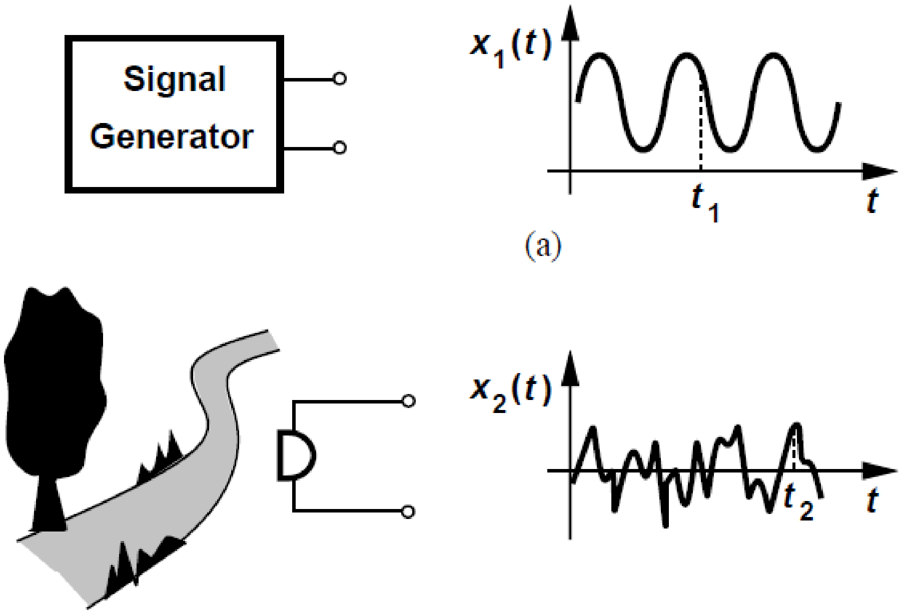
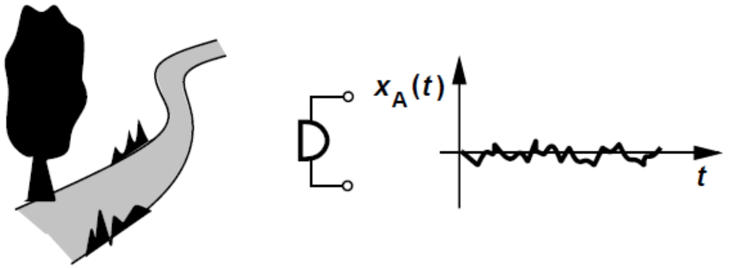
Chapter 7: Noise

Statistical Characteristics of Noise

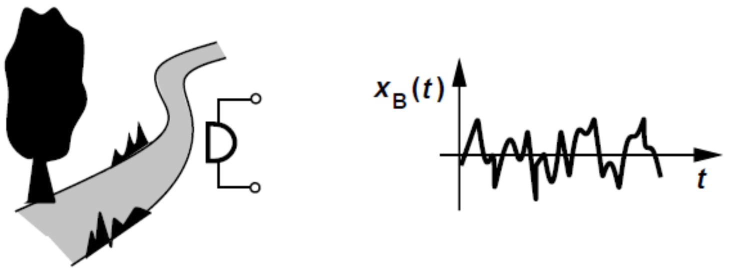


- Noise is a random process
- Value of $x_1(t_1)$ can be predicted from observed waveform, that of $x_2(t_2)$ cannot
  - Difference between deterministic and random phenomena
- Instantaneous value of noise in time domain is unpredictable

Statistical Characteristics of Noise



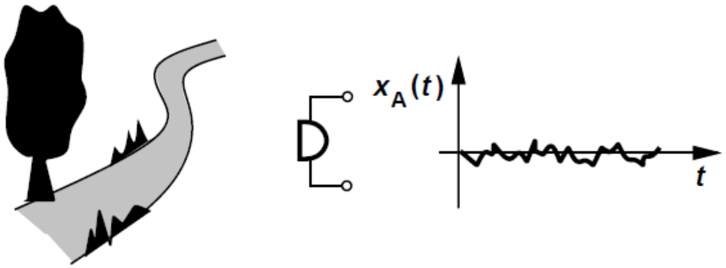
(a)



- Need for a "statistical model" for noise
- Average power of noise is predictable
  - Applicable to most sources of noise in circuits
- Average power delivered by a periodic voltage *v(t)* with period *T* to a load resistance R_L is defined as

$$P_{av} = \frac{1}{T}\int_{-T/2}^{+T/2} \frac{v^2(t)}{R_L} dt$$

Statistical Characteristics of Noise



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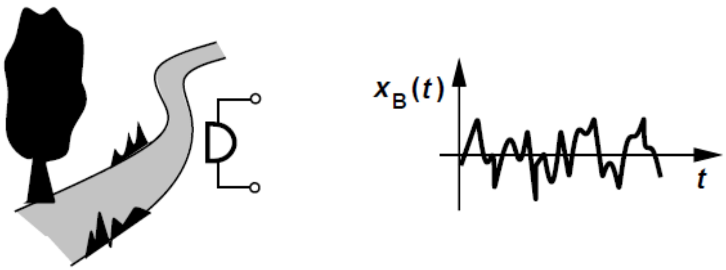
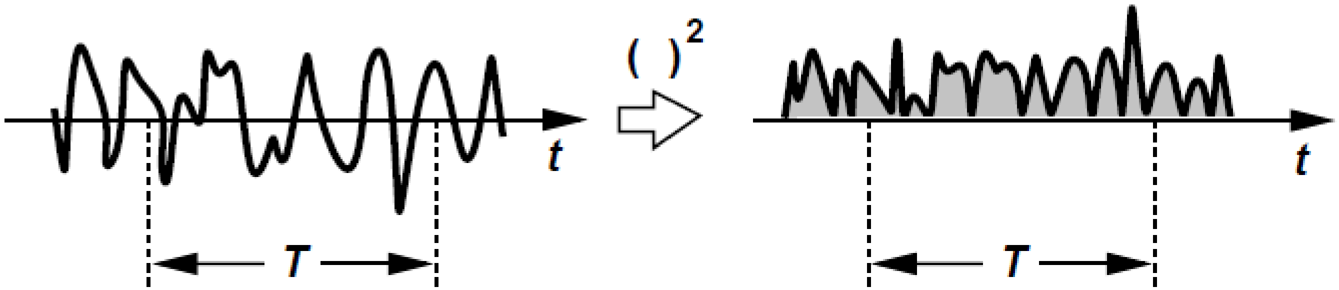


- **For a random signal (aperiodic), measurement must be carried out over a long time**

$$P_{av} = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} \frac{x^2(t)}{R_L} dt$$

- ***x(t)* is a voltage quantity**
- **$x_A(t)$ delivers more power to a resistive load than $x_B(t)$**

Statistical Characteristics of Noise



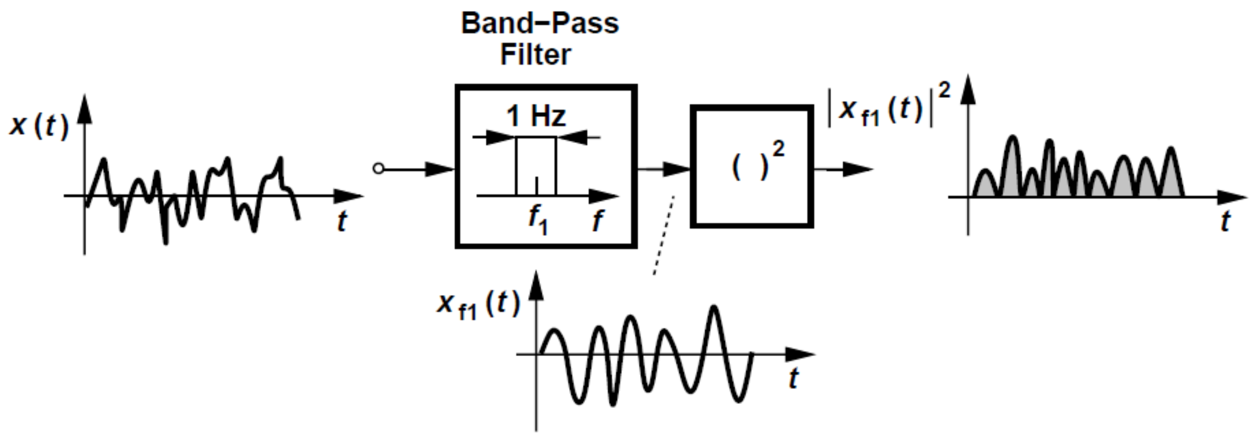
- To calculate average power of (noise) signal *x(t)*
  - Square the signal
  - Find area under resulting waveform for a long period *T*
  - Normalize area to *T*
- For simpler calculations, P_{av} is defined as

$$P_{av} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t)dt$$

- P_{av} is expressed in V^2 rather than W
- RMS voltage for noise can be defined as $\sqrt{P_{av}}$

Noise Spectrum

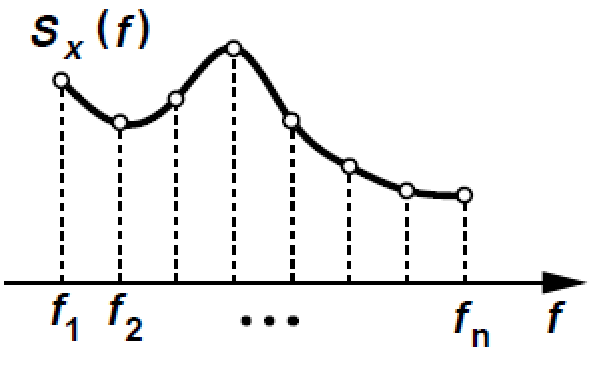
- Spectrum describes the *frequency content* of noise
- Also called Power Spectral Density (PSD)
  - Shows how much power signal carries at each frequency
- PSD $S_x(f)$ of a noise waveform $x(t)$ is defined as the average power carried by $x(t)$ in a 1-Hz bandwidth around f
- Fig. (a) shows calculation of $S_x(f_1)$, i.e., power contained in a specific frequency f_1



(a)

Noise Spectrum

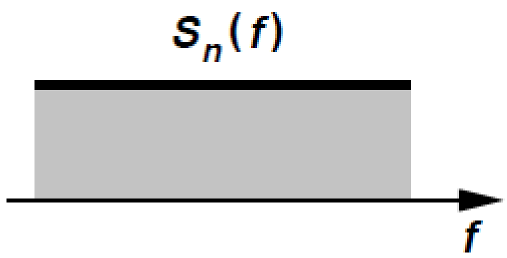
- Repeating previous procedure with bandpass filters with different center frequencies, the overall PSD $S_x(f)$ can be constructed [Fig. (b)]
- $S_x(f)$ Represents the power carried by signal (or noise) at all frequencies, i.e., total power
  - Generally measured in Watts per Hertz
- In signal processing theory, PSD is defined as Fourier transform of Autocorrelation function
  - Two definitions are equivalent



(b)

Noise Spectrum

- As with P_{av}, it is customary to eliminate R_L from $S_x(f)$
- $S_x(f)$ is expressed in V^2/Hz rather than W/Hz
- Also common to take the square root of $S_x(f)$, expressing result in $V/\sqrt{Hz}$
- Common type of noise PSD is "white noise"
  - Displays same value at all frequencies
- White noise does not exist strictly speaking since total power carried by noise cannot be infinite
- Noise spectrum that is flat *in the band of interest* is usually called white



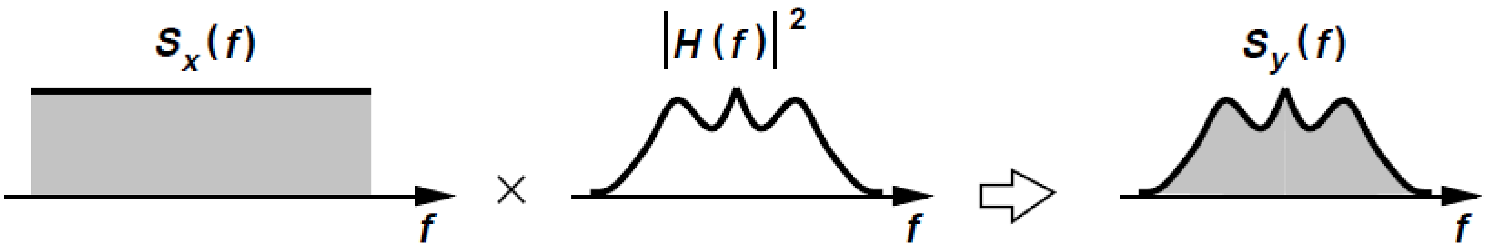
Theorem

- **If a signal with spectrum $S_x(f)$ is applied to a linear time-invariant (LTI) system with transfer function $H(s)$, then the output spectrum $S_Y(f)$ is given by**

$$S_Y(f) = S_x(f)|H(f)|^2$$

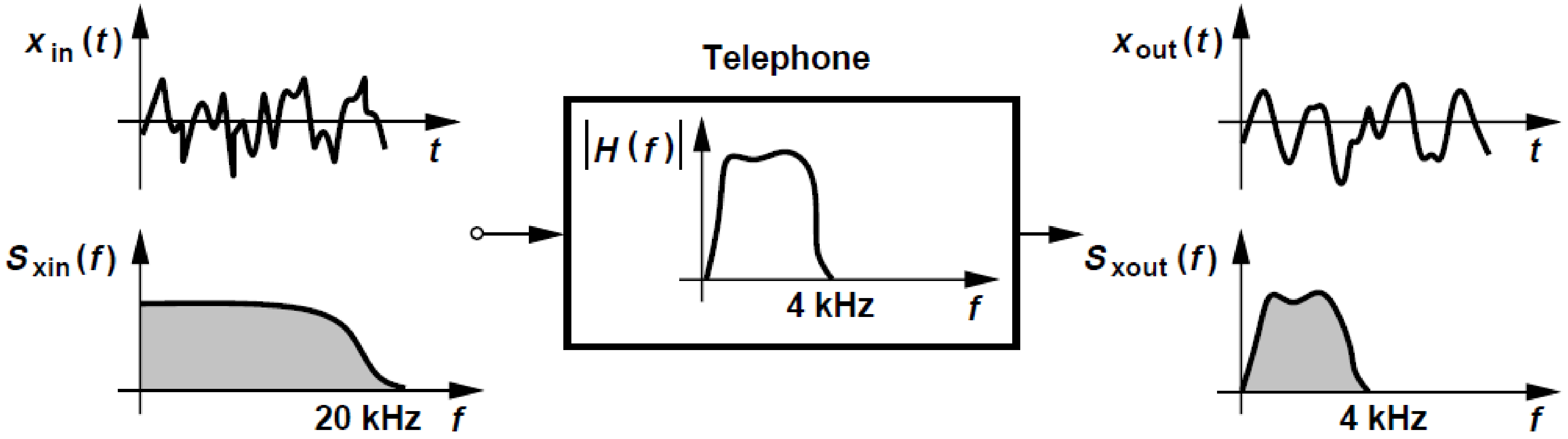
where $H(f) = H(s = j2\pi f)$

- **Spectrum of signal is "shaped" by the transfer function of the system (see Fig. below)**



Theorem: Example

- **Regular telephones have a bandwidth of approximately 4kHz and suppress higher frequency components in caller's voice**
- **Due to limited bandwidth, $x_{out}(t)$ exhibits slower changes than $x_{in}(t)$**
  - **Can be difficult to recognize the caller's voice**

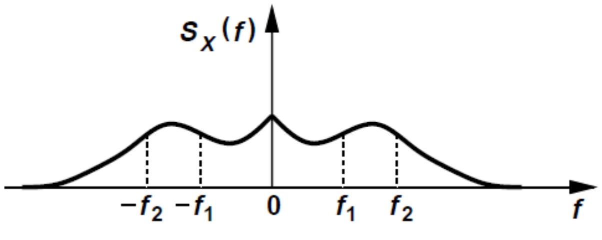


Two-sided and one-sided spectra

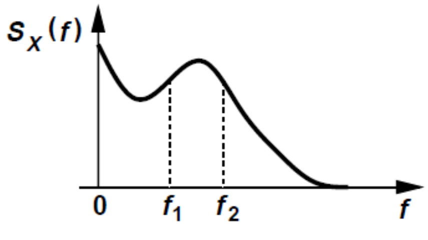
- $S_x(f)$ is an even function of f for real $x(t)$
- The total power carried by x(t) in the frequency range $[f_1, f_2]$ is equal to

$$P_{f1,f2} = \int_{-f_2}^{-f_1} S_x(f)df + \int_{+f_1}^{+f_2} S_x(f)df$$
$$= \int_{+f_1}^{+f_2} 2S_x(f)df.$$

- Negative-frequency part of the spectrum is folded around the vertical axis and added to the positive-frequency part
- Fig. (a) is called "two-sided" spectrum and Fig. (b) is called "one-sided" spectrum

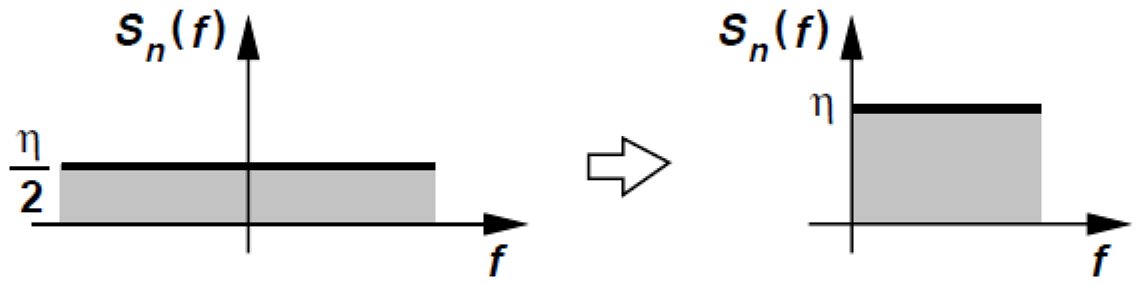


(a)



(b)

Two-sided and one-sided spectra



- **Two-sided white spectrum can be folded around the vertical axis to give a one-sided white spectrum as shown above**

- **Spectrum shows the power carried in a small bandwidth at each frequency, revealing how *fast* the waveform is expected to vary in the time domain**

Amplitude Distribution

- **Although instantaneous value of noise is unpredictable, it is possible to construct a "distribution" of the amplitude by observing the noise waveform for a long time**
- **The distribution indicates how often each value occurs**
- **Also called the "Probability Density Function" (PDF), the distribution of *x(t)* is defined as**

