|^2 \sum_{n=0}^{N-1} |\tilde{s}[n]|^2$ is the energy of the signal at the ith sensor and σ_i^2 is the noise variance at the ith sensor, then the signal to noise ratio averaged over M sensors is given by $10 \log\left(\frac{1}{M}\sum_{i=0}^{M-1} \frac{\mathcal{E}_{si}}{\sigma_i^2}\right)$. Here we assumed a single target and the maximization of the GLRT test statistic was done over (x, y). We used a grid search to maximize the test statistic and searched over a 50km $\times$ 50km region. The comparison receiver operating characteristics (ROC) curves are shown in Figure 2. It can be noticed that the performance of the GLRT is very much better than either of the two commonly used detectors. For a probability of false alarm $P_{FA} = 0.01$ the probability of detection P_D is about 0.1 for the maximum energy detector and the pair-wise maximum CAF detector but is 0.5 for the GLRT. Therefore, the GLRT which is a function of the energy at each sensor combined with the cross-sensor correlation is better than detectors that are solely based on either energy or cross-sensor correlation. Holt and Altes arrive at similar conclusion in [11] and [12].

The fact that GLRT uses both the energy and cross-correlation information is demonstrated in Figure 3. Here, the same emitter signal was used for the two cases of $M = 2$ and $M = 11$. A total of 32 samples were

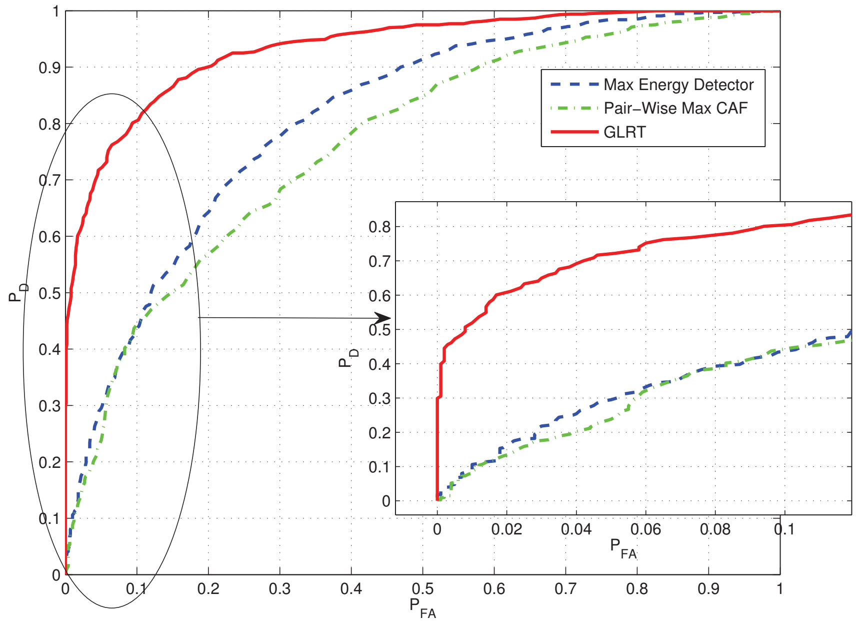

Figure 2. Comparison of GLRT against the maximum energy detector and the pair-wise maximum CAF detector

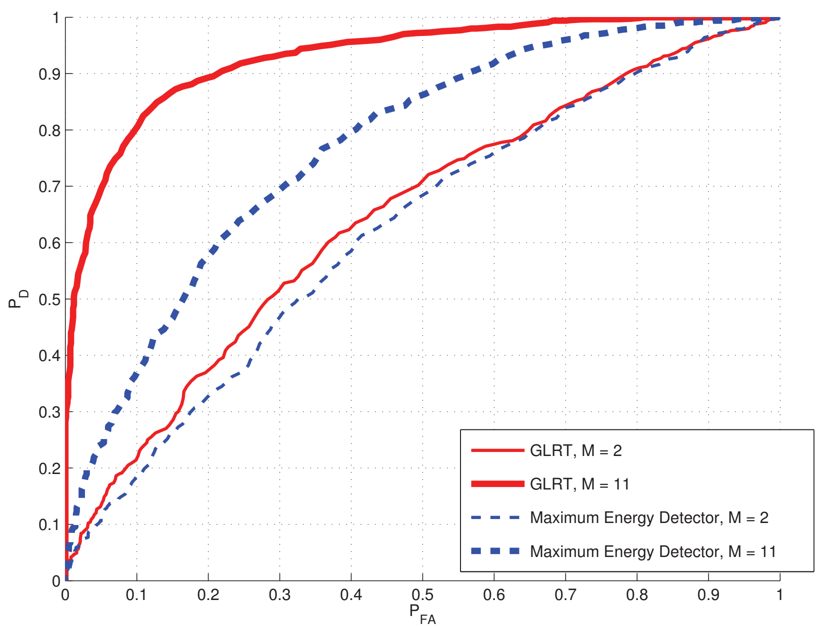

Figure 3. GLRT vs maximum energy detector for the two cases of M=2 and M=11.