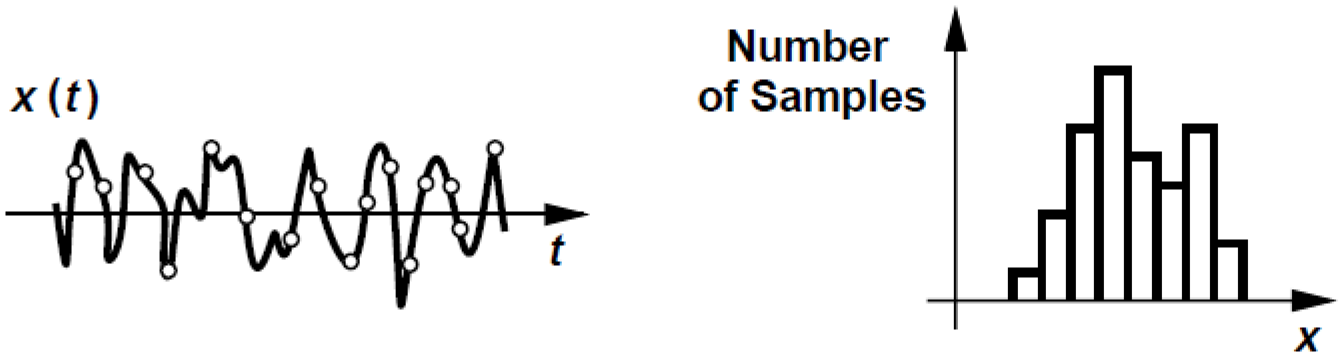
$$p_X(x)dx = \text{probability of } x < X < x + dx$$

where *X* is the measured value of *x(t)* at some point in time

Amplitude Distribution

- To estimate distribution, we sample *x*(t) at many points, construct bins of small width, choose bin height equal to number of samples whose value falls between two edges of the bin and normalize bin heights to total number of samples

- Fig. below shows an example amplitude distribution



- PDF provides no information as to how fast *x(t)* varies in the time domain, i.e., the frequency content of *x(t)*

Central Limit Theorem

- **The Central Limit Theorem states that if many independent random processes with arbitrary PDFs are added, the PDF of the sum approaches a Gaussian distribution**
- **Many natural phenomena exhibit Gaussian statistics**
  - **E.g., noise of a resistor that results from the random "walk" of a very large number of electrons, each with independent statistics**
- **The Gaussian PDF is defined as**

$$p_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x-m)^2}{2\sigma^2}$$

- ***σ* and *m* are the standard deviation and mean of the distribution respectively**
- **For a Gaussian distribution, *σ* is equal to the rms value of the noise**

Correlated and Uncorrelated Sources

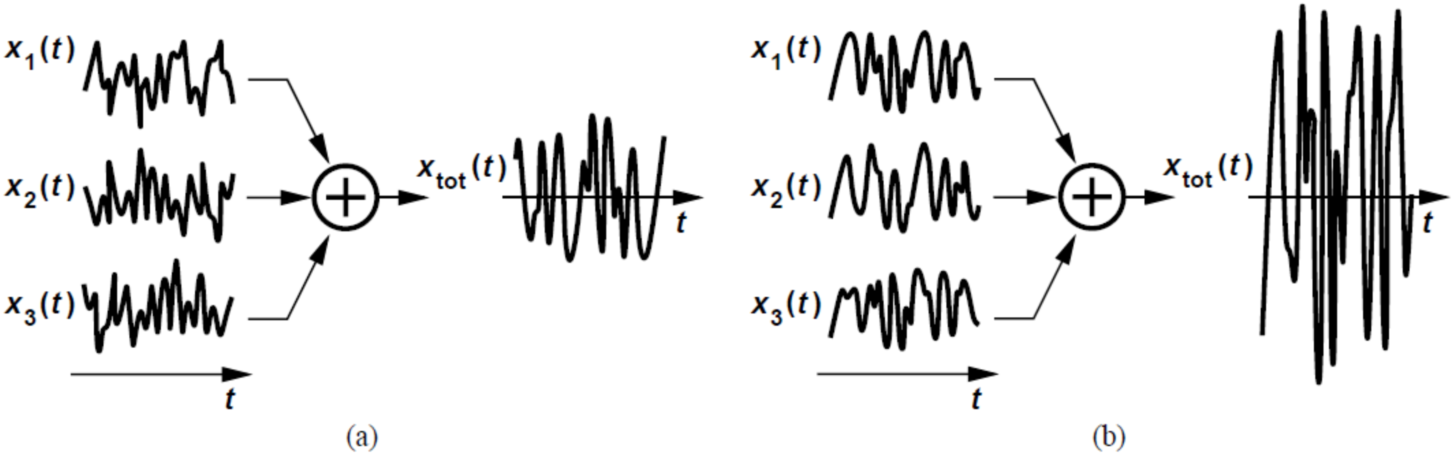
- **The Superposition Principle is applicable to deterministic voltages and currents, but not very suitable for random noise signals**
- **Average noise power is of more interest**
- **Consider the sum of two noise waveforms $x_1(t)$ and $x_2(t)$ and take the average of the resulting power**

$$
\begin{aligned}
P_{av} &= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} [x_1(t) + x_2(t)]^2 dt \\
&= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} x_1^2(t) dt + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} x_2^2(t) dt \\
&\quad + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} 2x_1(t)x_2(t) dt \\
&= P_{av1} + P_{av2} + \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{+T/2} 2x_1(t)x_2(t) dt,
\end{aligned}
$$

- **P_{av1} and P_{av2} denote the average power of $x_1(t)$ and $x_2(t)$ respectively**
- **Third term is called "correlation" between $x_1(t)$ and $x_2(t)$**

Correlated and Uncorrelated Sources

- **Correlation integral indicates how "similar" the two waveforms are**
- **Noise waveforms generated by independent devices are uncorrelated and integral vanishes to zero**
  - $P_{av} = P_{av1} + P_{av2}$
- **Superposition holds for the *power* of uncorrelated noise sources**
- **Total power of correlated noise sources [Fig. (a)] is higher than that of uncorrelated sources [Fig. (b)]**



Signal-to-Noise Ratio (SNR)

- **Signal-to-noise ratio (SNR) is defined as**

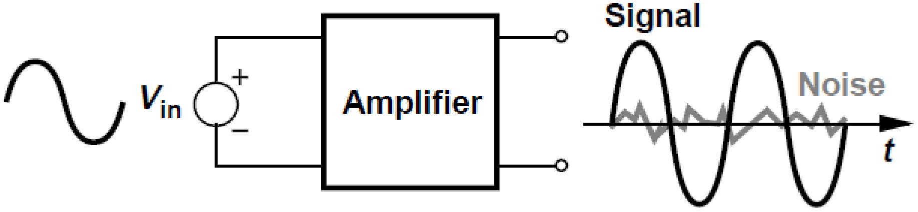
$$\mathrm{SNR} = \frac{P_{sig}}{P_{noise}}$$

- **SNR of a noise-corrupted signal should be high for it to be intelligible**
  - **Audio signals require a minimum SNR of 20 dB**
- **For a sinusoid with peak amplitude A, $P_{sig} = A^2/2$**
- **The total average power carried by noise is equal to the area under its spectrum**

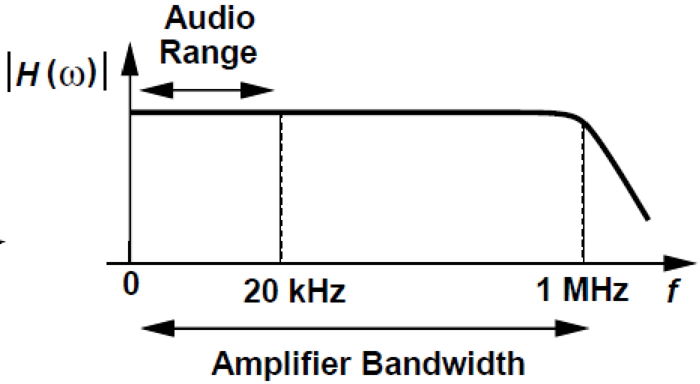
$$P_{noise} = \int_{-\infty}^{+\infty} S_{noise}(f)df.$$

- **P_{noise} can be very large if $S_{noise}(f)$ spans a wide frequency range**

Signal-to-Noise Ratio (SNR)



(a)



(b)

- Above amplifier [Fig. (a)] provides a bandwidth of 1 MHz while sensing an audio signal [Fig. (b)]
- Signal is corrupted by all noise components in the 1-MHz bandwidth
  - SNR may be unacceptably low
- Bandwidth of circuit must be limited to the minimum acceptable value to minimize total integrated noise power
  - Within the amplifier or by a series low-pass filter

Noise Analysis Procedure

- Output signal of a given circuit is corrupted by noise sources within the circuit
  - Interested in noise observed at the output
- Four steps:
  - Identify the sources of noise and note the spectrum of each
  - Find the transfer function from each noise source to the output
  - Use the theorem SY(f) = Sx(f)|H(f)|2 to calculate output noise spectrum contributed by each noise source
  - Add all the output spectra, accounting for correlated and uncorrelated sources
- Integrate the output noise spectrum from -∞ to +∞ to get total output noise power

Types of Noise

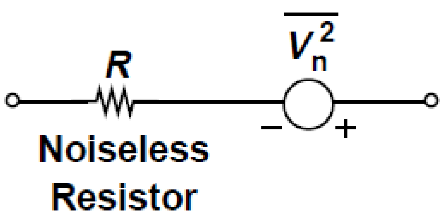
- **Analog signals processed by integrated circuits are corrupted by two types of noise**
  - **Device electronic noise**
  - **“Environmental” noise (Chapter 19)**

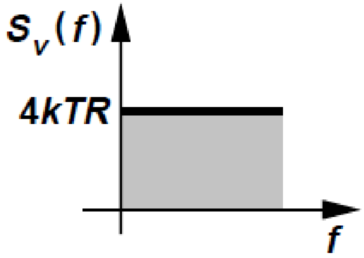
Resistor Thermal Noise

- Random motion of electrons in a conductor induces fluctuations in the voltage measured across it even though the average current is zero
- Thermal noise of a resistor *R* can be modeled by a series voltage source, with one-sided spectral density

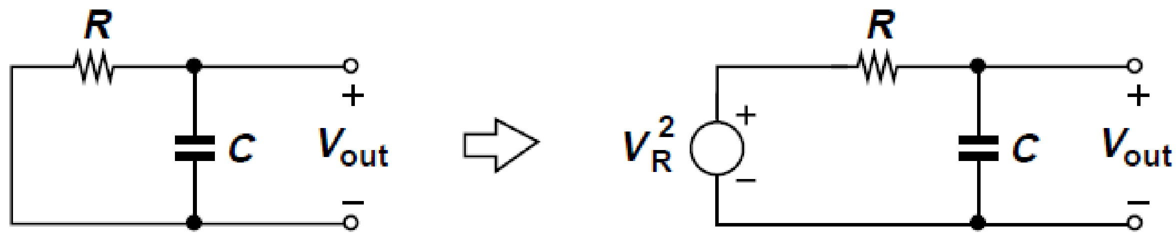
$$S_v(f) = 4kTR, \quad f \geq 0$$

- Here, $k = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant
- $S_v(f)$ is expressed in V^2/Hz, we also write $\overline{V_n^2} = 4kTR$
- For a 50-Ω resistor at T = 300 K, thermal noise is 8.28×10^{-19} V^2/Hz, or 0.91 nV/√Hz
- $S_v(f)$ is flat up to 100 THz, and is "white" for our purposes





Resistor Thermal Noise: Example



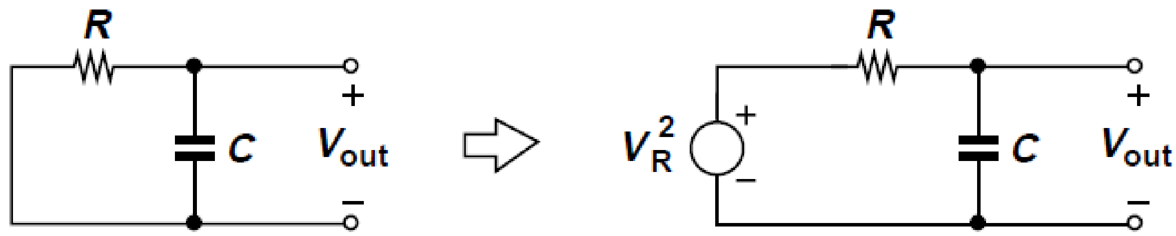
- <u>To find</u>: Noise spectrum and total noise power in V_{out}
- Solution: Noise spectrum of R is given by $S_v(f) = 4kTR$
- Modeling noise by a series voltage source V_R, transfer function from V_R to V_{out} is

$$\frac{V_{out}}{V_R}(s) = \frac{1}{RCs + 1}$$

- Using theorem, noise spectrum at the output $S_{out}(f)$ is

$$\begin{aligned} S_{out}(f) &= S_v(f)\left|\frac{V_{out}}{V_R}(j\omega)\right|^2 \\ &= 4kTR\frac{1}{4\pi^2 R^2 C^2 f^2 + 1} \end{aligned}$$

Resistor Thermal Noise: Example



- **White noise spectrum of the resistor is shaped by a low-pass characteristic**
- **Total noise power at the output is**

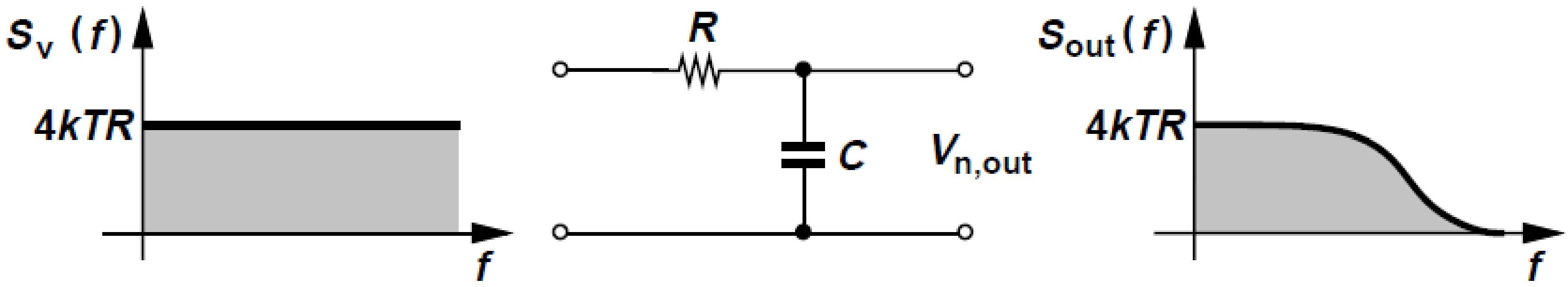
$$P_{n,out} = \int_0^\infty \frac{4kTR}{4\pi^2 R^2 C^2 f^2 + 1} df$$

- **The integral reduces to**

$$\begin{aligned} P_{n,out} &= \frac{2kT}{\pi C} \tan^{-1} u \Big|_{u=0}^{u=\infty} \\ &= \frac{kT}{C}. \end{aligned}$$

- **The unit of $P_{n,out}$ is V^2/Hz, $\sqrt{kT/C}$ may be considered as the total rms voltage measured at the output**

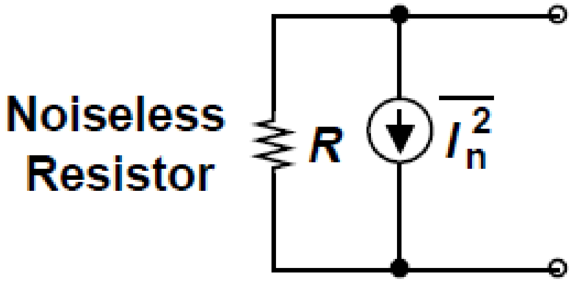
Resistor Thermal Noise: Example



- The *RC* low-pass filter shapes the noise spectrum of the resistor
- Total noise at the output (area under $S_{out}(f)$) is independent of the resistance *R*
- Intuitively, this is because for larger values of R, noise per unit bandwidth increases but the overall bandwidth of the circuit decreases
- *kT/C* noise can only be decreased by increasing R (if T is fixed)

Resistor Thermal Noise

- Thermal noise of a resistor can be represented by a parallel current source too

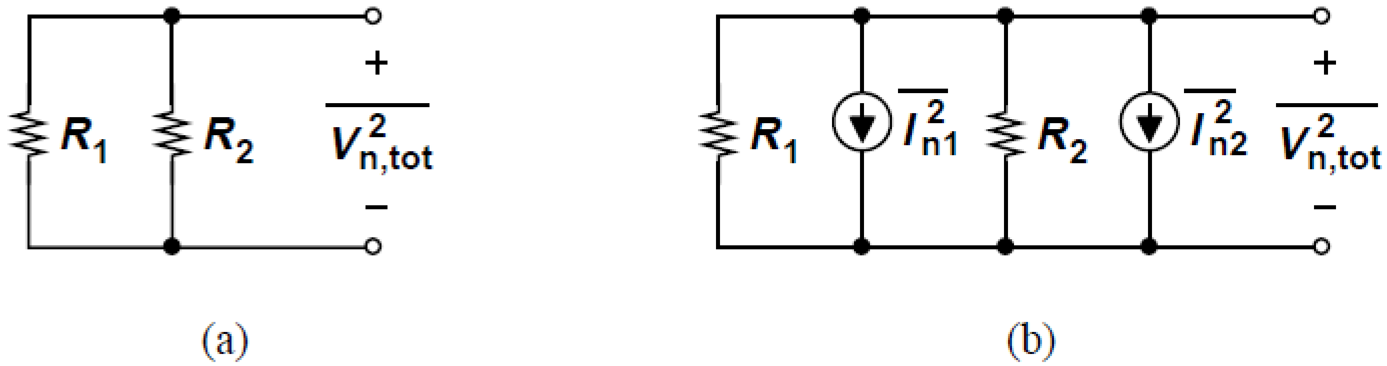


- This representation is equivalent to series voltage source representation with

$$\overline{I_n^2} = 4kT/R$$

- $\overline{I_n^2}$ is expressed in *A²/Hz*
- Depending on circuit topology, one model may lead to simpler calculations than the other

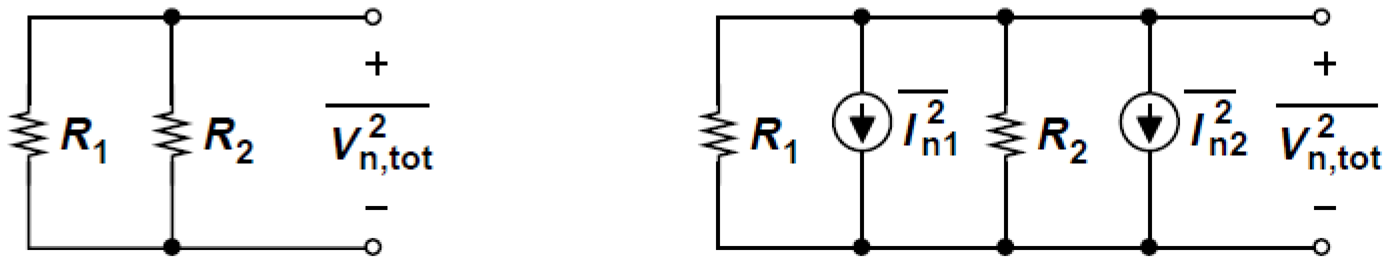
Resistor Thermal Noise: Example



- **<u>To find</u>: Equivalent noise voltage of two resistors in parallel, R_1 and R_2**
- **<u>Solution</u>:**
- **Each resistor exhibits equivalent noise current with spectral density *4kT/R***
- **Since the two noise sources are uncorrelated, we add the powers**

$$\begin{aligned} \overline{I_{n,tot}^2} &= \overline{I_{n1}^2} + \overline{I_{n2}^2} \\ &= 4kT\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \end{aligned}$$

Resistor Thermal Noise: Example



(a) (b)

- **Equivalent noise voltage is given by**

$$\overline{V_{n,tot}^2} = \overline{I_{n,tot}^2}(R_1||R_2)^2$$
$$= 4kT(R_1||R_2),$$

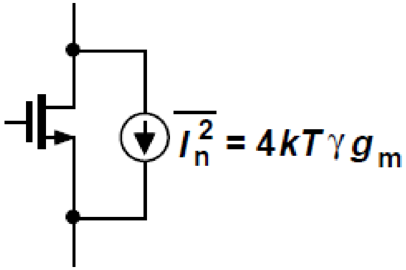
- **This notation assumes a 1-Hz bandwidth**

MOSFET Thermal Noise

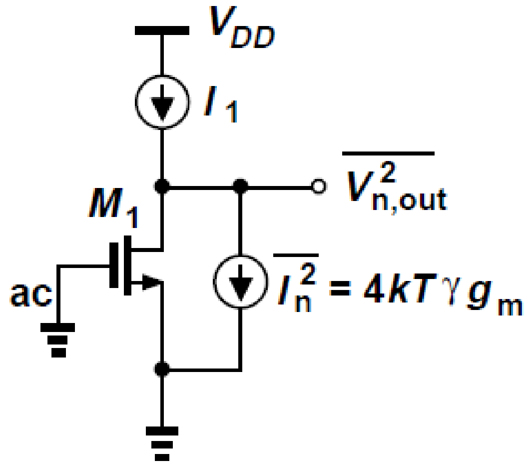
- MOS transistors exhibit thermal noise with the most significant source being the noise generated in the channel
- For long-channel MOS devices operating in saturation, the channel noise can be modeled by a current source connected between the drain and source terminals with a spectral density

$$\overline{I_n^2} = 4kT\gamma g_m$$

- The coefficient '*γ*' (not the body effect coefficient) is derived to be 2/3 for long-channel transistors and is higher for submicron MOSFETs
- As a rule of thumb, assume $\gamma = 1$



MOSFET Thermal Noise: Example

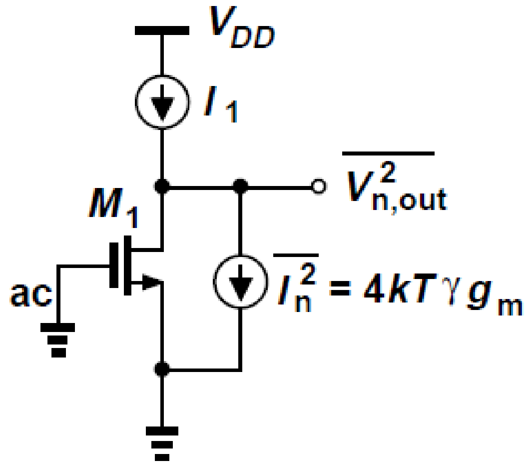


- **The maximum output noise occurs if the transistor sees only its own output impedance as the load, i.e., if the external load is an ideal current source**
- **Output noise voltage spectrum is given by**

$$S_{out}(f) = S_{in}(f)|H(f)|^2$$

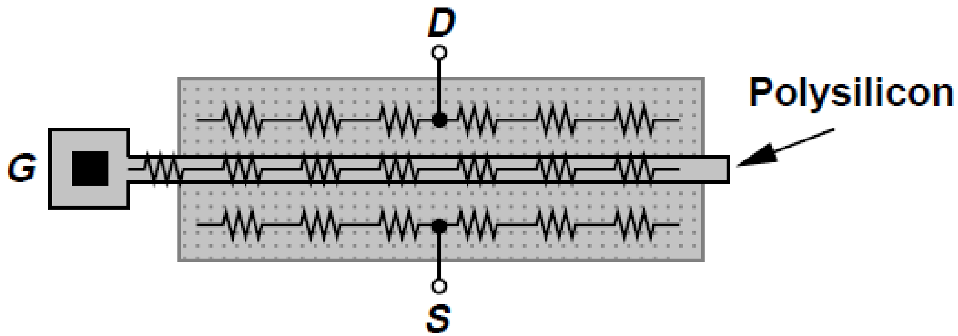
$$\begin{aligned} \overline{V_n^2} &= \overline{I_n^2} r_O^2 \\ &= (4kT\gamma g_m) r_O^2 \end{aligned}$$

MOSFET Thermal Noise: Example



- Noise current of a MOS transistor decreases if the transconductance drops
- Noise measured at the output of the circuit does not depend on where the input terminal is because input is set to zero for noise calculation
- The output resistance r_o does not produce noise because it is not a physical resistor

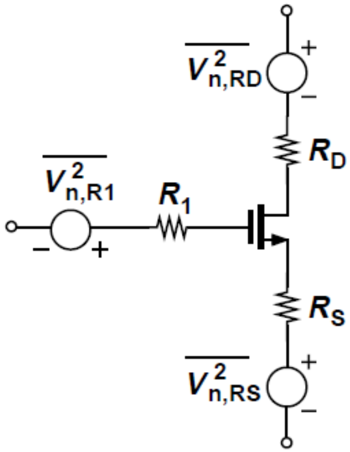
MOSFET Thermal Noise



(a)

- **Ohmic sections of a MOSFET have a finite resistivity and exhibit thermal noise [Fig. (a)]**
- **For a wide transistor, source and drain resistance is negligible whereas the gate distributes resistance may become noticeable**

MOSFET Thermal Noise



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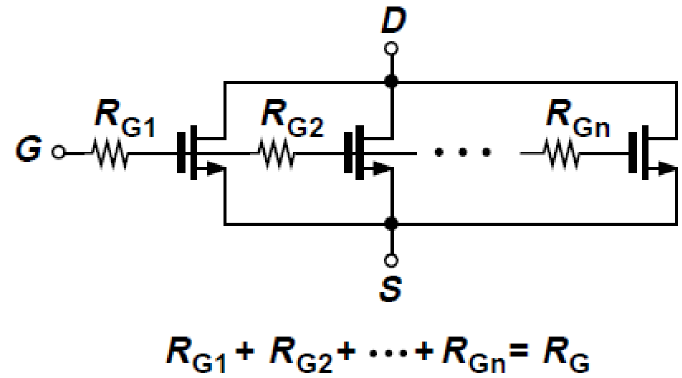
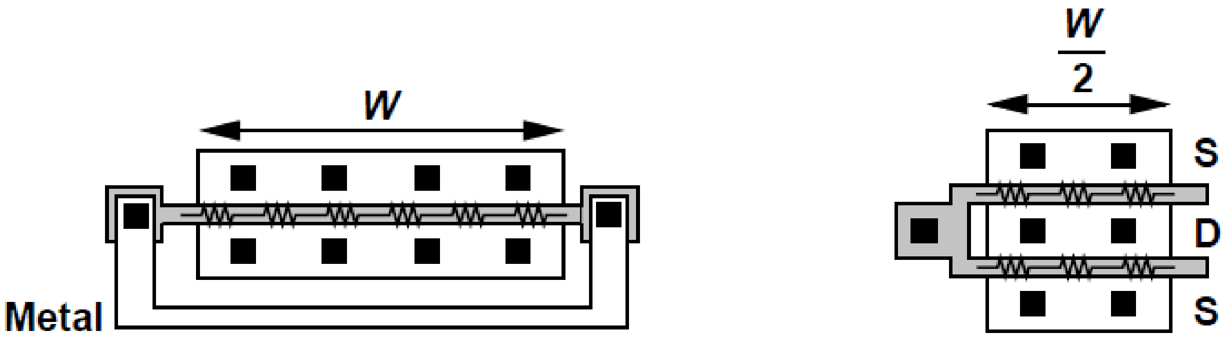


(c)

- In the noise model [Fig. (b)], the lumped resistance R_1 represents the distributed gate resistance
- In the distributed structure of Fig. (c), unit transistors near the left end see the noise of only a fraction of R_G whereas those near the right end see the noise of most of R_G
- It can be proved that $R_1 = R_G/3$ and hence the noise generated by gate resistance is $\overline{V^2_{nRG}} = 4kTR_G/3$

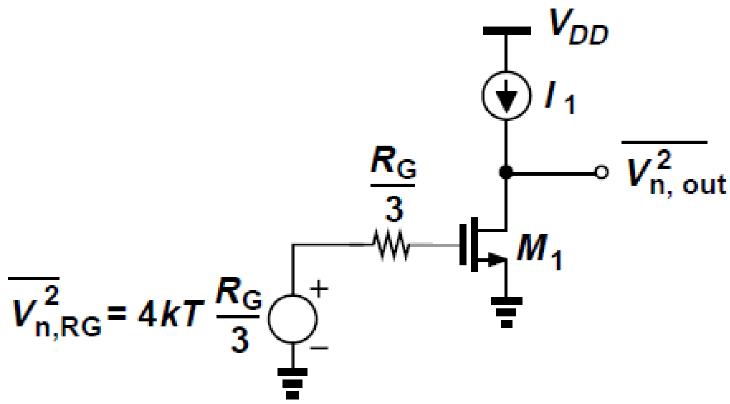
MOSFET Thermal Noise



(a) (b)

- Effect of R_G can be reduced by proper layout
- In Fig. (a), the two ends of the gate are shorted by a metal line, reducing the distributed resistance from R_G to $R_G/4$
- Alternatively, the transistor can be folded as in Fig. (b) so that each gate “finger” exhibits a resistance of $R_G/2$ yielding a total distributed resistance of $R_G/4$ for the composite transistor

MOSFET Thermal Noise: Example



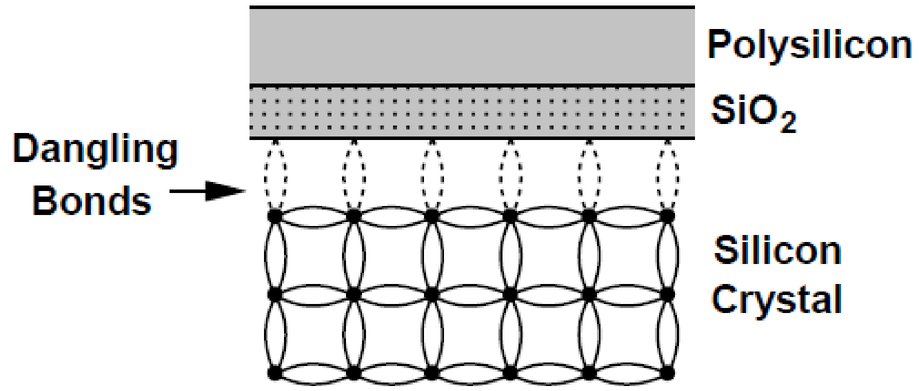
- If the total distributed gate resistance is R_G, the output noise voltage due to R_G is given by

$$\overline{V^2_{n,out}} = 4kT\frac{R_G}{3}(g_m r_O)^2$$

- For the gate resistance noise to be negligible, we must ensure

$$\frac{R_G}{3} \ll \frac{\gamma}{g_m}$$

Flicker Noise



- At the interface between the gate oxide and silicon substrate, many "dangling" bonds appear, giving rise to extra energy states
- Charge carriers moving at the interface are randomly trapped and later released by such energy states, introducing "flicker" noise in the drain current
- Other mechanisms in addition to trapping are believed to generate flicker noise

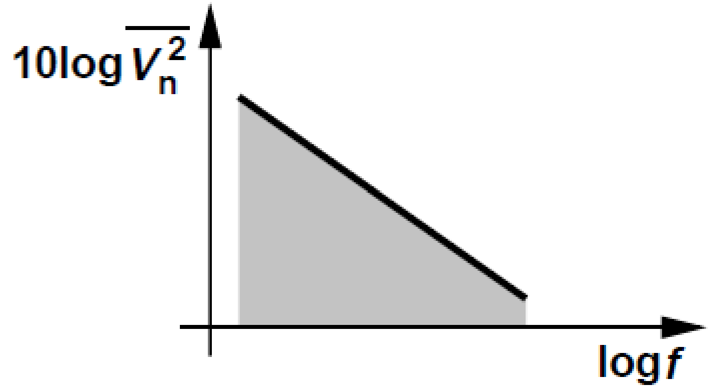
Flicker Noise

- **Average power of flicker noise cannot be predicted easily**
- **It varies depending on cleanness of oxide-silicon interface and from one CMOS technology to another**
- **Flicker noise is more easily modeled as a voltage source in series with the gate and in the saturation region, is roughly given by**

$$\overline{V_n^2} = \frac{K}{C_{ox}WL} \cdot \frac{1}{f}$$

- ***K* is a process-dependent constant on the order of 10^{-25} V^2F**

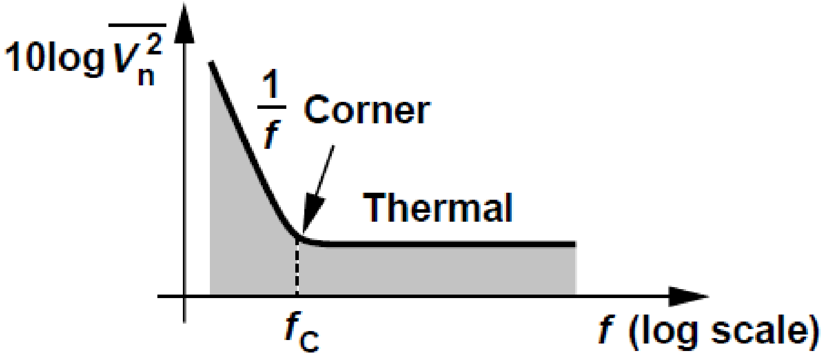
Flicker Noise



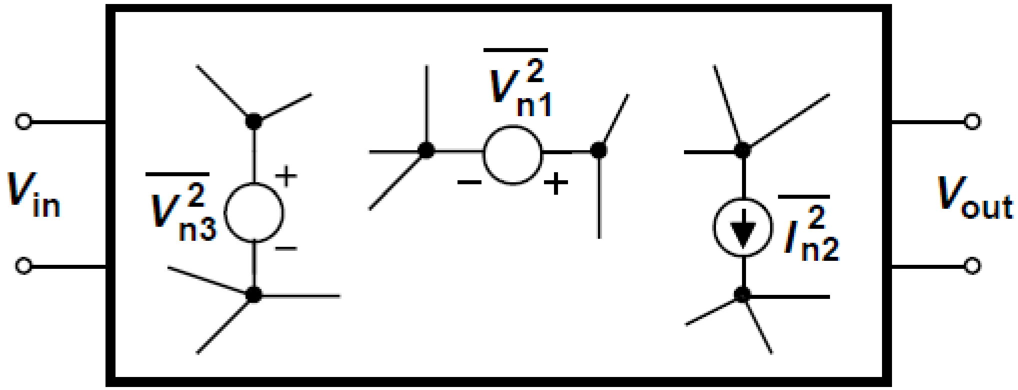
- The noise spectral density is inversely proportional to frequency
  - Trap and release phenomenon occurs at low frequencies more often
- Flicker noise is also called "*1/f*" noise
- To reduce *1/f* noise, device *area* must be increased
- Generally, PMOS devices exhibit less *1/f* noise than NMOS transistors
  - Holes are carried in a "buried" channel, at some distance from the oxide-silicon interface

Flicker Noise Corner Frequency

- **At low frequencies, the flicker noise power approaches infinity**
- **At very slow rates, flicker noise becomes indistinguishable from thermal drift or aging of devices**
  - **Noise component below the lowest frequency in the signal of interest does not corrupt it significantly**
- **Intersection point of thermal noise and flicker noise spectral densities is called "corner frequency" f_C**

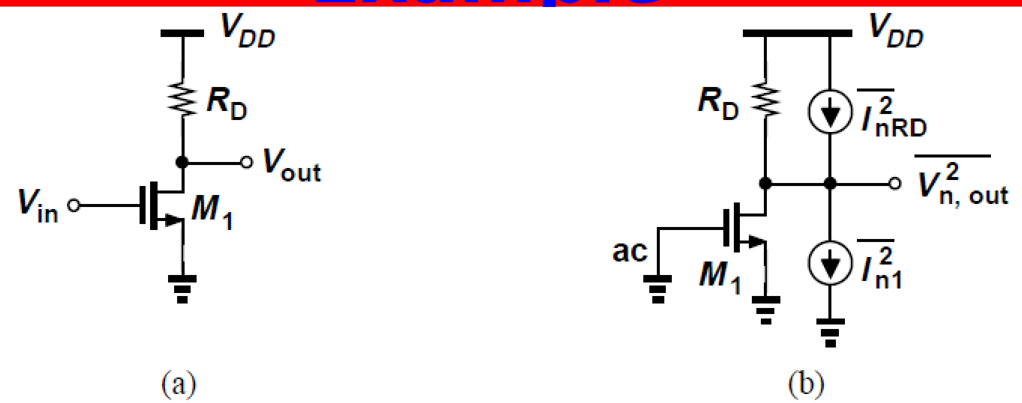


Representation of Noise in Circuits



- **To find the output noise, the input is set to zero and total noise is calculated at the output due to all the noise sources in the circuit**
- **This is how noise is measured in laboratories and in simulations**

Representation of Noise in Circuits: Example



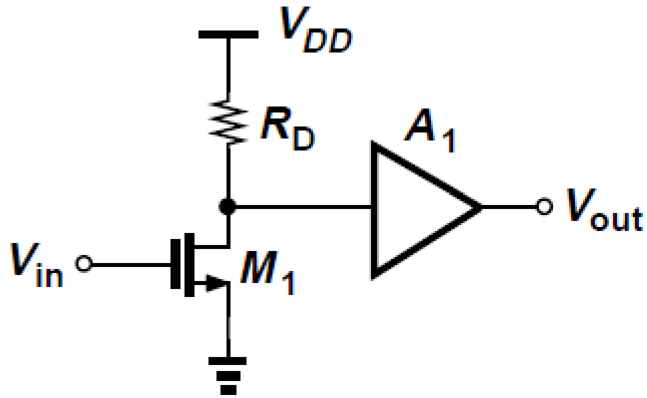
- **<u>To find</u>: Total output noise voltage of the common-source stage [Fig. (a)]**
- **Follow noise analysis procedure described earlier**
- **Thermal and flicker noise of M1 and thermal noise of RD are modeled as current sources [Fig. (b)]**

$$\overline{I^2_{n,th}} = 4kT\gamma g_m \qquad \overline{I^2_{n,1/f}} = Kg_m^2/(C_{ox}WLf) \qquad \overline{I^2_{n,RD}} = 4kT/R_D$$