collected. It should be noticed that when $M = 2$ the GLRT does only slightly better than the energy detector as there is very little cross-correlation information but when M is increased to 11 the performance of GLRT exceeds that of energy detector considerably. The average SNR at each sensor in the case where $M = 2$ is 4.96 dB while when $M = 11$ it is 6.61 dB. In real world, as the signal travels from the target to the sensor the signal power is attenuated, which is called the propagation loss. In our simulation, while generating the observations at each sensor, we accounted for the propagation loss due to spreading when computing the $\tilde{A}_i$s. Due to the physical setup that we have chosen, as we increased the number of sensors, the newly added sensors were placed closer to the target and so the average SNR per sensor increased.

4.2 Localization

The localization performance of the MLE is compared against the existing time-difference of arrival (TDOA) based localization technique, from hereon referred to as TDOA approach. In the TDOA approach, the sensors are grouped into pairs. From the possible $\binom{M}{2}$ pairs a set of pairs is chosen. The CAF for each pair is computed and the $\hat{\tau}_d$ that maximizes the CAF is the estimate for the TDOA between that pair of sensors. Since we are assuming that the sensors and the target are stationary, the Doppler shift is zero and we need only use cross-correlation instead of CAF. The time difference estimates are computed for the chosen $M-1$ pairs $\{(0,1),(0,2),\cdots,(0,M-1)\}$ to obtain the $M-1$ estimates $(\hat{\tau}_{d_1},\hat{\tau}_{d_2},\cdots,\hat{\tau}_{d_{M-1}})$. The true delays (τ_d) are computed as a function of the target location for each of the $M-1$ pairs of sensors as follows. For the sensor pair (i,j) and assuming a plane geometry

$$\tau_d = \frac{1}{c}\left(\sqrt{(x_i - x_T)^2 + (y_i - y_T)^2} - \sqrt{(x_j - x_T)^2 + (y_j - y_T)^2}\right)$$

where (x_i, y_i) and (x_j, y_j) are the locations of the ith and jth sensor and (x_T, y_T) is the location of the target. c is the speed of light. The difference in the estimated and computed delays is computed as the error

$$J(x_T, y_T) = \sum_{i=1}^{M-1} (\tau_{d_i} - \hat{\tau}_{d_i})^2$$

The values of target location $(\hat{x}_T, \hat{y}_T)$ that minimize J are the estimates of the target location. Figures 4 and 5 show the scatter plots for the TDOA approach and the MLE respectively for an average signal to noise ratio of 11dB. The MLE requires maximization of the eigenvalue and the TDOA requires minimization of the error. These optimizations were performed using grid search over a 24km $\times$ 24km region around the true target location which is at $x_T = 15$, $y_T = 20$ km. This is represented as a box in the figures. From these figures it is very clear that the MLE outperforms the TDOA approach. A total of 1000 simulations was run to generate the scatter plot. The mean and covariance of the target location estimates using both the TDOA and MLE approaches were computed. These were used to generate the error ellipses which are shown in Figure 6. It can be noticed that the TDOA is extremely biased.

5. CONCLUSIONS

We have modeled the problem of detection and localization of LPI signals and derived the GLRT detector and the MLE for target location and velocity for the model. We have shown that the GLRT uses both the energy and the cross-sensor correlation and thus outperforms the currently used detectors which only use either the energy or the cross correlation. While finding the GLRT, the MLE of the target location and velocity are also simultaneously computed. It is shown that the MLE outperforms the existing TDOA approach which pairs up the sensors and performs an inefficient pair-wise processing. We are also investigating the possibility of extending this technique to the detection of multiple targets.