- **Output noise voltage per unit bandwidth, added as power quantities is**

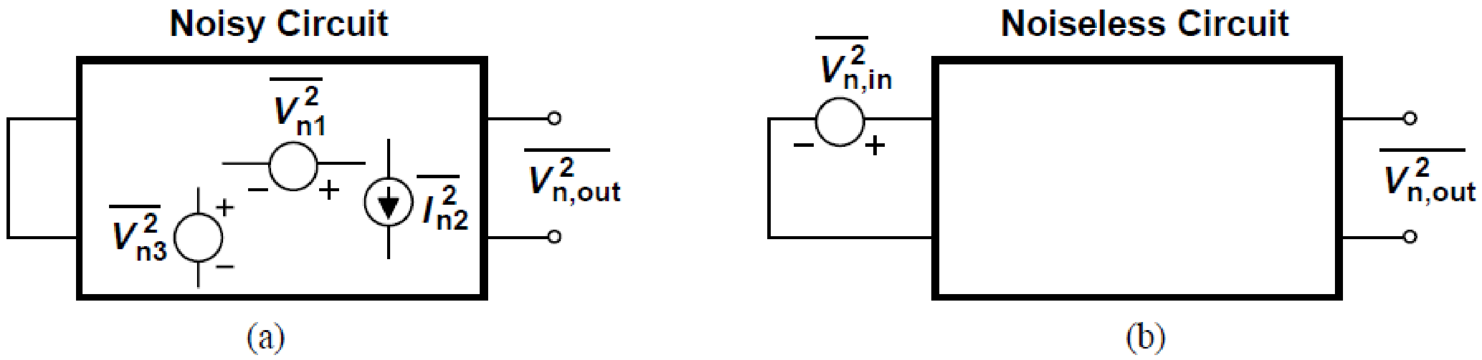
$$\overline{V^2_{n,out}} = \left(4kT\gamma g_m + \frac{K}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right)R_D^2$$

Input-Referred Noise



- **Output-referred noise does not allow a fair comparison of noise in different circuits since it depends on the gain**
- **In above figure, a CS stage is succeeded by a noiseless amplifier with voltage gain A_1, then the net output noise is now multiplied by A_1^2**
- **This may indicate that circuit becomes noisier as A_1 increases, which is incorrect since the signal level also increases proportionally, and net SNR at the output does not depend on A_1**

Input-Referred Noise

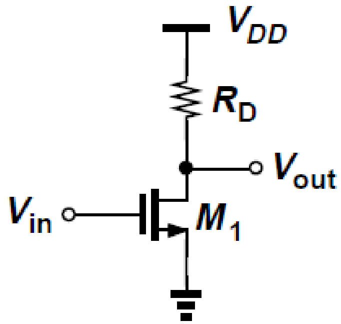


- **Input-referred noise represents the effect of all noise sources in the circuit by a single source $\overline{V_{n,in}^2}$, at the input such that the output noise in Fig. (b) is equal to that in Fig. (a)**
- **If the voltage gain is A_v, then we must have**

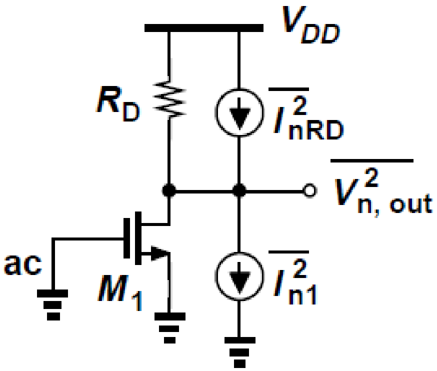
$$\overline{V_{n,out}^2} = A_v^2 \overline{V_{n,in}^2}$$

- **The input-referred noise voltage in this simple case is simply the output noise divided by the gain squared.**

Input-Referred Noise: Example



(a)



(b)

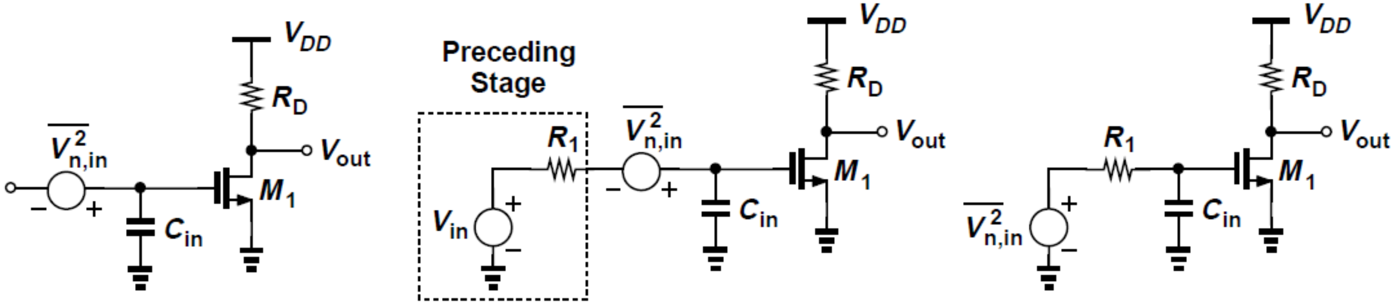
- **For the simple CS stage, the input-referred noise voltage is given by**

$$\begin{aligned}
\overline{V^2_{n,in}} &= \frac{\overline{V^2_{n,out}}}{A_v^2} \\
&= \left(4kT\gamma g_m + \frac{K}{C_{ox}WL}\cdot\frac{1}{f}\cdot g_m^2 + \frac{4kT}{R_D}\right) R_D^2 \frac{1}{g_m^2 R_D^2} \\
&= 4kT\frac{\gamma}{g_m} + \frac{K}{C_{ox}WL}\cdot\frac{1}{f} + \frac{4kT}{g_m^2 R_D}.
\end{aligned}$$

- **First and third terms combined can be viewed as thermal noise of an equivalent resistance R_T, so that the equivalent input-referred thermal noise is $4kTR_T$**

Input-Referred Noise

- Single voltage source in series with the input is an incomplete representation of the input-referred noise for a circuit with a finite input impedance and driven by a finite source impedance
- For the CS stage, the input-referred noise voltage is independent of the preceding stage



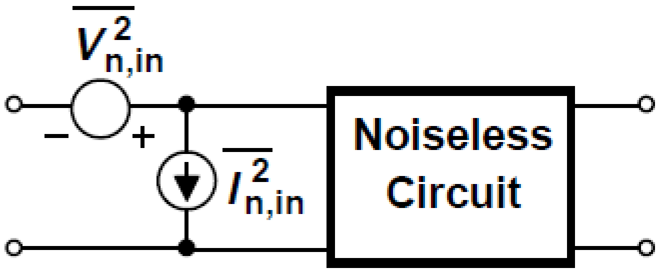
(a) (b) (c)

- If the preceding stage is modeled by a Thevenin equivalent with an output impedance of R_1, the output noise due to voltage division is

$$\overline{V^2_{n,out}} = \overline{V^2_{n,in}} \left| \frac{1}{R_1 C_{in} j\omega + 1} \right|^2 (g_m R_D)^2$$
$$= \frac{4kT\gamma g_m R_D^2}{R_1^2 C_{in}^2 \omega^2 + 1}.$$

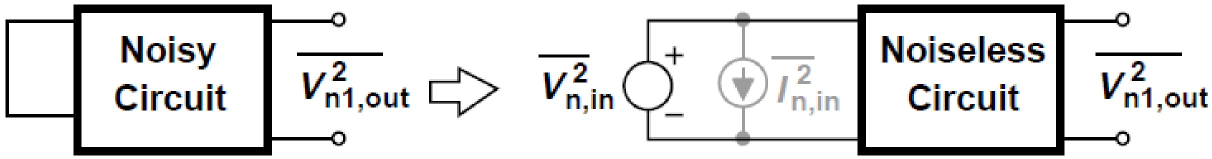
Input-Referred Noise

- The previous result is incorrect since the output noise due to M_1 does not diminish as R_1 increases
- To solve this issue, we model the input-referred noise by both a series voltage source and a parallel current source, so that if the output impedance of the previous stage assumes large values, thereby reducing the effect of $\overline{V_{n,in}^2}$, the noise current still flows through the finite impedance, producing noise at the input
- It can be proved that $\overline{V_{n,in}^2}$ and $\overline{I_{n,in}^2}$ are necessary and sufficient to represent the noise of any linear two-port circuit

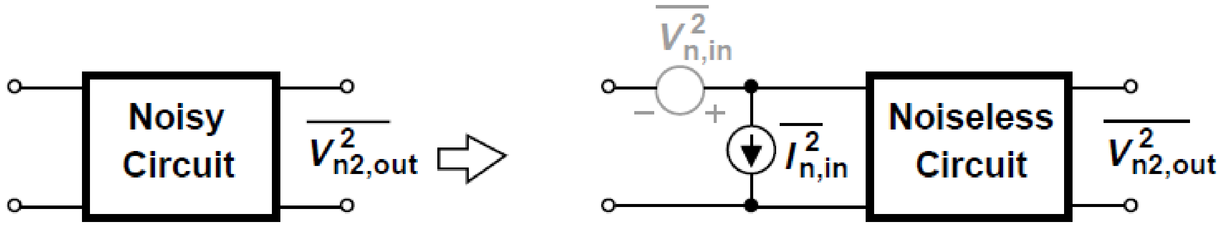


Input-Referred Noise

- To calculate $\overline{V^2_{n,in}}$ and $\overline{I^2_{n,in}}$, two extreme cases are considered: zero and infinite source impedances



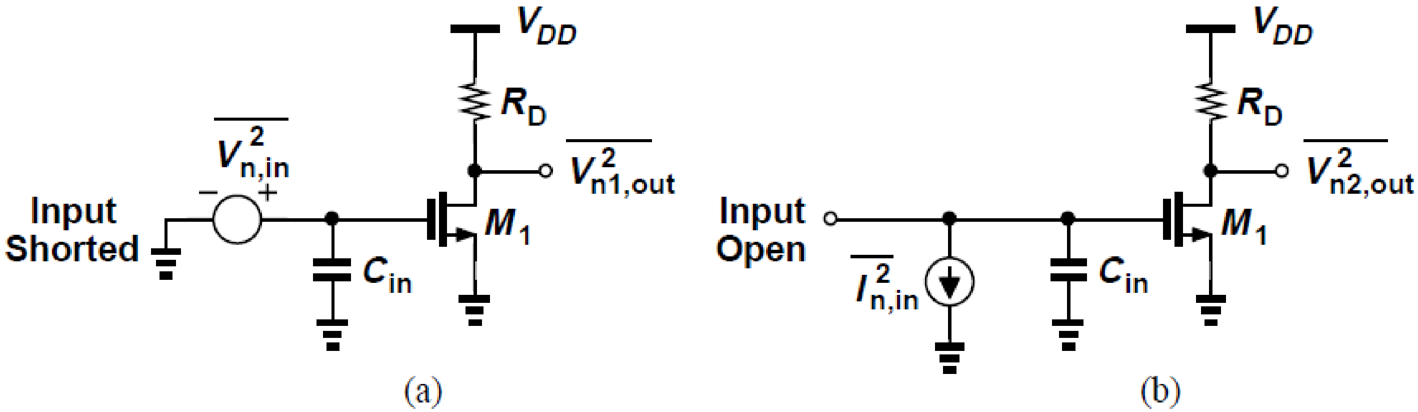
(a)



(b)

- If the source impedance is zero [Fig. (a)], $\overline{I^2_{n,in}}$ flows through $\overline{V^2_{n,in}}$ and has no effect at the output, i.e., the output noise measured arises solely from $\overline{V^2_{n,in}}$
- If the input is left open [Fig. (b)], then $\overline{V^2_{n,in}}$ has no effect and the output noise is only due to $\overline{I^2_{n,in}}$

Input-Referred Noise: Example



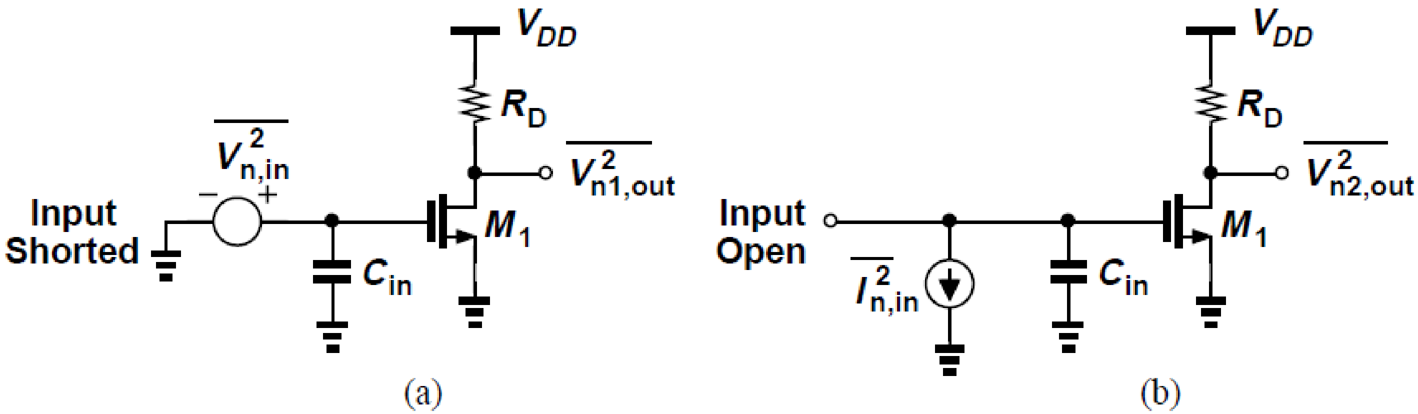
- For the circuit in Fig. (a), the input-referred noise voltage is simply

$$\overline{V^2_{n,in}} = 4kT\frac{\gamma}{g_m} + \frac{4kT}{g_m^2 R_D}$$

- To obtain the input-referred noise current, the input is left open and we find the output noise in terms of $\overline{I^2_{n,in}}$
- The noise current flows through C_{in}, generating at the output [Fig. (b)]

$$\overline{V^2_{n2,out}} = \overline{I^2_{n,in}}\left(\frac{1}{C_{in}\omega}\right)^2 g_m^2 R_D^2$$

Input-Referred Noise: Example



- **This value must be equal to the output of the noisy circuit when the input is left open [Fig. (b)]**

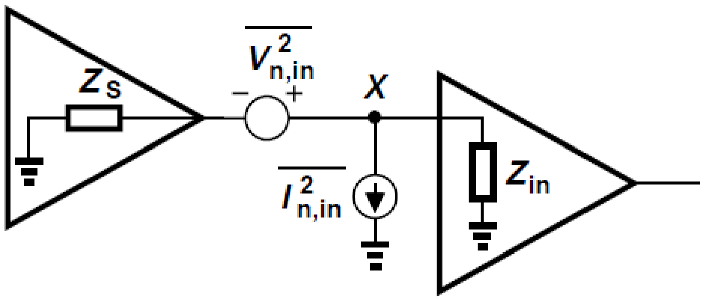
$$\overline{V^2_{n2,out}} = \left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2$$

- **Thus,**

$$\overline{I^2_{n,in}} = (C_{in}\omega)^2 \frac{4kT}{g_m^2}\left(\gamma g_m + \frac{1}{R_D}\right)$$

Input-Referred Noise

- The input noise current $I_{n,in}$, becomes significant for low enough values of the input impedance Z_{in}



- In above figure, Z_S denotes the output impedance of the preceding circuit; total noise voltage sensed by the second stage at node X is

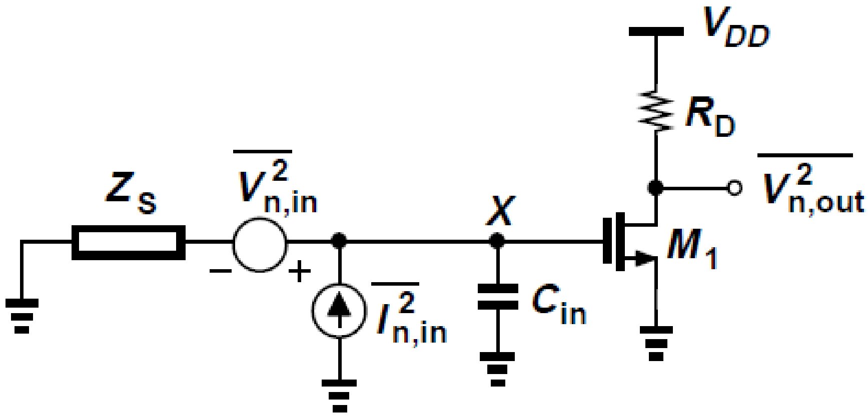
$$V_{n,X} = \frac{Z_{in}}{Z_{in} + Z_S} V_{n,in} + \frac{Z_{in} Z_S}{Z_{in} + Z_S} I_{n,in}$$

- If $\overline{I^2_{n,in}} |Z_S|^2 \ll \overline{V^2_{n,in}}$, the effect of $I_{n,in}$ is negligible
- Thus, the input-referred noise current can be neglected if

$$|Z_S|^2 \ll \frac{\overline{V^2_{n,in}}}{\overline{I^2_{n,in}}}$$

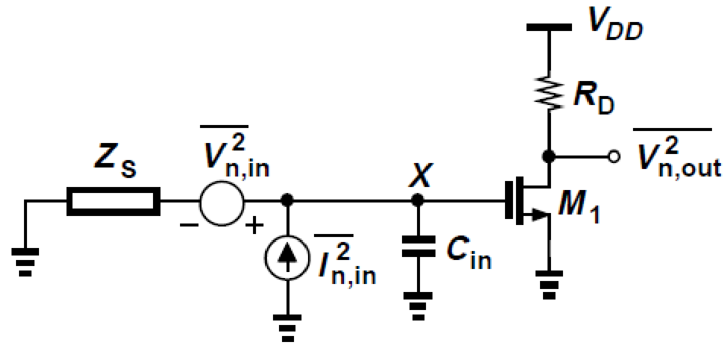
Input-Referred Noise: Correlation

- **Input-referred noise voltages and currents may be correlated**
- **Noise calculations must include correlations between the two**
- **Use of both a voltage source and a current source to represent the input-referred noise does not "count the noise twice"**



- **It can be proved that the output noise is correct for any source impedance Z_S, with both $\overline{V^2_{n,in}}$ and $\overline{I^2_{n,in}}$ included**

Input-Referred Noise: Correlation

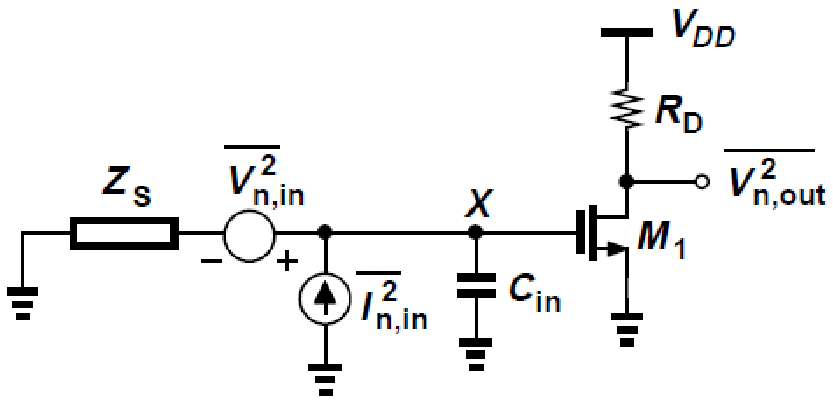


- Assuming Z_S is noiseless for simplicity, we first calculate the total noise voltage at the gate of M_1 due to $\overline{V^2_{n,in}}$ and $\overline{I^2_{n,in}}$
- Cannot apply superposition of powers since they are correlated, but can be applied to voltages and currents since the circuit is linear and time-invariant
- Therefore, if $V_{n,M1}$ denotes the gate-referred noise voltage of M_1 and $V_{n,RD}$ the noise voltage of R_D, then

$$V_{n,in} = V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD}$$

$$I_{n,in} = C_{in} s V_{n,M1} + \frac{C_{in} s}{g_m R_D} V_{n,RD}$$

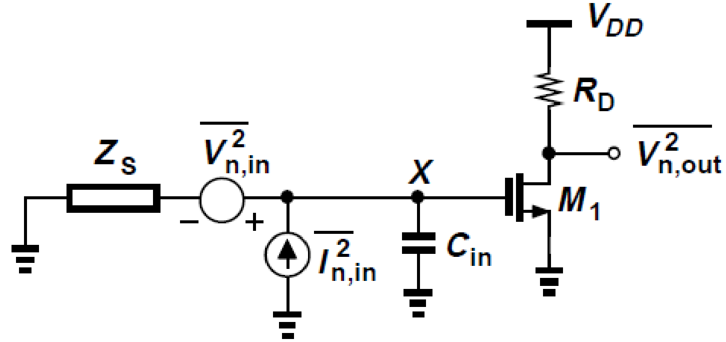
Input-Referred Noise: Correlation



- $V_{n,M1}$ and $V_{n,RD}$ appear in both $V_{n,in}$ and $I_{n,in}$, creating a strong correlation between the two
- Adding the contributions of $V_{n,in}$ and $I_{n,in}$ at node X, as if they were deterministic quantities, we have

$$
\begin{aligned}
V_{n,X} &= V_{n,in}\frac{\frac{1}{C_{in}s}}{\frac{1}{C_{in}s}+Z_S}+I_{n,in}\frac{\frac{Z_S}{C_{in}s}}{\frac{1}{C_{in}s}+Z_S} \\
&= \frac{V_{n,in}+I_{n,in}Z_S}{Z_SC_{in}s+1}.
\end{aligned}
$$

Input-Referred Noise: Correlation



- **Substituting for $V_{n,in}$ and $I_{n,in}$**

$$V_{n,X} = \frac{1}{Z_S C_{in} s + 1}\left[V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD} + C_{in} s Z_S \left(V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD}\right)\right]$$
$$= V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD}.$$

- **$V_{n,X}$ is indepen**
- **It follows that**

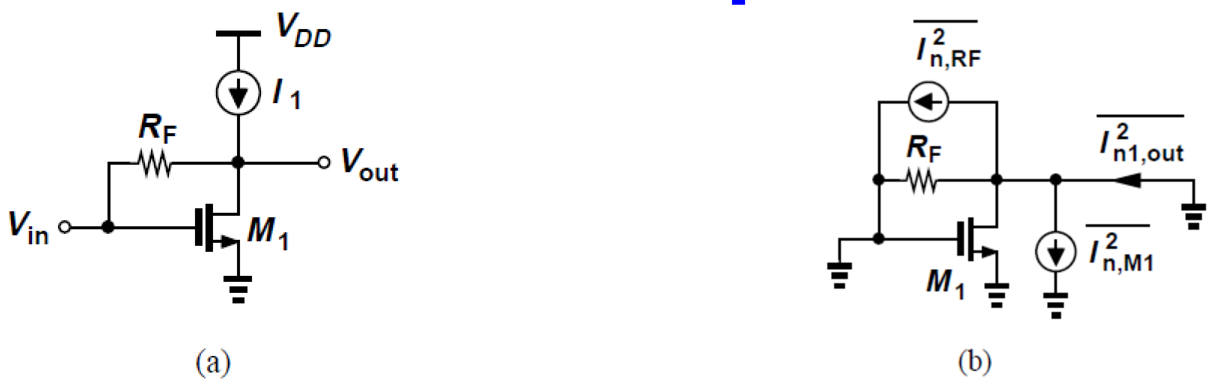
$$\overline{V^2_{n,out}} = g_m^2 R_D^2 \overline{V^2_{n,X}}$$
$$= 4kT\left(\gamma g_m + \frac{1}{R_D}\right) R_D^2$$

- **$V_{n,in}$ and $I_{n,in}$ do not double count the noise**

Input-Referred Noise: Correlation

- **In some cases, it is simpler to consider the output short-circuit noise *current*-rather than the output noise voltage**
- **This current is then multiplied by the circuit's output resistance to yield the output noise voltage or simply divided by a proper gain to give input-referred quantities**

Input-Referred Noise: Correlation (Example)



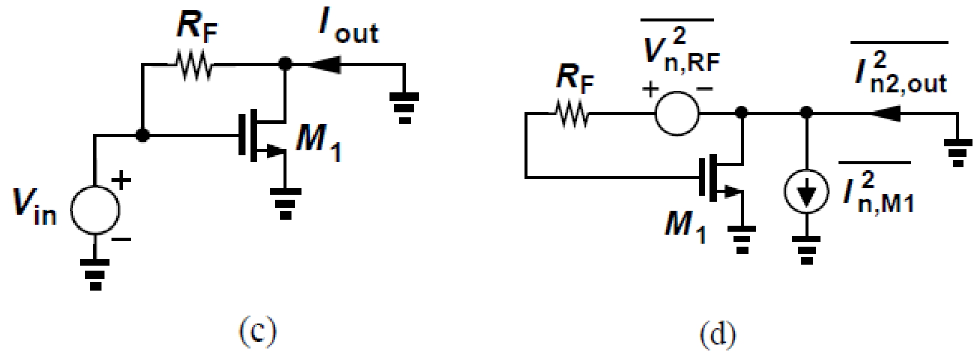
- <u>To find</u>: Input-referred noise voltage and current. Assume I_1 is noiseless and λ=0
- To compute the input-referred noise voltage, we short the input port [Fig. (b)]. Here, it is also possible to short the output port and hence

$$\overline{I^2_{n1,out}} = \frac{4kT}{R_F} + 4kT\gamma g_m$$

- The output impedance of the circuit with the input shorted is simply R_F, therefore,

$$\overline{V^2_{n1,out}} = \left(\frac{4kT}{R_F} + 4kT\gamma g_m\right) R_F^2$$

Input-Referred Noise: Correlation (Example)



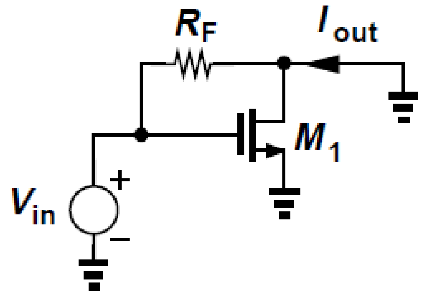
- **Input-referred noise voltage can be found by dividing previous equation by voltage gain or** $\overline{I^2_{n1,out}}$ **by** G_m^2
- **As shown in Fig. (c),**

$$G_m = \frac{I_{out}}{V_{in}} = g_m - \frac{1}{R_F}$$

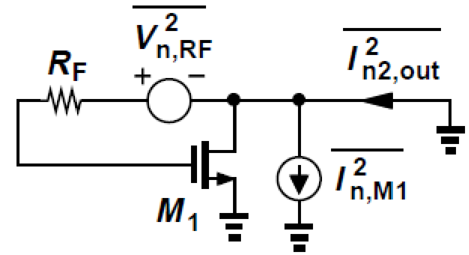
- **Therefore,**

$$\overline{V^2_{n,in}} = \frac{\frac{4kT}{R_F} + 4kT\gamma g_m}{(g_m - \frac{1}{R_F})^2}$$

Input-Referred Noise: Correlation (Example)



(c)



(d)

- **To find the input-referred noise current [Fig. (d)], we find the output noise current with the input left open**

$$\overline{I^2_{n2,out}} = 4kTR_F g_m^2 + 4kT\gamma g_m$$

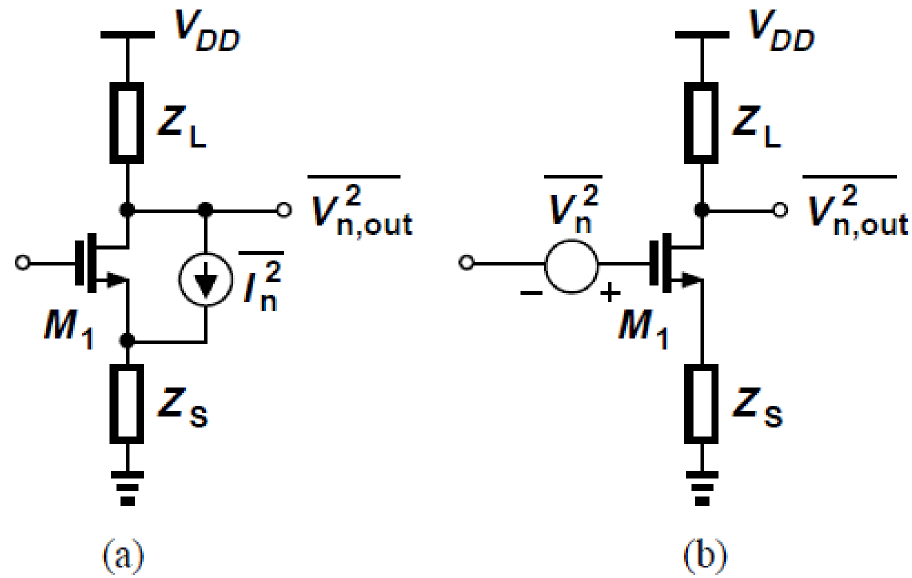
- **Next, we need to find the current gain of the circuit according to the arrangement in Fig. (c)**
- **Since,** $V_{GS} = I_{in}R_F$ **and** $I_D = g_m I_{in} R_F$**,**

$$\begin{aligned} I_{out} &= g_m R_F I_{in} - I_{in} \\ &= (g_m R_F - 1) I_{in} \end{aligned}$$

- **Thus,**

$$\overline{I^2_{n,in}} = \frac{4kTR_F g_m^2 + 4kT\gamma g_m}{(g_m R_F - 1)^2}$$

Noise in Single-Stage Amplifiers: Lemma



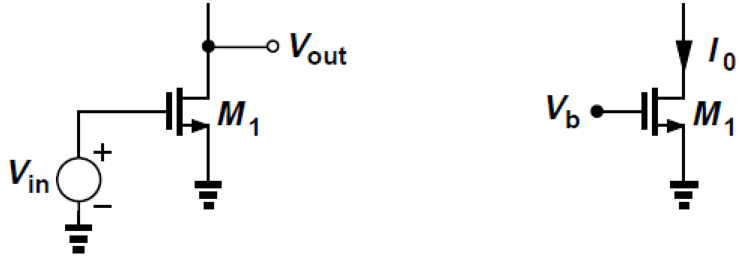
- **<u>Lemma</u>: The circuits in Fig. (a) and (b) are equivalent at low frequencies if $\overline{V_n^2} = \overline{I_n^2}/g_m^2$ and the circuits are driven by a finite impedance**
- **The noise source can be transformed from a drain-source current to a gate series voltage for arbitrary Z_S**

Common-Source Stage

- From a previous example, the input-referred noise voltage of a simple CS stage was found to be

$$\overline{V_{n,in}^2} = 4kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{K}{C_{ox}WL}\frac{1}{f}$$

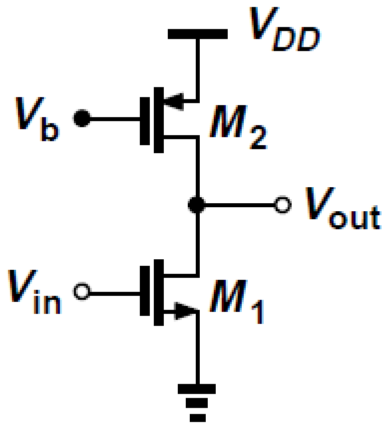
- From above expression, *4kTγ/g_m* is the thermal noise current expressed as a voltage in series with the gate



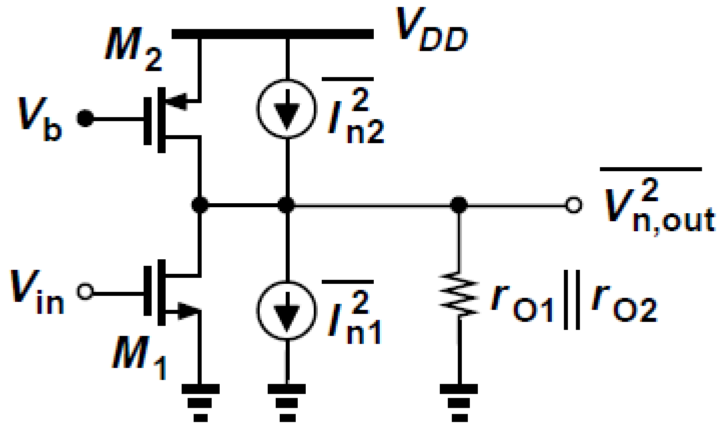
(a) (b)

- To reduce input-referred noise voltage, transconductance must be maximized if the transistor is to amplify a voltage signal applied to its gate [Fig. (a)] whereas it must be minimized if operating as a current source [Fig. (b)]

Common-Source Stage: Example



(a)

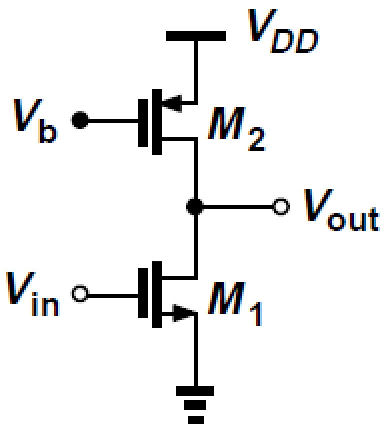


(b)

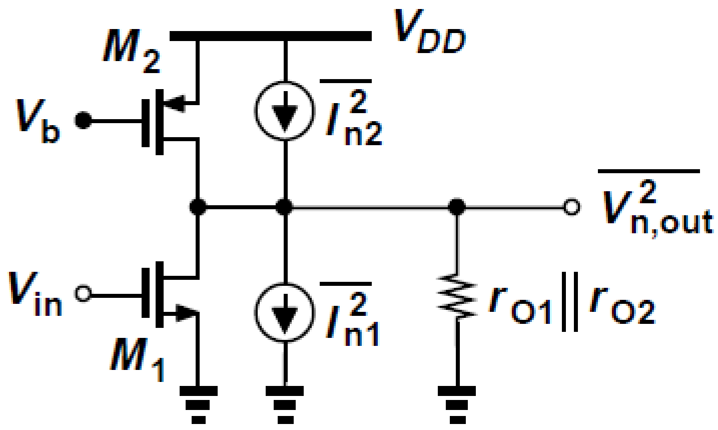
- **<u>To find</u>: Input-referred thermal noise, total output thermal noise with a load capacitance C_L**
- **Representing the thermal noise of M_1 and M_2 by current sources and noting that they are uncorrelated,**

$$\overline{V_{n,out}^2} = 4kT(\gamma g_{m1} + \gamma g_{m2})(r_{O1}||r_{O2})^2$$

Common-Source Stage: Example



(a)



(b)

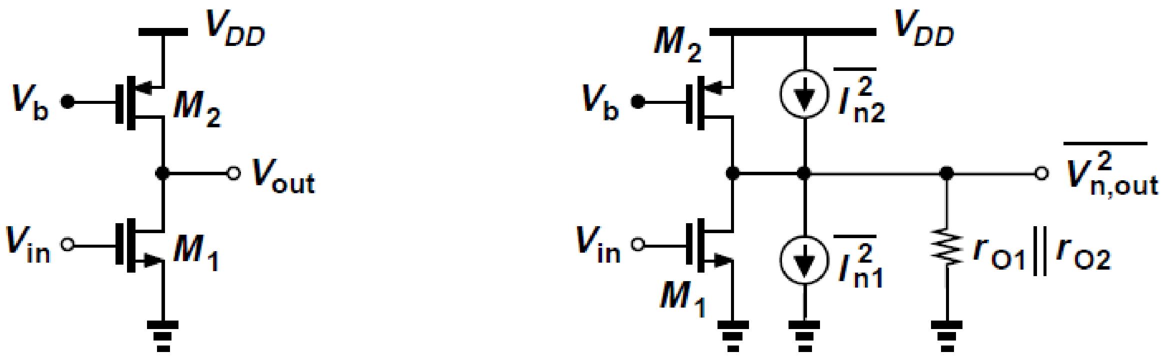
- **Since the voltage gain is equal to $g_{m1}(r_{O1}||r_{O2})$, total noise voltage referred to the gate of *M_1* is**

$$\overline{V_{n,in}^2} = 4kT(\gamma g_{m1} + \gamma g_{m2})\frac{1}{g_{m1}^2}$$

$$= 4kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right).$$

- **Thus, g_{m2} must be minimized because M_2 serves as a current source rather than a transconductor**

Common-Source Stage: Example



(a) (b)

- **Total output noise is**

$$\overline{V_{n,out,tot}^2} = \int_0^\infty 4kT\gamma(g_{m1}+g_{m2})(r_{O1}||r_{O2})^2 \frac{df}{1+(r_{O1}||r_{O2})^2 C_L^2 (2\pi f)^2}$$