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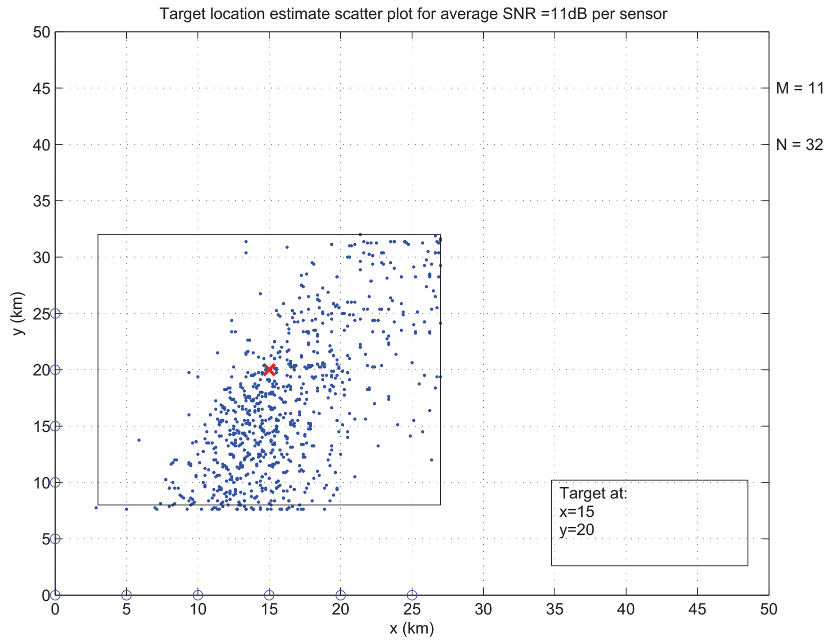

Figure 4. TDOA scatter plot.

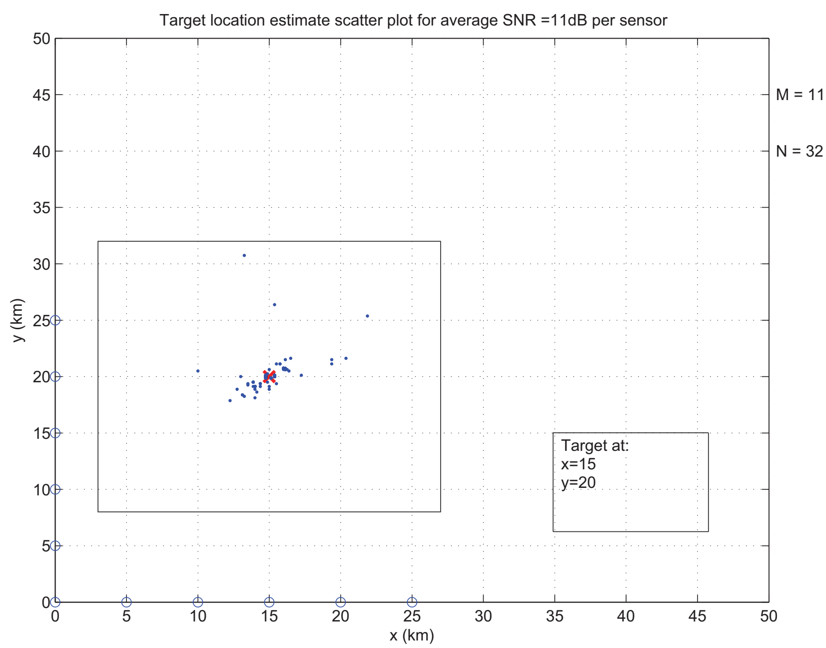

Figure 5. MLE scatter plot.

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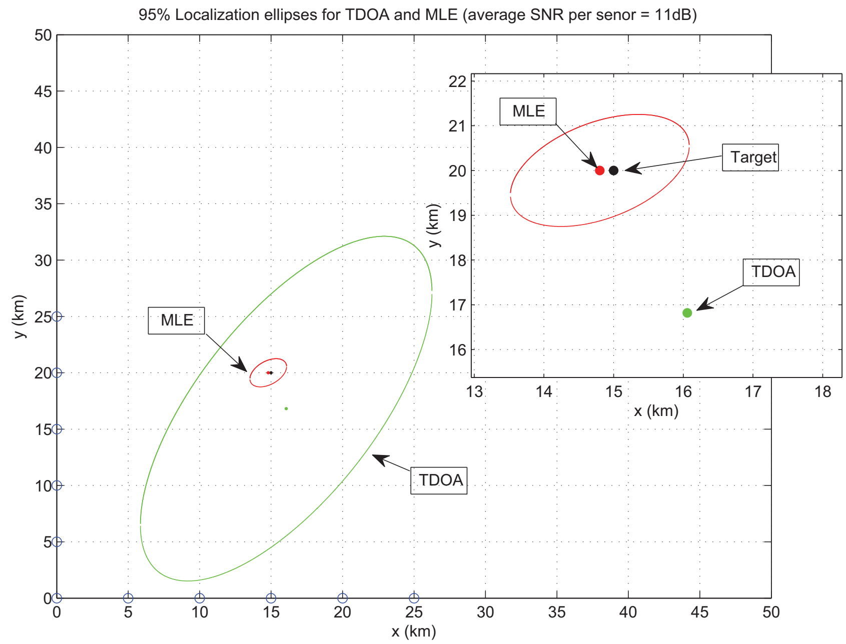

Figure 6. Localization performance comparison of MLE vs TDOA using the error ellipses.

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