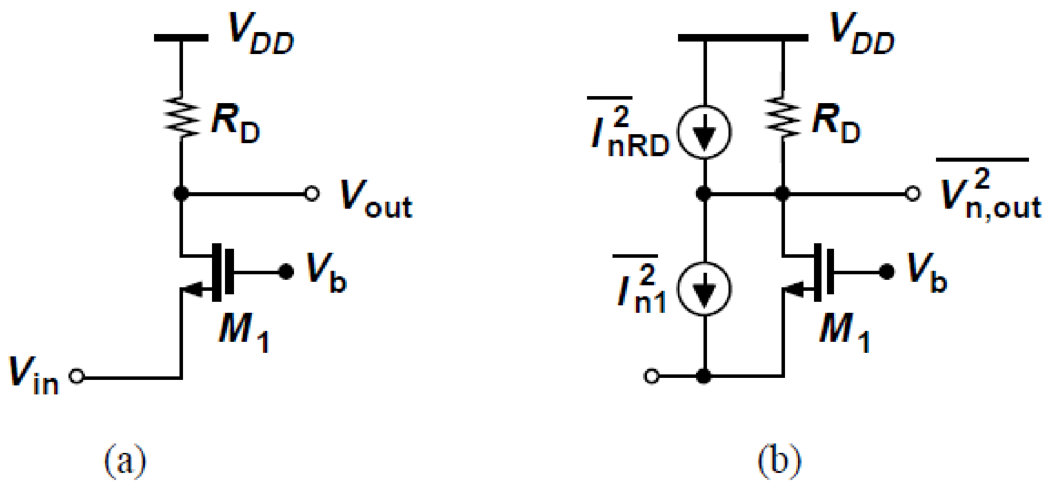
$$\overline{V_{n,out,tot}^2} = \gamma(g_{m1}+g_{m2})(r_{O1}||r_{O2})\frac{kT}{C_L}$$

- **A low-frequency input sinusoid of amplitude V_m yields an output amplitude equal to $g_{m1}(r_{O1}||r_{O2})V_m$ output SNR of**

$$SNR_{out} = \left[\frac{g_{m1}(r_{O1}||r_{O2})V_m}{\sqrt{2}}\right]^2 \cdot \frac{1}{\gamma(g_{m1}+g_{m2})(r_{O1}||r_{O2})(kT/C_L)}$$

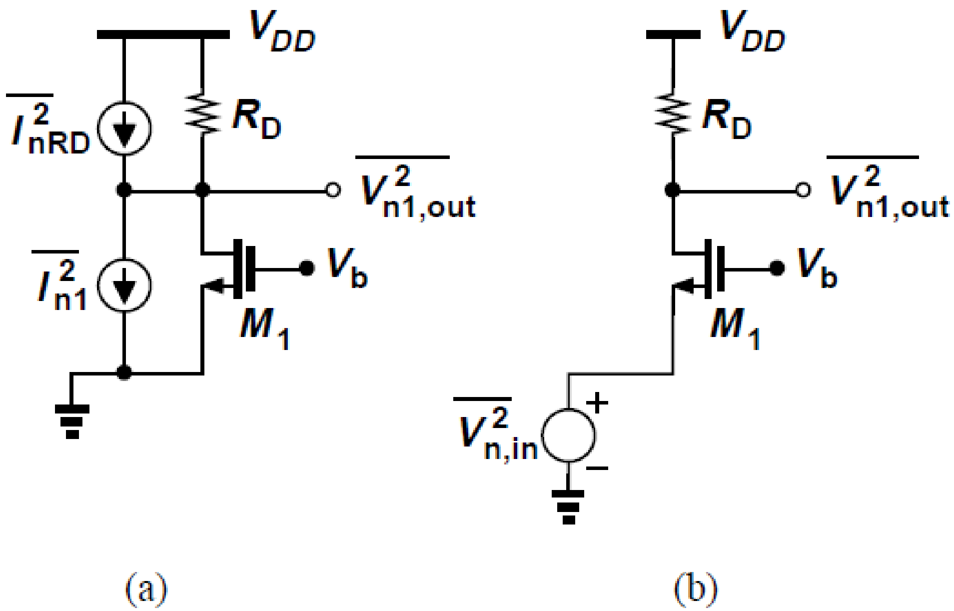
$$= \frac{C_L}{2\gamma kT}\cdot\frac{g_{m1}^2(r_{O1}||r_{O2})}{g_{m1}+g_{m2}}V_m^2.$$

Common-Gate Stage: Thermal Noise



- Neglecting channel-length modulation, we represent the thermal noise of M_1 and R_D by two current sources [Fig. (b)]
- Due to low input impedance of the circuit, the input-referred noise current is not negligible even at low frequencies

Common-Gate Stage: Thermal Noise

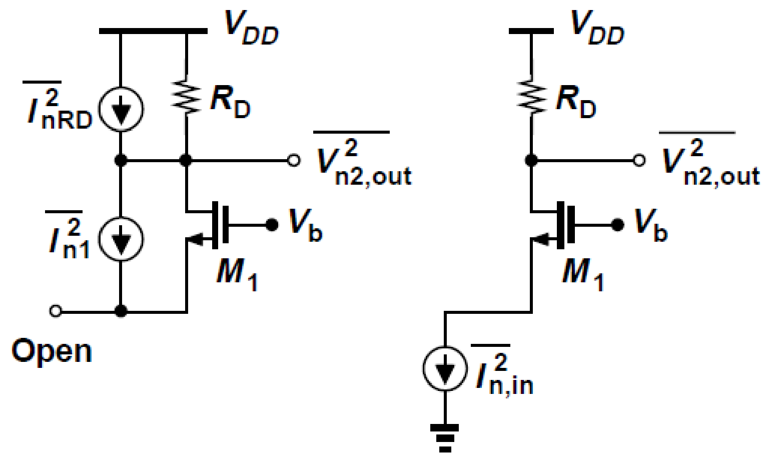


- **To calculate the input-referred noise voltage, we short the input to ground and equate the output noises of the circuits in Figs. (a) and (b)**

$$\left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2 = \overline{V^2_{n,in}}(g_m + g_{mb})^2 R_D^2$$

$$\overline{V^2_{n,in}} = \frac{4kT(\gamma g_m + 1/R_D)}{(g_m + g_{mb})^2}$$

Common-Gate Stage: Thermal Noise

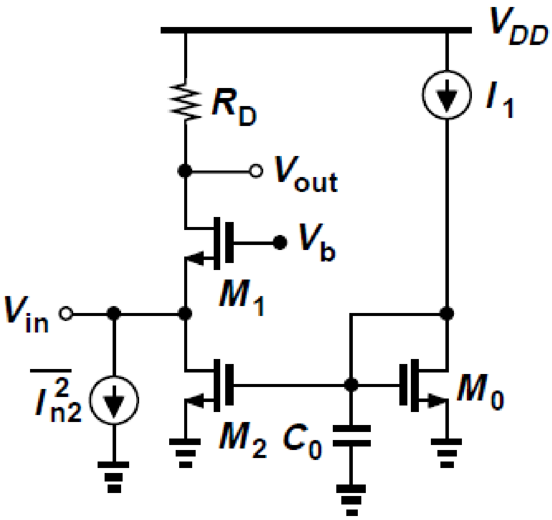


(c) (d)

- To calculate the input-referred noise current, we equate the output noises of the circuits in Figs. (c) and (d)
- I_{n1} produces no noise at the output since the sum of the currents at the source of M_1 is zero
- The output noise voltage is therefore $4kTR_D$ and hence $\overline{I^2_{n,in}}R^2_D = 4kTR_D$
- Thus,

$$\overline{I^2_{n,in}} = \frac{4kT}{R_D}$$

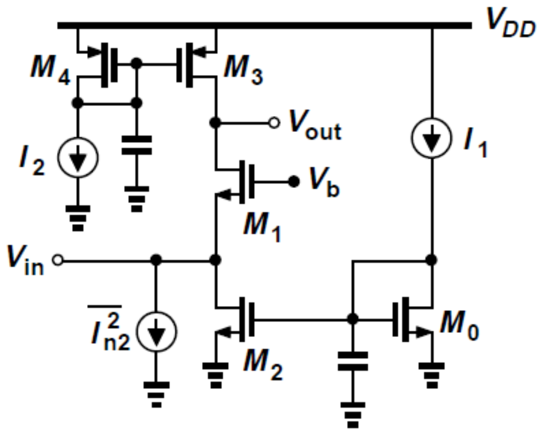
Common-Gate Stage: Thermal Noise

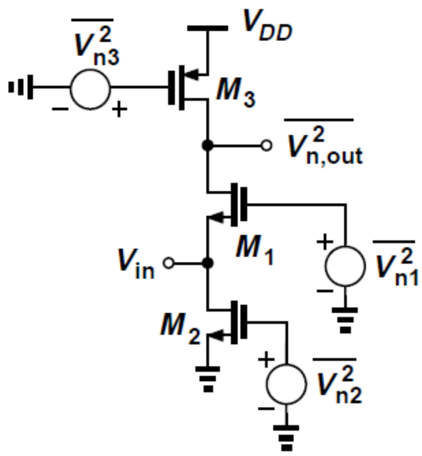


- Bias current source in the common-gate stage also contributes thermal noise
- Current mirror arrangement establishes bias current of M_1 as a multiple of I_1

- If input is shorted to ground, drain noise current of M_2 does not contribute to input-referred noise voltage
- If input is open, all of $\overline{I_{n2}^2}$ flows from M_1 and R_D, producing an output noise of $\overline{I_{n2}^2}R_D^2$ and hence an input-referred noise current of $\overline{I_{n2}^2}$
- It is desirable to minimize transconductance of M_2, at the cost of reduced output swing

Common-Gate Stage: Flicker Noise



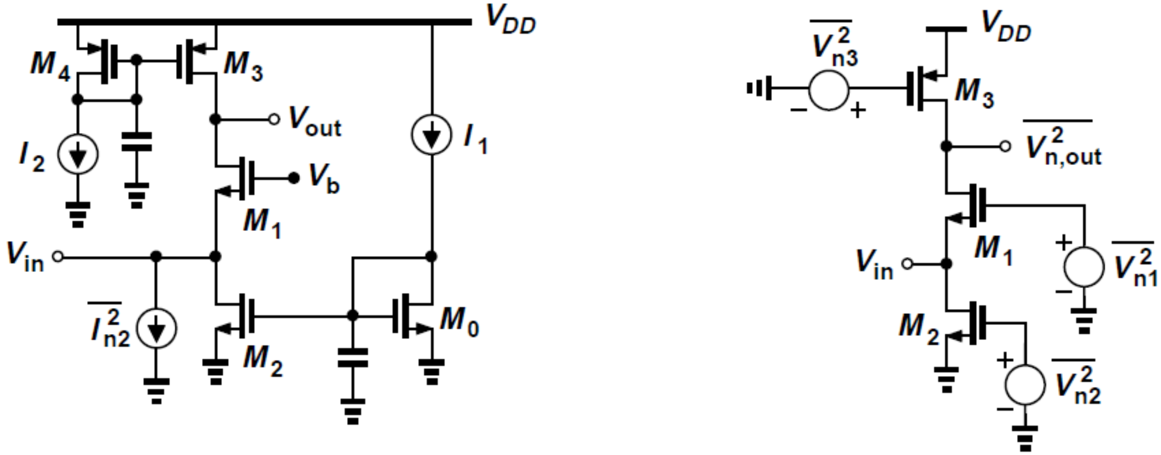


- **To compute input-referred 1/*f* noise voltage and current of above circuit, each 1/*f* noise generator is modeled by a voltage source in series with the gate of the corresponding transistor (1/*f* noise of M_0 and M_4 is neglected)**
- **With the input shorted to ground, we have**

$$\overline{V^2_{n1,out}} = \frac{1}{C_{ox} f}\left[\frac{g^2_{m1} K_N}{(WL)_1} + \frac{g^2_{m3} K_P}{(WL)_3}\right](r_{O1} \| r_{O3})^2$$

- **K_P and K_N denote flicker noise coefficients of NMOS and PMOS devices respectively**

Common-Gate Stage: Flicker Noise



- **Approximating the voltage gain as** $(g_{m1} + g_{mb1})(r_{O1}||r_{O3})$**,**

$$\overline{V^2_{n,in}} = \frac{1}{C_{ox}f}\left[\frac{g^2_{m1}K_N}{(WL)_1} + \frac{g^2_{m3}K_P}{(WL)_3}\right]\frac{1}{(g_{m1}+g_{mb1})^2}$$

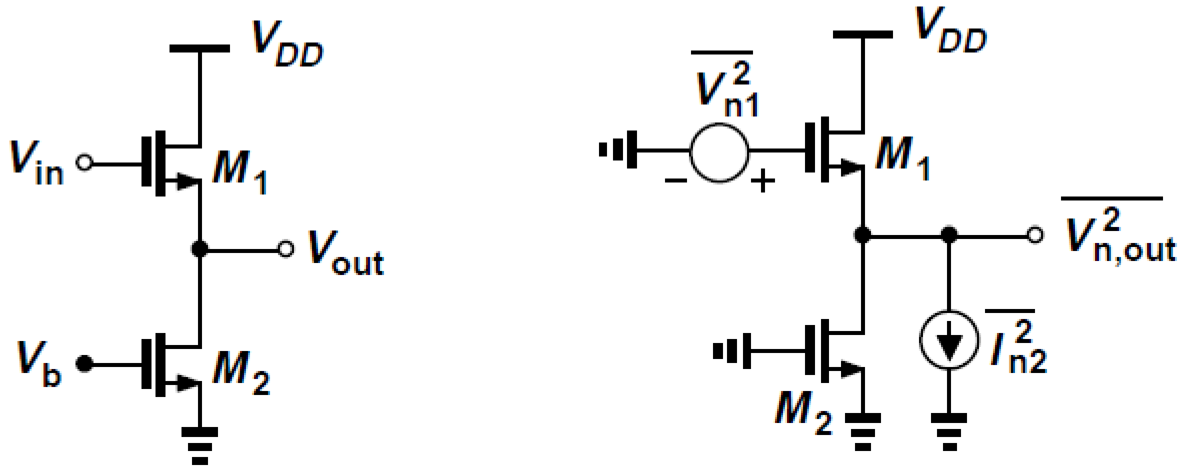
- **With the input open, the output noise is approximately**

$$\overline{V^2_{n2,out}} = \frac{1}{C_{ox}f}\left[\frac{g^2_{m2}K_N}{(WL)_2} + \frac{g^2_{m3}K_P}{(WL)_3}\right]R^2_{out}$$

- **It follows that**

$$\overline{I^2_{n,in}} = \frac{1}{C_{ox}f}\left[\frac{g^2_{m2}K_N}{(WL)_2} + \frac{g^2_{m3}K_P}{(WL)_3}\right]$$

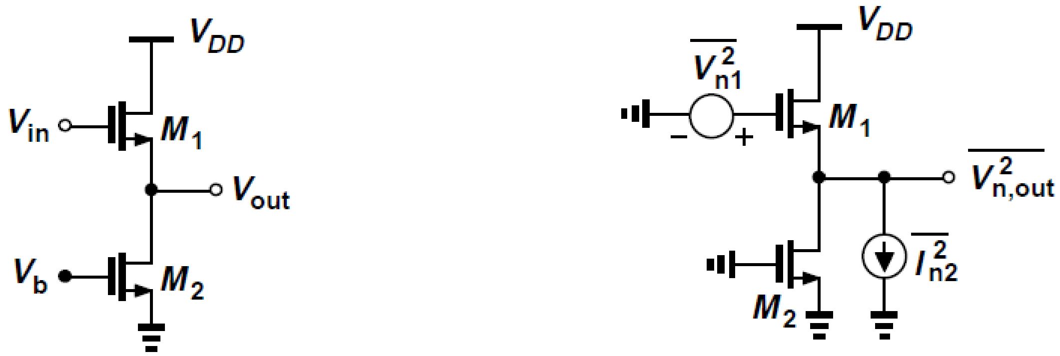
Source Followers: Thermal Noise



- **Since the input impedance of the source follower is quite high, the input-referred noise current can be neglected for moderate driving source impedances**
- **To compute the input-referred noise voltage, the output noise of *M_2* can be expressed as**

$$\overline{V_{n,out}^2}|_{M2} = \overline{I_{n2}^2}\left(\frac{1}{g_{m1}}\middle\|\frac{1}{g_{mb1}}\middle\|r_{O1}\|r_{O2}\right)^2$$

Source Followers: Thermal Noise



- **The voltage gain is (from Chapter 3),**

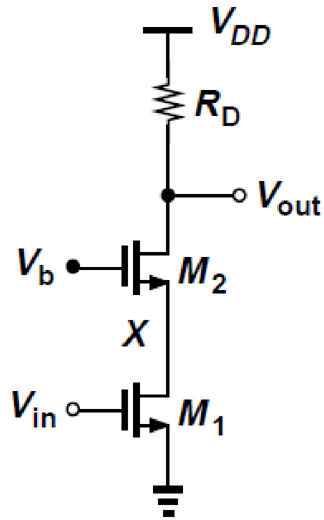
$$A_v = \frac{\frac{1}{g_{mb1}} \| r_{O1} \| r_{O2}}{\frac{1}{g_{mb1}} \| r_{O1} \| r_{O2} + \frac{1}{g_{m1}}}$$

- **Total input-referred noise voltage is**

$$\overline{V_{n,in}^2} = \overline{V_{n1}^2} + \frac{\overline{V_{n,out}^2}|_{M2}}{A_v^2}$$
$$= 4kT\gamma\left(\frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2}\right)$$

- **Source followers add noise to the input signal and provide a voltage gain less than unity**

Cascode Stage

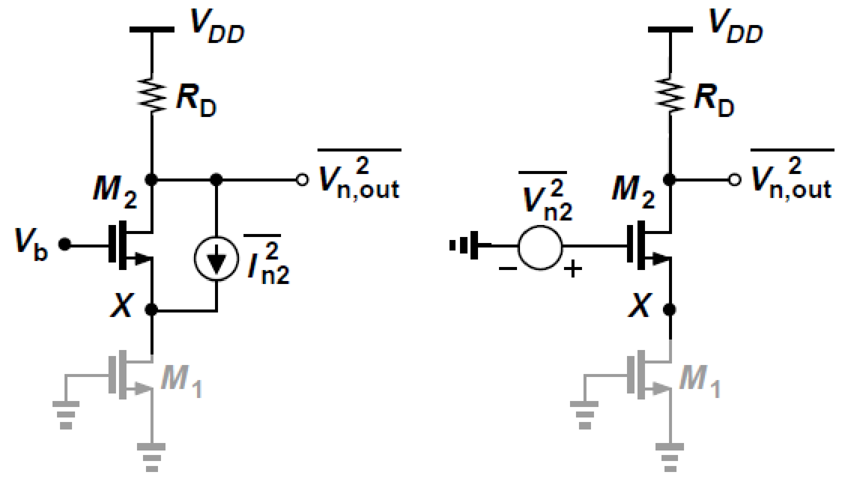


(a)

- In the cascode stage of Fig. (a), the noise currents of M_1 and R_D flow mostly through R_D at low frequencies
- Noise contributed by M_1 and R_D is quantified in a common-source stage

$$\overline{V^2_{n,in}}|_{M1,RD} = 4kT\left(\frac{\gamma}{g_{m1}} + \frac{1}{g^2_{m1}R_D}\right)$$

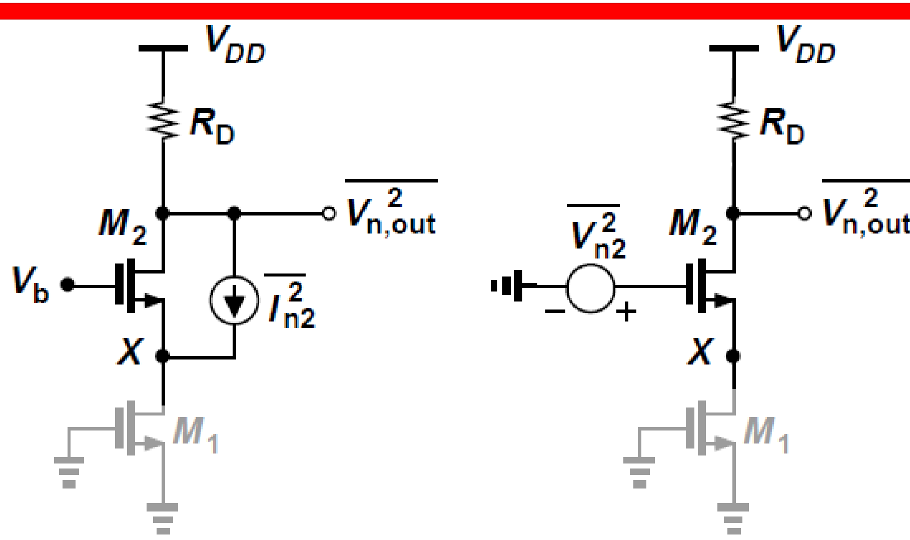
Cascode Stage



(b) (c)

- As shown in Fig. (b), M_2 contributes negligibly to noise at the output, especially at low frequencies
- If channel-length modulation of M_1 is neglected, then $I_{n2} + I_{D2} = 0$ and hence M_2 does not affect $V_{n,out}$
- From another perspective, in the equivalent circuit of Fig. (c), voltage gain from V_{n2} to the output is small if impedance at node X is large

Cascode Stage



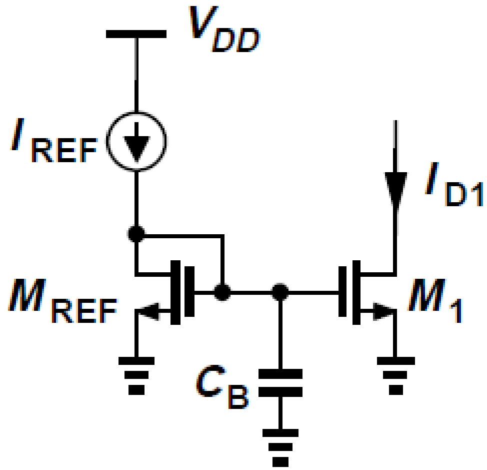
(b) (c)

- **At high frequencies, the total capacitance at node *X*, C_X gives rise to a gain, increasing the output noise**

$$\frac{V_{n,out}}{V_{n2}} \approx \frac{-R_D}{1/g_{m2} + 1/(C_X s)}$$

- **This capacitance also reduces the gain from the main input to the output by shunting the signal current produced by M_1 to ground**

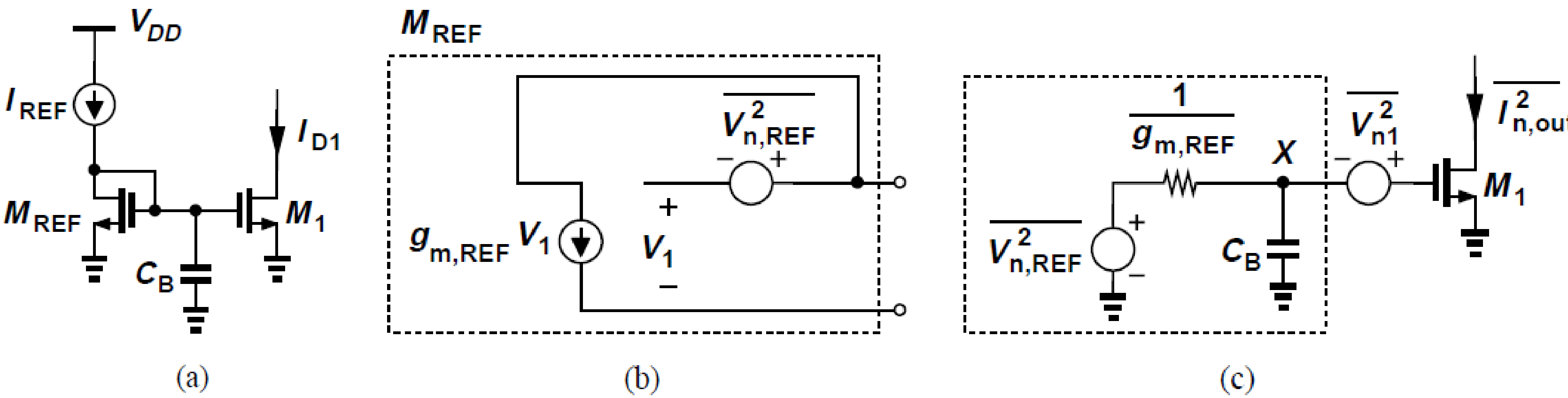
Noise in Current Mirrors



(a)

- **In the above current-mirror topology,** $(W/L)_1 = N(W/L)_{REF}$
- **The factor *N* is in the range of 5 to 10 to minimize power consumed by reference branch**
- **To determine the flicker noise in I_{D1}, we assume $\lambda = 0$ and I_{REF} is noiseless**

Noise in Current Mirrors

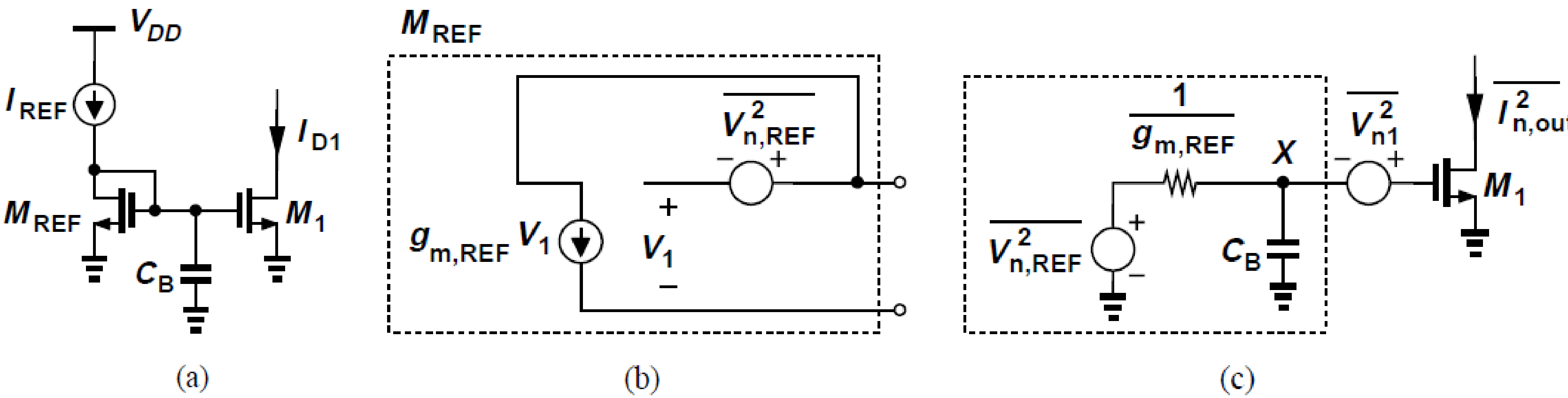


- In the Thevenin equivalent for M_{REF} and its flicker noise, the open-circuit voltage is $V_{n,REF}$ [Fig. (b)] and the Thevenin resistance is $1/g_{m,REF}$ [Fig. (c)]
- The noise voltage at node X and V_{n1} add and drive the gate of M_1 producing

$$\overline{I^2_{n,out}} = \left(\frac{g^2_{m,REF}}{C_B^2\omega^2 + g^2_{m,REF}}\overline{V^2_{n,REF}} + \overline{V^2_{n1}}\right) g^2_{m1}$$

- Since $(W/L)_1 = N(W/L)_{REF}$ and typically $L_1 = L_{REF}$, we observe that $\overline{V^2_{n,REF}} = N\overline{V^2_{n1}}$

Noise in Current Mirrors



- **It follows that**

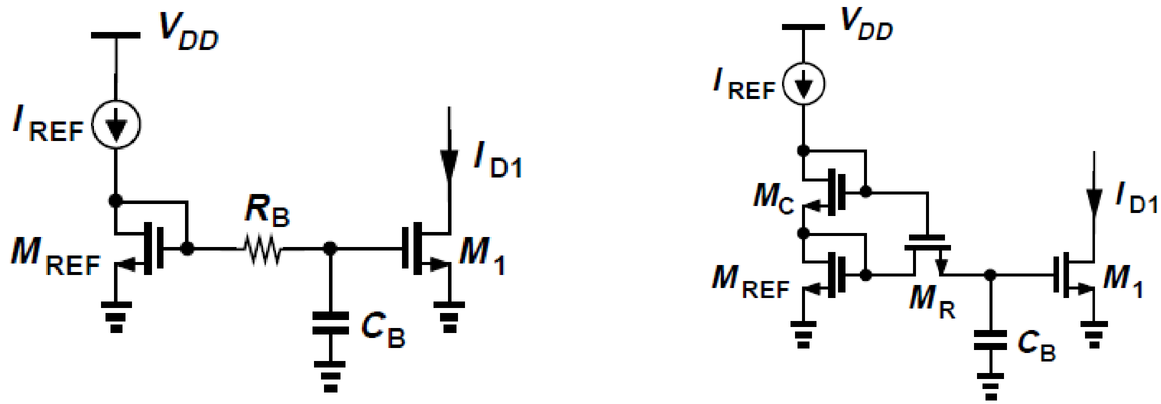
$$\overline{I^2_{n,out}} = \left(\frac{N g^2_{m,REF}}{C_B^2\omega^2 + g^2_{m,REF}} + 1 \right) g^2_{m1}\overline{V^2_{n1}}.$$

- **For the noise of the diode-connected device to be small, we need** $(N-1)g^2_{m,REF} \ll C_B^2\omega^2$ **and hence**

$$C_B^2 \gg \frac{(N-1)g^2_{m,REF}}{\omega^2}$$

- **This can lead to C_B being very high**

Noise in Current Mirrors



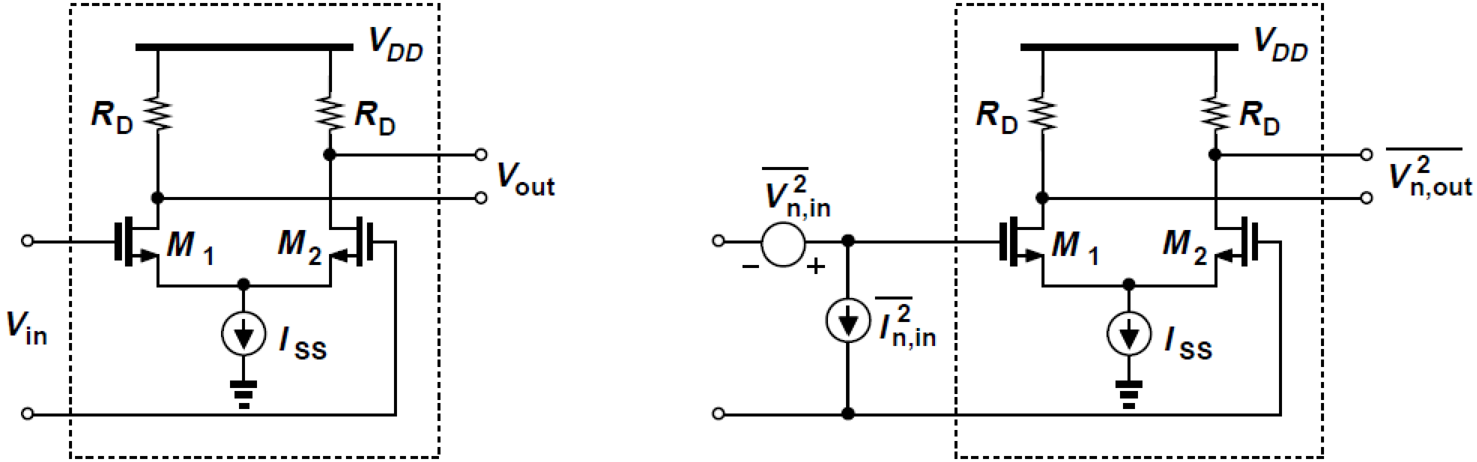
(a) (b)

- In order to reduce noise contributed by M_{REF} and avoid a large capacitor, we can insert a resistance between its gate and C_B, so that

$$\overline{I_{n,out}^2} = \left[\frac{g_{m,REF}^2}{(1+g_{m,REF}R_B)^2 C_B^2 \omega^2 + g_{m,REF}^2} (\overline{V_{n,REF}^2} + \overline{V_{n,RB}^2}) + \overline{V_{n1}^2} \right] g_{m1}^2$$

- R_B lowers the filter cutoff frequency but also contributes its own noise
- The MOS device M_R with a small overdrive provides a high resistance and occupies a moderate area [Fig. (b)]

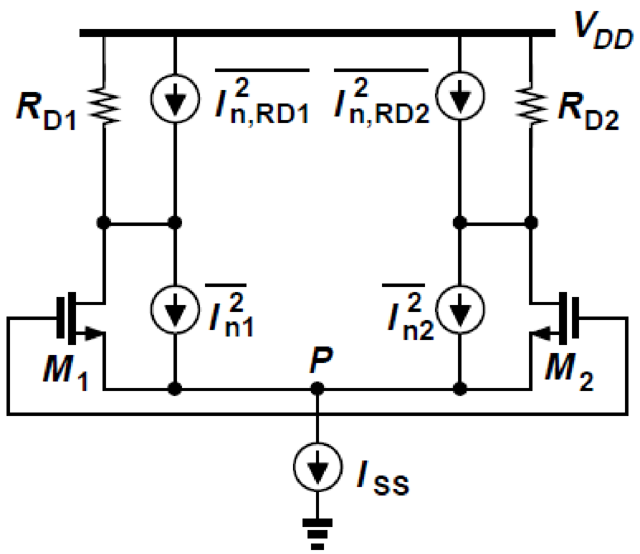
Noise in Differential Pairs



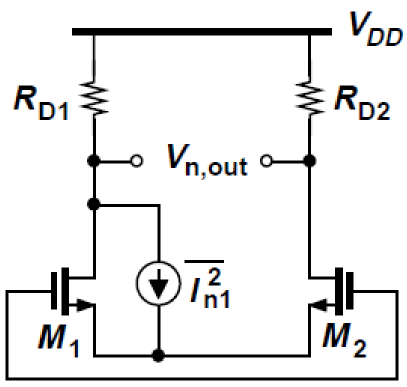
(a) (b)

- **As shown in Fig. (a), a differential pair can be viewed as a two-port circuit**
- **It is possible to model the overall noise as depicted in Fig. (b)**
- **For low-frequency operation, $\overline{I^2_{n,in}}$ is negligible**

Noise in Differential Pairs



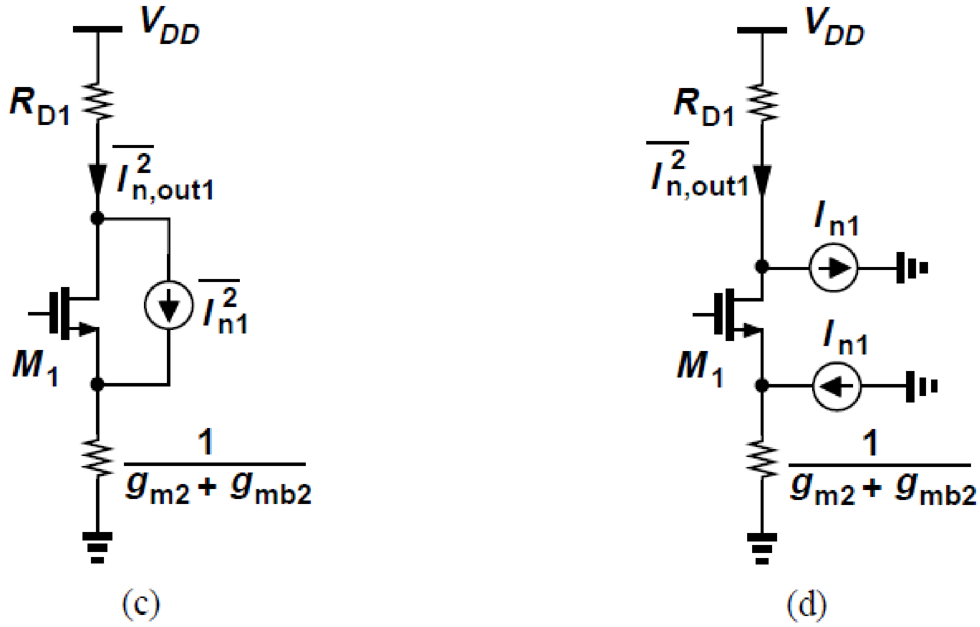
(a)



(b)

- **To calculate the thermal component $\overline{V^2_{n,in}}$, we first obtain the total output noise with the inputs shorted together [Fig. (a)]**
- **Since I_{n1} and I_{n2} are uncorrelated, node *P* cannot be considered a virtual ground, so cannot use half-circuit concept**
- **Derive contribution of each source individually [Fig. (b)]**

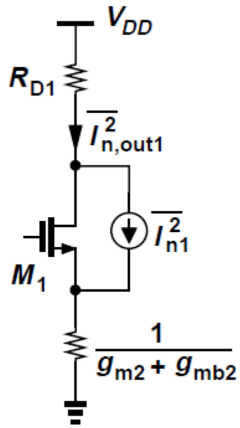
Noise in Differential Pairs



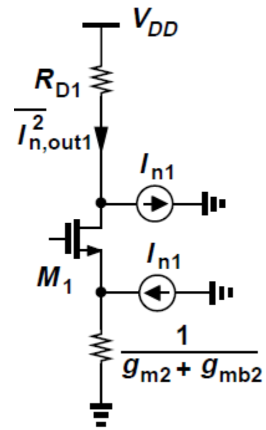
- **In Fig. (c), to calculate the contribution of I_{n1}, neglecting channel-length modulation, it can be proved that half of I_{n1} flows through R_{D1} and the other half through M_2 and R_{D2} [Fig. (d)]**
- **The differential output noise due to M_1 is equal to**

$$V_{n,out}|_{M1} = \frac{I_{n1}}{2} R_{D1} + \frac{I_{n1}}{2} R_{D2}$$

Noise in Differential Pairs



(c)



(d)

- If $R_{D1} = R_{D2} = R_D$,

$$\overline{V^2_{n,out}}|_{M1} = \overline{I^2_{n1}} R^2_D \qquad \overline{V^2_{n,out}}|_{M2} = \overline{I^2_{n2}} R^2_D$$

$$\overline{V^2_{n,out}}|_{M1,M2} = (\overline{I^2_{n1}} + \overline{I^2_{n2}}) R^2_D$$

- Thus
- Taking into a[ccount] R_{D2},

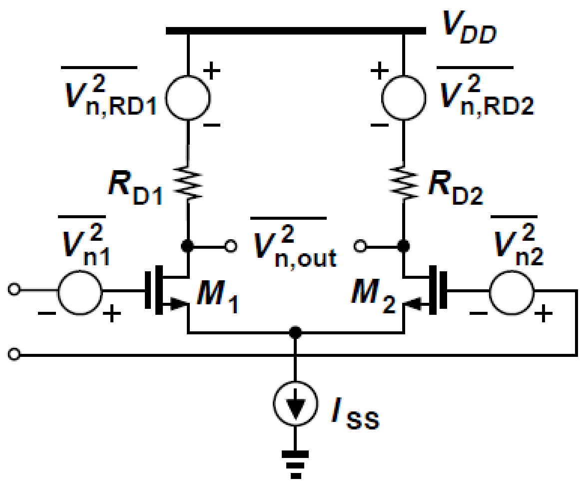
$$\overline{V^2_{n,out}} = \left(\overline{I^2_{n1}} + \overline{I^2_{n2}}\right) R^2_D + 2(4kTR_D)$$

$$= 8kT\left(\gamma g_m R^2_D + R_D\right).$$

- Diving by the squ[are of the differenti]al gain $g^2_m R^2_D$,

$$\overline{V^2_{n,in}} = 8kT\left(\frac{\gamma}{g_m} + \frac{1}{g^2_m R_D}\right)$$

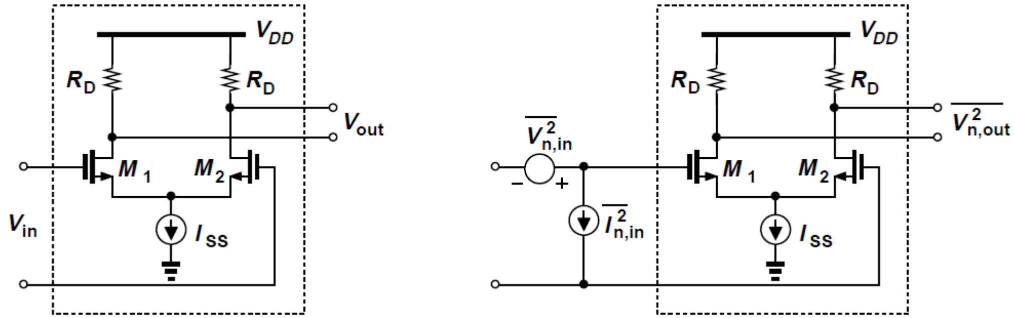
Noise in Differential Pairs



- **Input-referred noise voltage can also be calculated using the previous lemma**
- **The noise of M_1 and M_2 can be modeled as a voltage source in series with their gates**
- **The noise of R_{D1} and R_{D2} is divided by $g_m^2 R_D^2$ resulting in previously obtained equation**
- **Including 1/*f* noise.**

$$\overline{V^2_{n,in,tot}} = 8kT\left(\frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D}\right) + \frac{2K}{C_{ox}WL}\frac{1}{f}$$

Noise in Differential Pairs



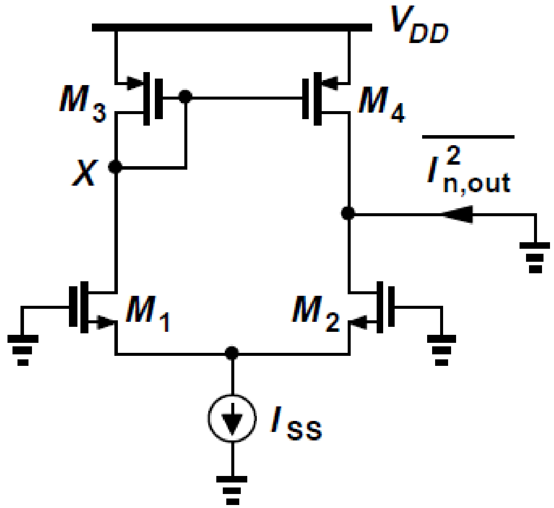
(a) (b)

- **If the differential input is zero and the circuit is symmetric, then the noise in I_{SS} divides equally between M_1 and M_2, and produces only a common-mode noise voltage at the output**
- **For a small differential input ΔV_{in}, we have**

$$\begin{aligned}\Delta I_{D1} - \Delta I_{D2} &= g_m \Delta V_{in} \\ &= \sqrt{2\mu_n C_{ox}\frac{W}{L}(\frac{I_{SS}+I_n}{2})}\Delta V_{in}\end{aligned}$$

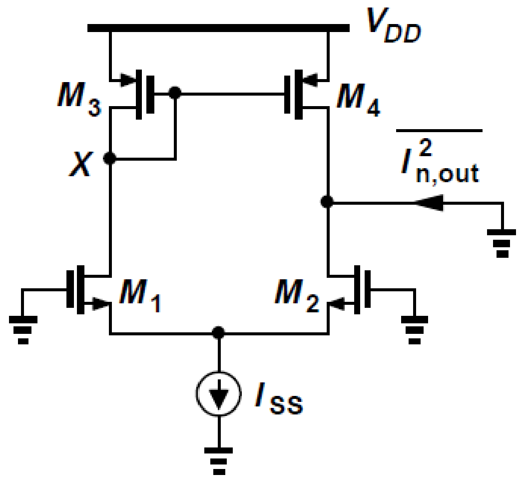
- **I_n denotes the noise in I_{SS} and I_n << I_{SS}**
- **As circuit departs from equilibrium, In is more unevenly divided, generating differential output noise**

Noise in five-transistor OTA



- **The Norton noise equivalent is sought by first computing the output short-circuit noise current**
- **This is then multiplied by the output resistance and divided by the gain to get input-referred noise voltage**
- **Transconductance is approximately $g_{m1,2}$**
- **Output noise current due to M_1 and M_2 is this transconductance multiplied by gate-referred noises of M_1 and M_2, i.e.,** $g_{m1,2}^2(4kT\gamma/g_{m1} + 4kT\gamma/g_{m2})$

Noise in five-transistor OTA



- The noise current of M_3 primarily circulates through the diode-connected impedance $1/g_{m3}$, producing a voltage at the gate of M_4 with spectral density $4kT\gamma/g_{m3}$

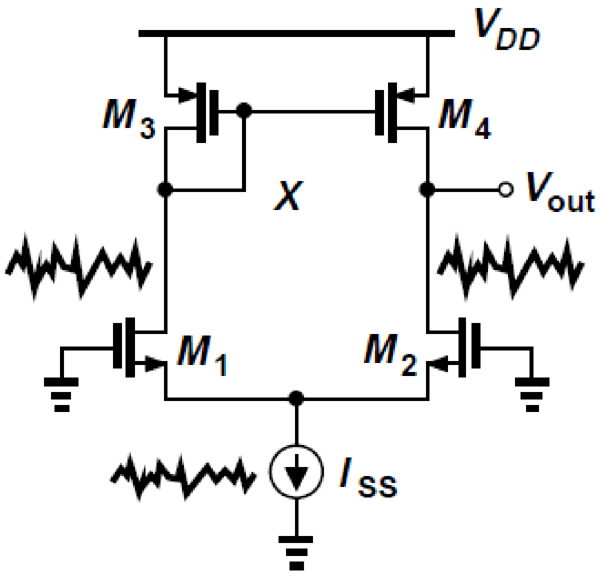
- This noise is multiplied by g_{m4}^2 as it emerges from the drain of M_4; the noise current of M_4 also flows directly through the output short-circuit, thus

$$\overline{I_{n,out}^2} = 4kT\gamma(2g_{m1,2} + 2g_{m3,4})$$

- Multiplying this noise by $R_{out}^2 \approx (r_{O1,2}||r_{O3,4})^2$ and dividing the result by $A_v^2 = G_m^2 R_{out}^2$, the total input-referred noise is

$$\overline{V_{n,in}^2} = 8kT\gamma\left(\frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2}\right)$$

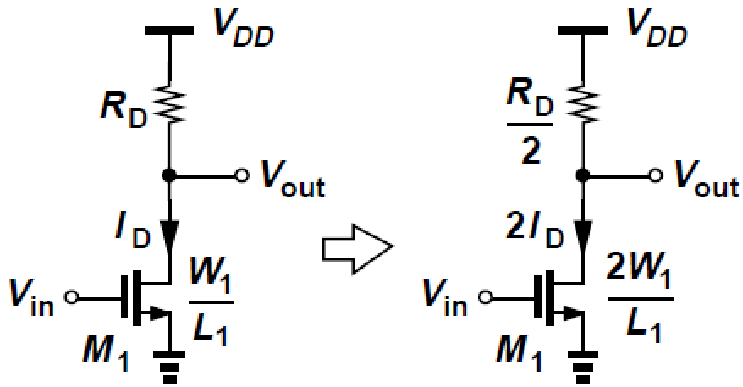
Noise in five-transistor OTA



- The output voltage in the OTA V_{out} is equal to V_X
- If I_{SS} fluctuates, so do V_X and V_{out}
- Since the tail noise current In splits equally between M_1 and M_2, the noise voltage at X is given by $\overline{I_n^2}/(4g_{m3}^2)$ and so is the noise voltage at the output

Noise-Power Tradeoff

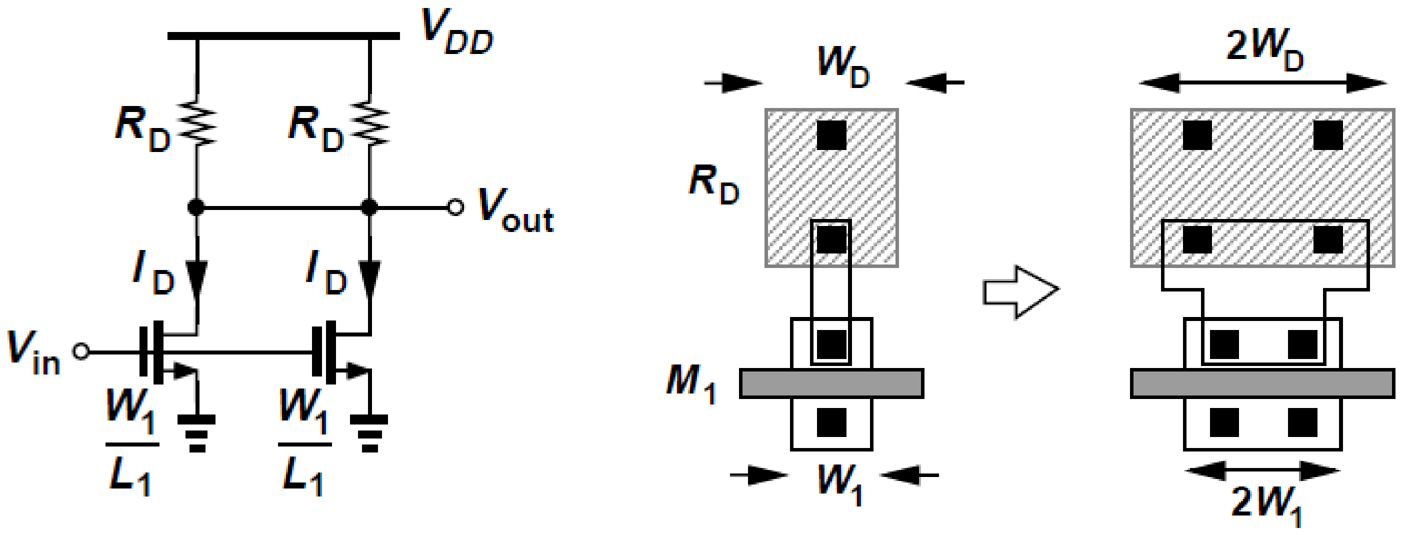
- Noise contributed by transistors “in the signal path” is inversely proportional to their transconductance
  - Suggests a tradeoff between noise and power consumption



(a)

- In the simple CS stage of Fig.(a), we double *W/L* and I_{D1} and halve R_D, maintaining voltage gain and output swing
- Input-referred thermal and flicker noise power is halved, at the cost of power consumption

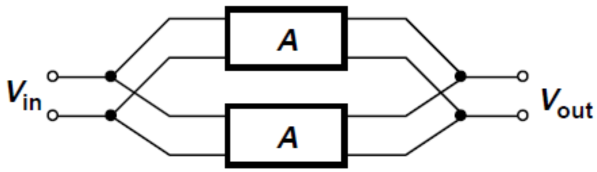
Noise-Power Tradeoff



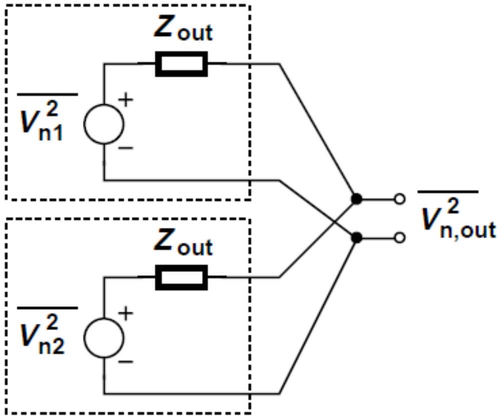
(b) (c)

- Called “linear scaling”, the earlier transformation can be viewed as placing two instances of the original circuit in *parallel* [Fig. (b)]
- Alternatively, it can be said that the widths of the transistor and the resistor are doubled [Fig. (c)]

Noise-Power Tradeoff



(a)



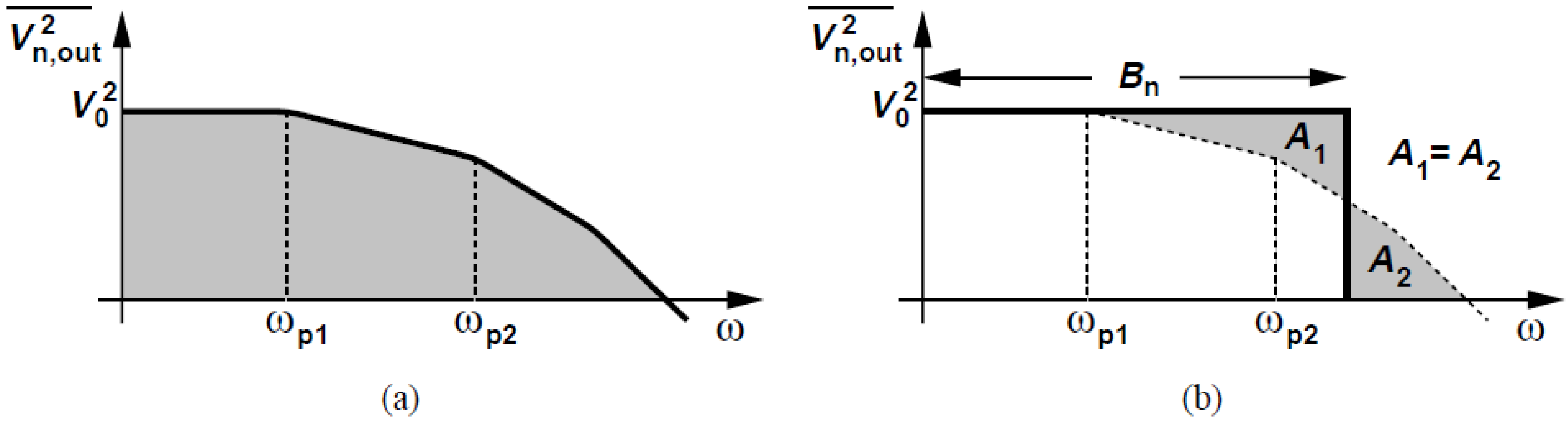
(b)

- **In general, if two instances of a circuit are placed in parallel, the output noise power is halved [Fig. (a)]**
- **Proved by setting the input to zero and constructing a Thevenin equivalent for each [Fig. (b)]**
- **Since $V_{n1,out}$ and $V_{n2,out}$ are uncorrelated, we can use superposition of powers to write**

$$\overline{V_{n,out}^2} = \frac{\overline{V_{n1,out}^2}}{4} + \frac{\overline{V_{n2,out}^2}}{4}$$

$$= \frac{\overline{V_{n1,out}^2}}{2}.$$

Noise Bandwidth



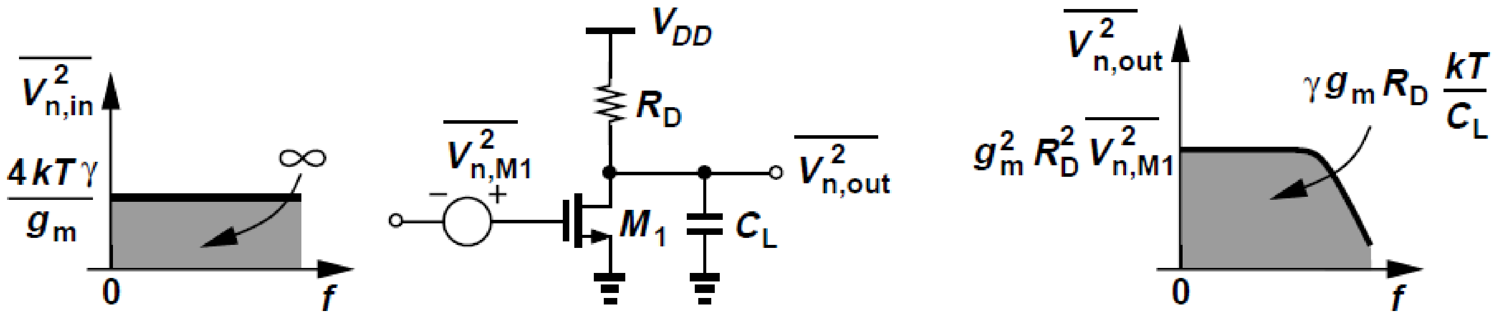
- **Total noise corrupting a signal in a circuit results from all frequency components in the bandwidth of the circuit**
- **For a multipole system with noise spectrum as in Fig.(a), total output noise is**

$$\overline{V^2_{n,out,tot}} = \int_0^\infty \overline{V^2_{n,out}} df.$$

- **As shown in Fig.(b), the total noise can also be expressed as** $V_0^2 \cdot B_n$ **, where the bandwidth** ***B****n***, called the "noise bandwidth" chosen is such that**

$$V_0^2 \cdot B_n = \int_0^\infty \overline{V^2_{n,out}} df$$

Problem of Input Noise Integration



- In the CS stage above, where it is assumed $\lambda = 0$ and noise of R_D is neglected with only thermal noise of M_1 considered
- Output noise spectrum is the amplified and low-pass filtered noise of M_1, easily lending to integration
- Input-referred noise voltage, however, is simply $\overline{V^2_{n,M1}}$, carrying an *infinite* power and prohibiting integration
- For fair comparison of different designs, we can divide the integrated output noise by the low-frequency gain, for example

$$\overline{V^2_{n,in,tot}} = \gamma g_m R_D \frac{kT}{C_L} \cdot \frac{1}{g_m^2 R_D^2} = \frac{\gamma}{g_m R_D} \frac{kT}{C_L}$$